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tary flattening, though its elasticity has
shape.

is imperfect, however; and the results
fore not exactly what they have been
Thus we have seen that, when an elas-
es another at rest, the latter is set in
ction, whilst the former remains station-
lly the case with ivory balls, however;
ce that both balls move after the impact;
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ction of the balls on the cloth-covered
of the d other causes of dis-
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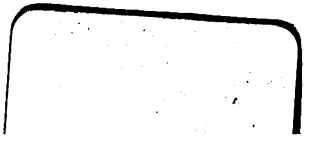

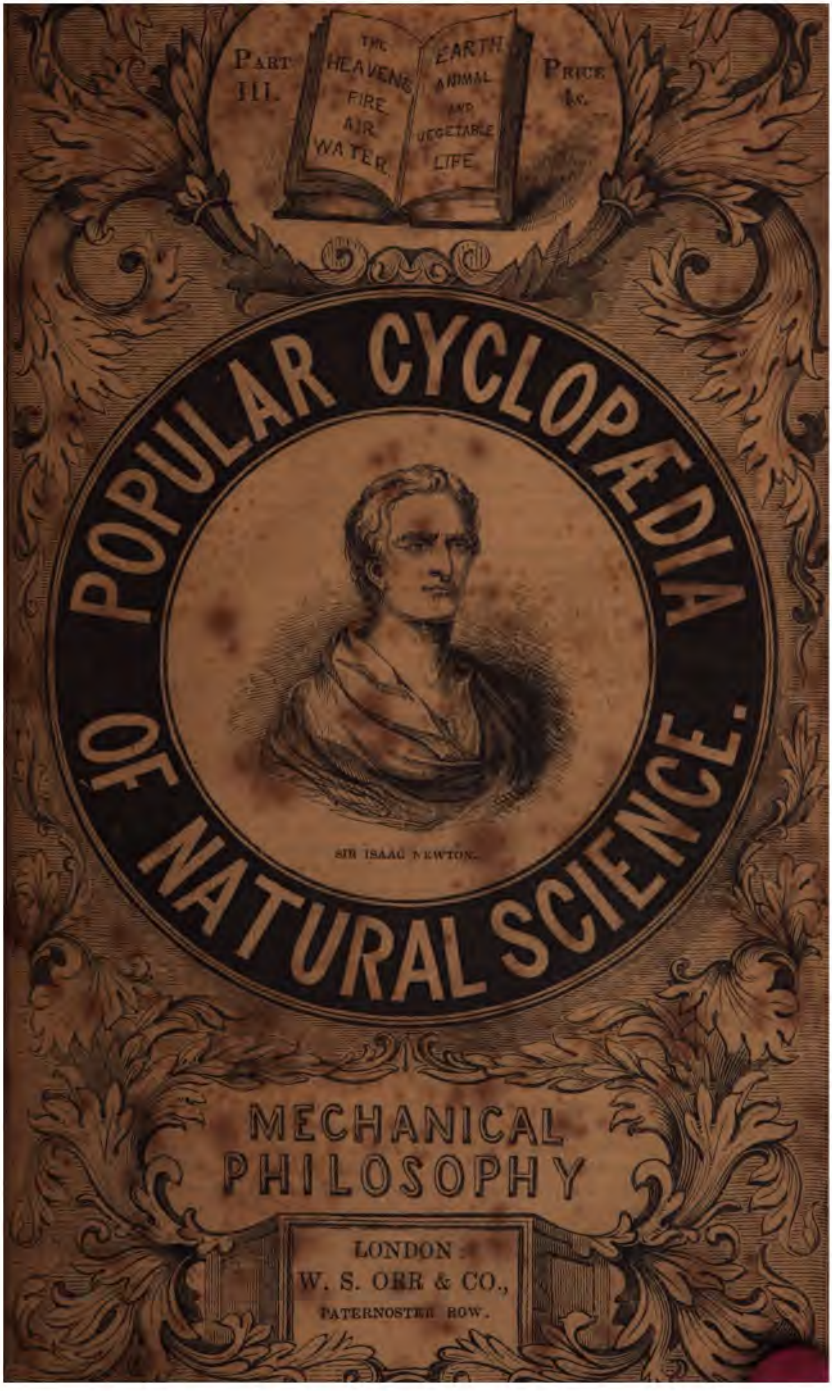
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POPULAR CYCLOPÆDIA OF NATURAL SCIENCE.

By W. B. CARPENTER, M.D.,

AUTHOR OF "PRINCIPLES OF GENERAL AND COMPARATIVE PHYSIOLOGY," AND "PRINCIPLES OF HUMAN PHYSIOLOGY."

THE omission of the study of Natural Science from the course of instruction generally followed in this country, appears a neglect of one of the most important means of intellectual discipline and moral improvement which the Almighty has placed within our reach. This study, rightly pursued, has for its object to enable us to read with understanding in that bright volume of Creation, the pages of which are daily and hourly unrolled before us. To every one do "the Heavens declare the glory of God," whilst "the Earth" shows itself "full of His goodness;" but most of all to him who has learned to interpret the wonders they display; and to trace, in the glorious works by which he is surrounded, His power, wisdom, and love.

As a means of intellectual discipline, the study of Natural Science is perhaps second to none. Habits of accurate, discriminating, unprejudiced observation; of cautious reasoning, and of sound judgment; together with a fearless love of Truth, are cultivated by it. Moreover, it has the advantage of interesting the pupil much more than the greater part of the ordinary routine of instruction; and will tend to increase his desire for the attainment of valuable knowledge of any description. Further, at the present day, when Science is constantly furnishing some new and important improvement in those Arts which minister so much to our comfort, and in their turn contribute in various modes to the expansion of the intellect, the practical benefit of a general acquaintance with its principles is sufficiently evident. And whatever portion may be destitute of such direct application, will be found to have uses of its own, in furnishing subjects for the healthy occupation of the thoughts, and objects of pursuit as rational and interesting relaxation.

No works at present before the public appear to be altogether suitable to this purpose; the greater number of *strictly Elementary* Treatises on Natural Science being little else than abridgements of larger works; so that they are much behind the present state of science, and are for the most part but inaccurate copies of one another, executed in a mechanical spirit, and destitute of the striking novelties which scientific research is constantly bringing into view. The Publishers have therefore determined upon bringing out a connected series of Treatises upon the principal departments of Natural Science, designed and executed in accordance with the views above expressed. It is intended that these should all be *founded* upon such knowledge as every person of ordinary capacity possesses, and that the attention of the reader should be directed in the first instance to phenomena of constant occurrence around him; and it is hoped that by a judicious mode of treating the subject, principles may be gradually developed in such a manner as to render them fully comprehensible, and their more extended application thus marked out. "The neglecting to form a proper connection with previously-acquired knowledge; the undervaluing the results of

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MECHANICAL PHILOSOPHY

AND

ITS APPLICATION TO THE ARTS.

BY

WILLIAM B. CARPENTER, M.D.

AUTHOR OF "PRINCIPLES OF GENERAL AND COMPARATIVE PHYSIOLOGY," AND "PRINCIPLES OF HUMAN PHYSIOLOGY."

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PREFATORY NOTICE.

THE following little work is intended to occupy a place somewhat intermediate between the simpler elementary treatises on mechanical science, and the more elaborate works of a professedly philosophical character. With the former, it has this in common—that the Author has aimed to render it adapted to those who have no preliminary knowledge of the subject; and to carry on his readers, step by step, from the known to the unknown, without requiring from them more than an accurate acquaintance with the ground over which they have already passed. To the latter, the Author ventures to hope it may be compared in this—that the highest principles of the science are introduced, and exhibited in their connexion with each other, and with the phenomena they govern. He has thus endeavoured to give to his little work the character of a philosophical treatise; whilst, by the number of examples and illustrations which he has introduced, he has aimed at preventing what might have otherwise been its dry and abstruse appearance. In this attempt, he has merely followed out the plan which has proved successful in the former volume of the present Series.

The only preliminary knowledge required for the comprehension of the whole of this treatise, is an acquaintance with ordinary Arithmetic, and with the rudiments of Geometry. It is intended

to give, however, as an Appendix to the second part, explanations of such geometrical terms as may be perplexing to those who are entirely ignorant of Mathematics.

The mere beginner is recommended not to attempt to read the work straight through; but to omit Chapters V., VII., IX., and XII., until an accurate acquaintance has been formed with the rest.

The Author is well aware, that some parts of this treatise may be deemed verbose and diffuse; and that the numerical illustrations may be thought to be too frequently introduced. To this he would simply reply, that some experience as a teacher has convinced him, of the necessity of explanations much more ample, and of illustrations much more complete, than those ordinarily given in elementary works; and that he has endeavoured to commit to paper the instructions which, when delivered verbally, were successful in impressing the minds of his pupils.

In conclusion he would state, that, although a large part of the contents of this treatise is the common property of all writers on the subject, many statements and illustrations may claim some degree of novelty. He thinks it right to add, that he is under particular obligation to the works of Professors Moseley and Daniell, of King's College, London.

CHAPTER I.

OF MATTER AND ITS PROPERTIES.

1. The universe in which we are placed, and the changes which are continually taking place in it, offer to the inquiring mind of Man unlimited scope for the exercise of the faculties with which he has been endowed. And, by the wise ordination of the Creator, the minds of different men are so constituted, that they have a natural taste for different objects of pursuit; so that there is scarcely a subject of inquiry, in the eager pursuit of which there are not many engaged. Thus one devotes himself to observing the appearances and the motions of those heavenly bodies, whose constant but regular changes have, from the remotest ages, been sources of wondering interest; and the observations thus collected serve as the basis, on which the philosopher is enabled to found a beautiful system of reasoning that explains all their complex movements (although these seem more irregular, the more closely they are observed,) upon a few simple principles. Another considers the changes which have taken place in the mass of our own globe: he seeks for observations respecting these changes,—whether they result from the action of heat, as in the case of volcanoes and earthquakes,——or from that of water, as when new islands are formed at the mouths of rivers by the sediment brought down in their streams,—or from the agency of living beings, as when islands of large extent are raised above the surface of the ocean by the labours of innumerable coral-polypes; and from these he reasons as to the causes of those past changes in the condition of the earth's surface, which, there is abundant reason to know, must have been very numerous and important. Another directs his attention to the living inhabitants of our globe, seeks to become acquainted with

their almost countless forms, and examines their internal structure; or he observes their mode of life, watches their habits, and studies the changes which they are continually undergoing; and from the information thus obtained, he endeavours to ascertain the principles which have governed their construction, or (in other words) to find out the plan upon which the Creator has been pleased to form and arrange their parts, so as to produce the wonderful variety which we behold, whilst each is adapted with equal perfectness to the objects it has to fulfil.

2. In all these studies, and in many more that might be enumerated, we are occupied with *matter* under some of its forms. By *matter* we understand the *material* of which the universe is built up; and with this we become acquainted through our senses, which inform us of its properties; and as we find these to vary in different cases, we say that there are different forms of matter. We know just as much of any form, or state of matter, as our senses can communicate to us; and no more. A person deficient in one or more of the senses, is shut out from any exact knowledge of the properties, of which those senses inform us. Thus a blind man, who has been blind from birth, can form no proper idea of colour, nor a deaf man of sound. There are some bodies, of whose existence we are informed by numerous senses; whilst others are known to us through one only. Thus we receive information respecting the properties of a piece of cinnamon by the sight, the touch, the taste, and the smell; whilst those of a piece of flint are known to us only through the sight and the touch: we are aware of the existence of the planets and stars (which do not impress us with any perceptible warmth) only by the sense of sight; and there are many gases which are perfectly transparent, and do not possess colour, so that they cannot be distinguished by the eye, yet are at once recognized by the smell. Hence our knowledge of the peculiarities of different kinds of matter is limited by the nature of our senses; but the information which these communicate to us respecting it is so abundant, as to serve all the purposes for which we can require such knowledge. It is wonderful how many ideas may be obtained respecting the world around, even by the sense of touch alone. Instances have

occurred of children who were at the same time blind and deaf, and who possessed but little smell or taste; who yet, under the instruction of intelligent persons, have gained a large amount of information, and have been able even to comprehend the meaning of language.

3. This is the less surprising, however, when it is remembered that our first knowledge of *matter* is derived from the *touch*. It is through this sense that we gain the idea of that property which is common to every form of matter, and which gives us, in fact, our notion of it,—namely, its *resistance*. The child strikes its head against the table; it feels a hard blow, because the table is a solid body, and the resistance it makes is complete. We press backwards our hands against the water in swimming; and feel an amount of resistance which is sufficient to urge forwards our body. We run quickly in a dress which presents a large surface to the air; and we feel that its resistance makes our movement less rapid. This resistance, then, is common to all bodies,—whether solid, liquid, or aeriform;* but it is greater in the solid than in the liquid, and greater in the liquid than in air or gas. The cause is this. It is the fundamental property of matter, that no two particles, whatever may be their nature, can occupy the same space at the same time; we cannot, therefore, cause one particle or one body to occupy the place of another, without moving or displacing it. Now in solid bodies, the particles are so firmly held together, that we cannot move one alone without separating it from the rest; and an impulse given to one, if it be sufficiently strong, will move the whole mass. In liquids and gases, on the contrary, the particles have a free movement upon one another; so that we can displace any part, leaving the rest as it was; and we cannot move the whole by a force applied to a part only.

4. These truths are made evident by simple illustrations. If we desire to make a hole in a piece of hard iron, we cannot do it by driving an awl into it; for the awl merely pushes the particles to one side and the other, to make its own way; and the particles of iron hold together so firmly that they cannot be

* Having the condition of air or gas.

thus displaced. In order to perforate the iron, we must employ an instrument,—the drill,—which shall cut or separate from the rest the particles that occupy the situation in which we desire our hole to be. Thus we see that the iron is a very hard or solid body. On the other hand, when we dip the finger into a basin of water, we have no difficulty in displacing its particles; for they roll over one another so readily, that those previously filling the space which the hand is made to occupy, are pushed to one side and the other to make room for it. This is still more readily accomplished in the case of aeriform bodies, whose particles hold to each other so very loosely, that their displacement is accomplished without any difficulty. Thus we can move a stick rapidly backwards and forwards in air, without being much restrained by its resistance; although, through the whole movement, the particles of air are being continually pushed aside, as the stick advances to take their place. But in water this movement is much more difficult, since the particles are not so readily made to move upon one another. And between liquids and the hardest bodies, there are many intermediate states. Thus mud or dough may not be liquid enough to flow, or to spread itself out from a mass along a flat surface; and yet it shall not be solid enough to hold together when one portion is pulled from the rest, or to offer firm resistance to a blunt stick, or to the hand pushed into it. It may be still more solid, so that its parts shall adhere enough for the mass to hold together pretty firmly, when an attempt is made to separate them by pulling them asunder; yet it shall still be easily penetrated by a sharp stick, or by any instrument that has not a large surface for it to resist. Or it may be more fluid, so that a mass may spread itself out into a flat surface, or *run*, though slowly; yet its resistance to any substance moving through it, shall be much greater than that offered by water or any other ordinary liquid.

5. Though the term *fluid* is commonly regarded as having the same meaning with *liquid*, the scientific use of the two is different; and it is important to regard the distinction. The term *fluid* includes *all* those bodies of which the particles move readily upon one another,—in other words, all those which are

not solid. But there is this important distinction among fluid bodies; that in some, the particles have still a certain tendency to hold together, so that they always occupy the same space, however much the form of that space may vary; whilst in others, the particles have so little tendency to remain attached, that they always have a disposition to separate as far as they are permitted to do, and thus occupy the whole of any space within which they are confined. A given weight of water, for instance, will occupy exactly the same quantity of space, whatever may be the form or size of the vessel which holds it; and if we take away a portion, the remainder has no tendency to increase in bulk, to occupy the room of that which has been removed. But a given weight of air may be made to spread itself equally through the whole of a vessel of any size; and if a portion be taken away, the remainder will spread itself still more, so that still every part of the space shall contain the same amount of it. Again, if we fill a bladder or bag *quite* full of water, and strike it, we can make no more impression upon it by a blow (that is, supposing the walls of the bag not to be capable of extending themselves) than we could upon a solid body; but if filled with air of any kind, the bladder or bag will yield to the blow, returning again, however, to its original form and size as soon as the pressure is removed.* Hence we see that there is in reality as great a distinction between these two classes of bodies, as there is between fluids and solids: and they are scientifically divided into *elastic* fluids, namely, aeriform bodies, gases, and vapours, and *non-elastic* fluids or liquids.

6. Now with regard to *solid* bodies, we may first notice a circumstance familiar to all—that they possess different degrees of hardness. Thus, as "hard as flint," is a common proverb. The diamond is probably the hardest of all substances, for it cannot be cut or scratched by any other; so that the lapidary is obliged to use diamond powder, in the same manner as he uses emery in other cases (§. 179.), for cutting and polishing this gem. Glass, agate, and various precious stones, are other instances of very hard substances, of which the particles are so firmly united

* This property is known as *elasticity*.—See Chap. II.

to each other, that we cannot displace them by any mechanical force, without altogether detaching them. But lead and wood are also solid substances; and yet they may both be pierced with sharp-pointed instruments, which shall push away their particles to one side and the other, without separating them from the mass. There are different degrees of solidity, therefore, as there are different degrees of fluidity. This is seen in another way. The hardest substances cannot be diminished in bulk by any amount of pressure that can be applied to them; but many of the softer metals can be made much harder by hammering; so that the same weight shall be contained in a less space. Wood is far more compressible than any other solid substance; for, by a sufficient amount of force, a piece of light wood may be made to occupy less than half its ordinary bulk, so that it shall sink in water. This is due, however, to the fact, that wood has a *texture* very different from that of solids in general, being composed of a number of tubes of different sizes,* which in green wood are filled with fluid, but in dry wood contain only air: so that there is nothing to prevent the sides of these from being forced together by pressure.

7. Liquids are but very little compressible, although their particles seem to be at a greater distance from each other than those of solids. Indeed it was for a long time uncertain whether they could be compressed at all. The question has been decided, however, by the very accurate experiments of the Danish philosopher, Œrsted. His apparatus was so constructed as to measure accurately a change in the bulk of the fluid experimented on, that should be no more than one-millionth of the whole. With this he ascertained that, by every addition of pressure, the bulk of the water is diminished in a very minute degree; the proportion being such, that an additional weight of 15 lbs. upon every square inch of the surface pressed on, will diminish the bulk of the water by about 46 millionths of the whole. Other liquids are also found to be compressible in various proportions, some more and some less than this; mercury (quicksilver), which is the heaviest fluid known, being the least

* See Treatise on Vegetable Physiology, Chap. V.

compressible; and ether, which is the lightest, being the most so.

8. Elastic or aeriform fluids, are extremely compressible; and the more they have been compressed, the greater is the additional pressure necessary to diminish their bulk. Thus, a certain weight of air has a very different bulk, according as it is near the earth's surface, or a mile or two above it. This is easily understood, by considering that, if a high pile of horse-hair mattresses were made by laying one upon another, those at the bottom of the pile would be reduced in thickness by the weight of those above. Now the weight of the column of air that presses upon each square inch of the surface of the earth, or of any body upon it, is about 15 lbs.; and the particles of air at the bottom of the atmosphere, therefore, are pressed together by a force of that amount. But on the top of a high mountain, a large part of the atmosphere is below, and a much smaller portion above; so that the pressure from above is not nearly so great, and the same weight of air will here occupy a much larger space. Now it has been ascertained by a very simple experiment, that common air (as well as all other gases) is compressed into half its bulk, whenever the pressure is doubled; so that if a quantity of air, which is compressed on the surface of the globe by a weight above it of 15 lbs. on every square inch, be pressed with just as much more, it will be reduced to half its dimensions. If it be compressed by three times its ordinary weight, it is reduced to one-third of its usual bulk. The elasticity of air, or any other gas, is such that, when it has been compressed, it seeks (as it were) to get free with a force equal to that which has been applied to it; so that, if we were to endeavour to condense a large bulk of air into a small vessel whose walls were not sufficiently strong, it would be burst open; and in order to reduce any quantity of air to a smaller bulk, we have first to overcome the outward pressure which it is exerting. By means of a very strong apparatus, Œrsted has succeeded in compressing air to 1-65th of its usual bulk; and it must then have exercised 65 times its usual pressure, so that the vessel to hold air thus condensed, must have been capable of resisting a force of (15×65)

975 lbs. on each square inch of its surface. All other gases can be compressed in the same proportion. The gas, for instance, which serves to light our streets and houses may be so compressed, that a quantity sufficient to burn for a whole night may be contained within a vessel of the size of a large ordinary jar. In the making of soda-water, again, a gas of a different kind is very much compressed; and when in this state, water may be made to take in a large quantity of it; but when the pressure is removed, by the loosening of the cork, the greater part of the gas escapes rapidly from the water, causing the effervescence which renders this beverage so refreshing.

9. Although the bulk of solids and liquids can be but little changed by pressure, yet even the most incompressible are made to contract in size by cold, and to expand by heat. This change, however, is much greater in aeriform fluids than in solids and liquids; and whilst the degree of change in the latter is different for almost every substance, it is the same for all gases. These expand or increase in bulk, to the amount of 1-480th of the whole, for every degree of Fahrenheit's thermometer (the one in ordinary use in this country). The most remarkable effect of heat upon bodies, however, is its tendency to make them change their form, from the solid to the liquid, and from the liquid to the gaseous. The change of a solid into a liquid is usually termed its *melting*, *fusion*, or *liquefaction*; whilst the change from the liquid to the gaseous state is called *vaporization*,—when made rapidly, *boiling*,—or if made slowly, *evaporation*. Each substance requires a peculiar amount of heat, for such change of form; and although this is sometimes the greatest that we can employ, yet in other instances it is so little as to be below the ordinary temperature of the air, or of our bodies, and is therefore commonly regarded by us as cold. Thus we are accustomed to regard the liquid state as the natural condition of *water*; because it is only when there is a much less degree of heat in the air than usual, that water passes into the solid form by freezing. But in the Arctic regions, and on the tops of high mountains, even in the torrid zone, the frost is perpetual, and water is never found but in the state of ice, except when the summer's sun

slightly thaws the surface. Hence water cannot be said to differ in this respect from bodies which are solid at *our* common temperature, except that its melting point is lower than theirs.

10. There are very few liquids which may not become solid by a sufficient degree of cold; thus mercury, which retains its liquid state under a degree of cold far more intense than that which freezes water, is congealed at last into a solid mass, the characters of which are in all respects analogous to those of the metals in general. Hence mercury, though fluid at the ordinary temperatures at which the other metals are solid, has, like them, a true melting or fusing point; and there are parts of the globe, where the cold of the ordinary winter is so extreme, that mercury is kept by it in the solid form. We might imagine that a person living all the year round in a temperature low enough to keep mercury frozen (if such a case were possible), would think the liquefaction or fusing of mercury as great a curiosity as *we* are accustomed to regard its freezing. There are some liquids which have hitherto resisted all attempts to change them to the solid form; but there is much argument from analogy for the belief, that they all might be so changed, if we could produce a sufficient degree of cold, or rather withdraw a sufficient amount of heat.

11. There is not usually much change of bulk in the liquefaction of bodies; nor in their freezing or congelation. Generally speaking, the bulk of a body in the liquid form is greater than that of the same weight in the solid; for the solid body goes on expanding (or increasing in bulk), under the influence of heat, until it is just about to become fluid, and then a further expansion takes place. But there is a remarkable exception in the case of water, which takes up more room in the solid state than in the liquid; so that an equal bulk of it is lighter. It is a very familiar fact, that water in freezing expands, so as to burst any pipe or vessel within which it may be at all closely confined; and it is to the same cause, that we owe the floating of ice upon water, since its formation would otherwise take place at the bottom.

12. The expansion of liquids under the influence of heat increases very rapidly as the temperature is raised; and is

particularly great when the liquid is heated nearly to its boiling point. The change of bulk is then very great and sudden; for all vapours have many times the bulk of the liquids from which they rose. Thus a pint of water would produce 1694 pints of steam at the ordinary pressure. Though the vaporization of fluids takes place chiefly under the influence of heat, yet the quantity of heat required to produce it is very different under different degrees of pressure. Thus, if we take water at the ordinary pressure as the standard, we should find that any additional pressure (such as would be produced if the vessel were kept tightly closed) would render an additional quantity of heat necessary to convert it into steam; whilst, on the other hand, the removal of the ordinary pressure of the air, will cause water to boil at a much lower temperature, as happens on the top of high mountains, or as may be easily shown by the air-pump. Under pressure of the most powerful kind, water has been heated to such a degree, that the iron vessel which contained it was red-hot throughout; and if the pressure had been withdrawn in a very slight degree, the water would have immediately passed into the condition of steam, which, by its very great elasticity at high temperatures, would have blown the vessel to pieces. On the other hand, when the pressure of the air has been entirely removed, water boils at a temperature not above that which it naturally has on a hot day. This fact is one of great practical importance; for the boiling of many fluids which are injured by too high a temperature, is now successfully conducted in vessels from which the air has been partly exhausted*.

13. Our knowledge of the combined influence of temperature and pressure on the condition of certain bodies, has received many important additions within the last few years. It was formerly the custom to divide aeriform bodies into two classes,—vapours, and gases: the former were regarded as liquids in an aeriform state; whilst to the latter this state was supposed to be natural, it not being imagined that *they* could ever exist in the liquid form. Many kinds of gas, however, are capable of being reduced to the fluid form by pressure only; and others by cold

* The particulars will be given when HEAT comes to be treated of in detail.

and pressure combined; and there is one,—carbonic acid gas, (the fixed air of soda-water, champagne, &c.)—which has even been converted into the solid form. It is probable, then, that all gases might be made to assume the liquid or even the solid condition, if they could be subjected to the united influence of cold and pressure to a sufficient degree; and that they differ in nothing from those aeriform fluids which are ordinarily termed vapours, except in the very low temperature at which the liquids are vaporized.

14. Thus we have strong reason to believe, that the solid, liquid, and gaseous states may be common to all bodies; and that the reason why there is so great a variety in the conditions in which we usually find them, is simply,—that every kind of substance changes its state at certain fixed temperatures. So that whilst some (as metals in general) are solid at ordinary temperatures, they become liquid at higher, and gaseous at higher still; others, which are liquid at ordinary temperatures (as mercury and water), may become solid at lower temperatures, and pass into vapour at higher; and lastly, others (as carbonic acid) which have a gaseous condition at ordinary temperatures, may be turned into liquids or even solids, by an increase of pressure, or by a diminution of heat.

15. We see, in these different conditions, the influence of two different forces acting on the particles of matter;—a force which draws or attracts them together, and a force which tends to separate them. The former, which is termed the *attraction of cohesion*, operates only at very small distances; so that, if the particles be very widely separated, they have no cohesion whatever. On the other hand, the force which tends to separate them is one by which the particles *repel* each other; and this repulsive force seems to be *heat*, or at any rate to be produced by the action of heat. For, as we have just seen, heat causes the particles of bodies to undergo a gradual separation from one another, before their form is entirely changed. This is peculiarly shown in the case of glass, and many of the metals, which become very soft and easily moulded into any shape, before they actually melt. The action of the cohesive force is best seen in solid

bodies, of which the particles are so close together, that this force can have its full influence. It is weakened in the liquid, by the increase of the repulsive force, which has in some degree separated the particles; but still it is strong enough to prevent the repulsive force from carrying these particles wide apart from each other. In aeriform bodies, the repulsive force has been so much increased, as to carry the particles beyond the influence of each other's attraction, and, consequently, they have no cohesion whatever, but tend to separate as widely as they are allowed to do.

16. The knowledge of these principles, then, enables us to understand the influence of heat and cold, and of diminished or increased pressure, in changing the form of bodies. When a solid body is heated, the repulsion of its particles is increased,—they are separated from one another to such a degree, that their cohesive attraction is greatly weakened, so that they can move freely upon one another; but this is not destroyed, for it keeps them from flying asunder. But when the temperature is raised still further, the repulsive force is increased by the added heat, and the particles become separated so widely, that their cohesion is altogether destroyed; and the same effect may be produced by the simple removal of the external pressure, which has an obvious influence in keeping together the particles of the liquid. On the other hand, an increase of pressure applied to a gas, may, without any withdrawal of heat, bring the particles so closely together, that they come within the influence of each other's cohesive attraction, and thus they may be made to assume the fluid form; but the same effect is produced with a much less amount of pressure, when the repulsion of the particles has been diminished by lowering the temperature.

17. When the change from the liquid to the solid form occurs slowly, and other circumstances are favourable, the particles have a tendency to arrange themselves in certain very regular modes; producing those forms, bounded by straight lines and angles, which are termed crystals. These are most familiar to us in the spangles of hoar-frost or the flakes of snow; but water is not the only substance that produces them in the act of congealing. Thus, if we melt a mass of sulphur (brimstone), or of

the metal bismuth, in an earthen pot or crucible, carefully watching it whilst it *slowly* cools, until a crust has formed on its surface, and then pierce this crust, and allow the liquid portion remaining within to flow out,—we shall find the cavity studded with beautiful crystals, those of the bismuth being exact cubes, and those of the sulphur being needle-shaped, with six sides.

18. But crystallization may be made to take place in a very beautiful and regular mode, in many substances which could not be thus made to exhibit it; still the same principle governs it,—the *gradual* change from the liquid to the solid state, which gives the particles time to arrange themselves with regularity. This happens when substances that have been dissolved in water or other liquids are set at liberty by the cooling or the evaporation of the liquid. Thus, if we put a quantity of common salt into boiling water, continually adding until it will take up no more, (this is called a *saturated solution*,) and then allow it to cool, we shall find a large part of the salt deposited in small crystals at the bottom; for water when cold cannot dissolve so much of the salt as when warm, and cannot therefore retain what *has been* dissolved. The same effect is produced by dissolving in cold water as much salt as it can take up, and then letting the water gradually evaporate or dry away; as the salt does not go off in vapour, it must be left behind in the solid form, and this form is crystalline. A very simple experiment enables any one to see crystallization in the very act of going on. Let a Florence flask (such as olive oil is usually sold in) be nearly filled with water, and let as much Glauber-salt be then added as the water will dissolve when heated to boiling. If the flask be corked whilst the water is hot, and be not disturbed in cooling, the salt will remain dissolved; but as soon as the cork is taken out, crystallization will begin, and will rapidly spread through the whole mass. If (as sometimes occurs) it should not at once take place, it may be produced by shaking the flask, or by dropping into it any small body that is capable of being wetted by the fluid; the best for the purpose seems to be a crystal of the salt itself.

19. It generally happens, however, that crystallization which

takes place very rapidly has a confused character; whilst the same substance, when more slowly consolidated, may present large and very regular crystals. This difference is well seen in the case of loaf-sugar and sugar-candy. The process of crystallization is also favoured by the presence of any centre, or *nucleus*, round which the particles may arrange themselves; thus, in the manufacture of sugar-candy, verdigrise, blue vitriol, and other crystalline substances, strings, wires, or strips of wood are placed across the vessels holding the fluid, and may very commonly be found in the centre of large groups of the crystals. The best kind of nucleus consists of a crystal of the body itself, which seems to attract other particles of the same kind; and it is remarkable that it will choose (as it were) out of two substances dissolved in the fluid, the particles of its own kind. Thus, if we dissolve in the same water Glauber-salt and nitre, and then divide the fluid into two portions, suspending a crystal of nitre in the one and a crystal of Glauber-salt in the other, the nitre only will crystallize in the former part, and the Glauber-salt in the latter. This tendency enables us to obtain very large, regular crystals from smaller ones, by proper management. If we take the largest and most regular that can be obtained by the ordinary process, and place them in a saturated solution of the same salt, a gradual deposite of new particles will take place on their surfaces; but as this addition does not take place to the surface that is in contact with the bottom of the vessel, the crystal must be turned every day to preserve its regularity of form. It has been remarked that if a solution, containing crystals of different sizes, be exposed to changes of temperature, the large crystals will grow at the expense of the small ones. This takes place in the following manner: when the temperature of the liquid rises, it dissolves a portion of the salt, and diminishes the size of all the crystals in a like proportion; but when the temperature falls, it must part with this addition, and the newly deposited matter settles upon the large crystals in much greater proportion than upon the small; so that the matter of which these last are composed is gradually taken up and deposited upon the larger.

20. It has long been known that particular forms of crystallization are peculiar to certain kinds of substances; and these forms often enable the Mineralogist to distinguish them by the eye alone.* Some minerals occur in several different forms, but these may be shown to have a certain connexion with each other. As to the cause which governs this regular arrangement of particles, philosophers are still in doubt; and it is undesirable to discuss, in this place, the speculations which they have put forth. There can be no doubt, however, that it results from a certain regular operation of the cohesive attraction; since we find that all bodies which slowly pass from the liquid to the solid form, have a tendency to assume it. Many substances are found in beautiful crystalline forms, which the chemist has not until recently been able to imitate; since he cannot either melt these substances and allow them to crystallize in cooling, or dissolve them in water and cause them to crystallize from the solution. Of this kind is silex, the substance of which flint is composed; the large and beautiful crystals of silex, which are known under the name of *quartz*, or rock-crystal, are inferior in hardness only to diamond. Now silex cannot be crystallized in the ordinary way; for it obstinately resists being melted by itself, and it cannot be dissolved in water in any but the minutest proportion. But it may be dissolved in water containing a strong alkali, with which it enters into combination; and from this solution it may be separated in a crystalline form, by the prolonged action of a very feeble current of electricity. There are still many crystals, however, which cannot yet be imitated by art; amongst these is the diamond, which is nothing else than crystallized carbon,—the substance of which charcoal and coke are almost entirely composed.

21. The slow and undisturbed action of the cohesive force, not only produces the regularity of the external forms of crystals, but also an internal structure which is equally regular. This is evident from the fact, that they may be split in certain directions, whilst they will resist almost any force applied to them in others; this has been long known to diamond-cutters,

* See Treatise on MINERALOGY.

but the fact has been more recently applied in the science of Mineralogy, and has given to it an entirely new character. The directions in which crystals are readily split, are termed the planes of its *cleavage*. This tendency is very important in a practical view; for it is owing to it that crystalline substances, however dense they may be, are so brittle; and that substances which are more or less perfectly crystalline are usually brittle in proportion as they possess this structure. Thus, cast-iron is very hard, presenting a strong resistance to any forces that would separate its particles; but it is at the same time very brittle, on account of its partly crystalline character. On the other hand, wrought or malleable iron, which has had its character completely changed by exposure to the air when melted, and by being hammered when softened by heat, has a fibrous texture; and whilst it is more easily cut, or bored, or worn down by continual rubbing, it is much less brittle. This difference in the characters of these two well-known forms of iron has given rise to much discussion, as to which kind was preferable for the *rails* of rail-roads. Cast-iron, from its superior hardness, is less rapidly worn down, but it is liable to be broken by any sudden jar; whilst wrought-iron, being much more tough, can withstand such action better, but is more rapidly worn away. The injury that would result, however, from the breaking of a rail under a train in rapid motion, is so great, that wrought-iron rails are now in almost universal use, in spite of their additional costliness and more rapid wear.

22. The fibrous structure on which depends the toughness of malleable or wrought iron, is liable to disappear under peculiar circumstances; and to give place to a crystalline structure, which will (like that of cast-iron) be accompanied with great brittleness.* This change depends upon a new internal arrangement of the particles; and may take place without any alteration in the external form of the substance. Thus, a wrought-iron furnace-bar, of whatever quality it may have originally been, is invariably converted, within a short time, into crystallized iron, by the alternate heating and cooling to which it is exposed; and

* See a Paper on this subject by Mr. C. Hood, in the *Philosophical Magazine* for August 1842.

the effect may be still more speedily produced, by heating and *rapidly* cooling (as by quenching a few times in water) any piece of wrought-iron. The same brittleness is produced by continually hammering a bar of iron at a low temperature. If it be hammered at welding heat, the very contrary result is obtained; but it is often found, in the manufacture of wrought-iron bars, that one portion has became quite brittle from being hammered too long after it has partly cooled, whilst the rest possesses the highest degree of toughness. The effect appears to depend upon a peculiar state of vibration into which the particles are thrown by the blows: and this vibration does not take place when the iron is softened by heat. If a small bar of good tough iron be suspended, and struck continually with small hand-hammers, so that a constant vibration is kept up, it becomes so extremely brittle, after the experiment has been continued for some considerable time, as to fall to pieces under the light blows of the hand-hammer, presenting throughout its structure a highly crystalline appearance. Any continual *jarring* will produce the same effect. A piston-rod has been known to undergo this change, in consequenee of a ceaseless jarring to which it was subject, from not being fixed tightly into the piston; it broke short off, close to the piston, and presented at its fracture a highly crystalline appearance, whilst at a short distance it possessed the tough fibrous character, which (there was good reason to believe) originally belonged to the whole rod. It is, probably, to this cause that we are not unfrequently to attribute the breaking of the iron axles of carriages, carts, railroad carriages, &c. That some such change must have taken place in their interior structure seems evident from the fact, that, in many instances, they have been used for years with much heavier loads; and that they have at last broken without any apparent cause, under lighter burdens and less strain than they have formerly borne. In these cases, the crystalline structure does not prevail equally through the whole axle, but is found in the highest degree in the part where the jar is most felt by it. The causes of this change are not yet properly understood. It takes place much more rapidly in the axles of railway-carriages, than in those of common road-vehicles; and there

is reason to believe that the electricity and magnetism which are produced in the working of the former have a share in the effect. However this may be, the knowledge of the possibility of this important change should cause great attention to the strength of the axles, in order to avoid such lamentable accidents as those which recently occurred from this cause on the Versailles and Birmingham Railways.

23. A crystalline arrangement may be shown to exist in many substances, when the external form has no regularity, by some means which shall remove the particles that surround the crystals, without injuring them. The simplest of these means is the action of a fluid in which the substance can be dissolved; for the attraction of the particles of the crystals for each other seems to direct the mode in which the fluid acts upon the substance; so that the irregularities are removed, and their forms become apparent. Thus, if we take a mass of alum, of sufficient size, having no regular external form, but made up within of a confused mass of crystals of various shapes, and place this in water, the fluid will act upon it at first in all directions alike; but as soon as the solution is nearly saturated, its dissolving power is diminished, and it acts only in the directions where the substance most readily yields to it. Under these circumstances, its surface will become embossed with crystals, presenting an immense variety of geometrical figures, stamped or carved, as it were, upon its substance. Many other substances may be acted upon in a similar way by fluids that will dissolve them; thus, beautiful crystalline forms may be produced upon metallic surfaces by the action of weak acids; and these may be applied to ornamental purposes. An aspect of this kind, given to common tin-plate, is known under the name of *moirée métallique*; this at one time excited much attention and admiration; but the manufacture has been brought into disrepute, by the very cheapness arising from the simplicity of the operation.

24. Having now considered the operation of attraction between particles of the same kind, in producing their *cohesion* to each other, we have next to speak of the action of the same force in producing the *adhesion* of particles of different kinds. In

scientific language, the former is said to be *homogeneous* (from two Greek words, signifying *same kind*), whilst the latter is said to be *heterogeneous* (*other* or *different kinds*). This kind of attraction is scarcely less important than the former; since, whilst cohesive attraction gives to masses of particles their form and density, and causes them to remain in the same condition as long as they are undisturbed by any other influence, the attraction which they have for the particles of other substances is the frequent occasion of an entire change in their form and arrangement. Thus the homogeneous attraction of the particles of sugar, when free to act by itself, causes them to arrange themselves in a crystalline form; but when we bring that sugar into contact with water, the heterogeneous attraction of the particles of sugar and water is so strong, as to overcome the former; and the particles of sugar are consequently separated from each other, and diffused through the whole mass of the water. The first stage of the process consists in the *wetting* of the surface by the fluid; this is caused by the feebler action of the same force. Thus we can readily wet with water a surface of glass; and less readily, a surface of polished iron; but neither of these surfaces can be wetted with mercury. No *adhesion* exists between mercury and polished iron or glass; and it has not the least power of dissolving either of these substances. When mercury is spread over the ordinary surface of the metal platinum, it does not wet it; but it may be made to do so completely, by carefully cleaning the surface. If, for a plate of platinum, we substitute tin or lead, either metal will be immediately and completely wetted by the mercury; and if the quantity of the fluid be sufficient, it will dissolve the solid in a short time. It seems, then, that the adhesion of the surfaces is the first operation of heterogeneous attraction; and that, where this occurs, there is generally a power on the part of the fluid to dissolve the solid, though sometimes in very small amount. The case of glass and water is not an exception to this rule; for though we do not usually regard glass as at all soluble in water, yet we may show it to be so, by placing very finely-powdered glass upon turmeric

paper,* and then moistening it; the paper will receive a brown tinge from the alkali which glass contains.

25. The heterogeneous attraction of liquids for solids appears, therefore, to be the cause why the cohesion of the latter may be destroyed by a liquid, and their particles completely separated and diffused through it. The act of solution is usually much facilitated and hastened by additional heat; and this is at once understood, when it is remembered that heat weakens the cohesion of the particles of the solid, so that the resistance to the heterogeneous attraction of the fluid is much diminished. This power of *solution* is obviously one of immense importance, as well in the economy of nature, as in the arts of man. The diffusion of salt through the waters of the ocean, the reduction to the liquid form of the various substances which are required by the chemist and the dyer, and the ordinary sweetening of our tea and coffee, are familiar examples of its operation.

26. But there are other operations, scarcely less important, which result from the simple *adhesion* of solids and fluids, without any change of form. Of these, the most striking is that which is ordinarily known as *capillary attraction*. If we dip a glass tube of very small bore (hence termed a *capillary* tube, from *capilla* a hair) into a liquid that is capable of wetting it, we shall see that the liquid rises in the tube to a certain height above the surrounding level; and that it is drawn up against the surface of the glass, both inside and outside the tube. Here the heterogeneous attraction between the glass and the liquid is sufficient to overcome, to a certain extent, not only the cohesion of the latter, but the force of gravitation, which tends to preserve the ordinary level. The smaller the tube, (provided its diameter be not too small for the liquid to rise into it,) the greater will be the height of the surface

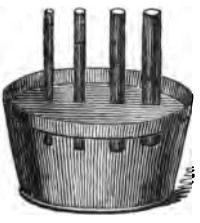


FIG. 1.

* A paper dyed yellow with turmeric; it is much used in Chemistry as a test for the presence of alkalies, which turn it brown.

within it; and the larger the tube, the more nearly will the surface within be upon the level of the surface without. In all instances, the surface will be lower towards the centre of the tube, so as to be concave. This is easily understood, when it is remembered that the attraction of the glass for the liquid will only operate upon the particles that are near the surface where they touch; and, therefore, that the more distant the centre of the tube from the circumference, the less attraction will be exercised upon the central part of the column of liquid. The

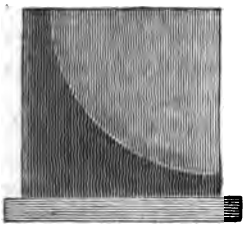


FIG. 2.

operation of this principle is very beautifully shown by a simple experiment. If we take two pieces of flat glass of the same size, and fix them in such a manner that they shall be in contact along one side, but shall be a little separated on the other,—when they are placed with the lower side in coloured water, the liquid rises up between them, and ascends higher in proportion as the glasses are nearer together, forming a regular curve, which is known to mathematicians as the *hyperbola*.

27. This tendency of liquids to ascend through tubes or spaces of narrow dimensions, is extremely important in many of our ordinary processes, as well as in the economy of nature. Thus, it is by means of the capillary action of the soil, that water is imbibed by it, and is carried to the roots of plants. The absorption of water by a piece of sponge, or by the towel with which we dry ourselves, takes place on the same principle. Where the fluid, which is drawn up by capillary tubes, is removed as fast as it is raised, a continual movement will be the result. Thus, a washerwoman who leaves an apron-string hanging down from one side of her tub at night, would probably find in the morning that a great part of the water had been drained by it upon the floor; for as the string becomes charged with liquid, its lower end will let it drop, and a continual withdrawal and fresh supply will take place, until the level of the fluid in the tub is much below that of the end of the string, in which case the fluid will be

drawn, by the force of gravity, rather into the tub again, than down the part of the string that is hanging over. In like manner, when we place a fresh cotton-wick in a lamp, the oil is speedily drawn up to its top, but it does not run over; it then ceases to move, until the lighting of the lamp exhausts the oil at the top of the wick, and this is continually supplied by fresh absorption below. The fibres of cotton may be replaced by a bundle of small wires, which will answer the same purpose, so long as they are kept clean. The natural tubes of plants are well adapted to raise fluid by their capillary action; this may be experimentally shown by dipping one end of a piece of the ordinary cane, into spirits of turpentine, which will rise to the other end, so that the vapour will take fire there.

28. The action of endosmose,* which takes place with such power through animal and vegetable membranes, and even through porous mineral substances, seems capable of being explained on the same principles. It is essential to its performances, that the two fluids should have a strong tendency to mix; and that one of them should rise more readily through capillary tubes than the other. Thus, when syrup is put into a bag of animal membrane, and this is dipped in water, the latter passes more readily than the syrup through the pores of the membrane, and is thus carried to the inner side; when it comes in contact with the syrup, it is drawn off by the attraction which that liquid has for it; and as the portion thus drawn off is being continually supplied from the water on the outer side, by the capillary action of the membrane, a large quantity of water is thus introduced into the bladder and mingled with the syrup. There is a similar current, however, in the opposite direction,—the syrup being conveyed through the pores of the membrane, and being drawn off by admixture with the water on the outer side; but this is weaker than the other, in proportion to the thickness of the syrup, which impedes its passage through the minute capillary tubes of the membrane.

29. Another instance in which capillary action is made serviceable, is in filtration. When we desire to separate fluids

* See VEGETABLE PHYSIOLOGY, § 117.

from impurities which are diffused through them, or to obtain the finely-divided particles of a solid which may be suspended in fluids, we make use of a filter of some porous substance, through which the fluid is caused to pass, leaving the solid particles behind. In ordinary filtration, the particles of the fluid, when transmitted to the under-surface of the filter, cohere together and form drops; over which the force of gravity has then sufficient power to cause them to fall. The action is much assisted, however, by any pressure which shall give the liquid an increased tendency to pass through the pores, which it may be otherwise slow in doing. This may be often conveniently accomplished by the force of gravity, as in the plan represented in the adjoining cut. The tube *a b c* being filled with sand, and receiving water at the higher end *a*, discharges it filtered at *c*; and, as the pressure of the water on the side *a b* would have carried it up on the other side, *b c*, to a height as great as that of *a*, the pressure of that part of the column in *a b* which is above the level of *c*, is over-balanced, and is therefore employed in forcing the liquid downwards through the filter. The filters used by druggists are now frequently constructed upon such a plan, that a high column of the liquid is continually pressing downwards, so as to hasten its passage through the filter; this is frequently of great importance; for the substance from which the liquid has to be cleared has often a tendency to *clog* the pores of the filter, and thus to obstruct its passage; and where the liquid easily evaporates (as

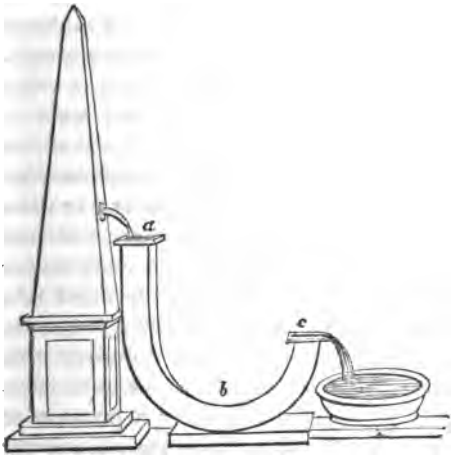


FIG. 3.

is the case with the *spirit* of tinctures), the long exposure of its surface to the air occasions a considerable loss. The filter may not only be used to separate fluids from solids, but to separate fluids that mix imperfectly from one another. To do this, it is only necessary to wet the filter in the first instance with the fluid we wish to draw off; that alone will be transmitted by it, the other remaining behind. In this manner, spirits of turpentine and water may be parted from each other.

30. The *force* which is capable of being produced by capillary attraction is much greater than would be generally supposed. It may be estimated by the opposition which it is capable of overcoming. Thus, a vessel of wire-gauze will hold a certain depth of water, if the meshes are sufficiently small; for each mesh will keep a drop of the fluid adhering to it by capillary attraction, so long as the weight of the water above is not sufficient to overcome this adhesion. For the same reason, a vessel of this kind will float in water, so long as it is not made to descend below a certain depth, depending on the size of the mesh. When endosmose takes place under the most favourable circumstances, it will raise a column of mercury to two or three times the height at which it stands in the barometer, and may thus sustain a weight of from 30 to 45 lbs. upon every square inch of surface. If a dry plug of wood be tightly fitted into one end of a stout tube of glass or porcelain, and a projecting portion be allowed to dip into water, the wood will swell with such force, by the entrance of the fluids into its pores, as to burst the tube, though it may be capable of resisting a pressure of more than 700 lbs. on the square inch. It is the same power which causes the stones of walls to be loosened by the growth of creeping plants, such as ivy; for the delicate fibrils of their roots being inserted into minute chinks between the stones, gradually become distended by capillary action, so as to separate these from their attachments. This power is turned to useful account in some parts of France and Germany, for separating large masses of stone from the solid rock, when this cleaves readily in particular directions. Holes are bored in its substance, into which wedges of dry wood are tightly driven; and when

these are exposed to moisture, they swell by capillary action with such force as to split the solid mass. The same plan is used for dividing a long piece thus obtained into shorter pieces, which are to be used as mill-stones.

31. A different set of phenomena occur when the liquid and solid have little or no attraction for each other, or when the force of *adhesion* is inferior to that of *cohesion*. A glass tube of small bore, when dipped into mercury, will not raise its level in the least degree, but will depress the column within it; and this in proportion to the smallness of the tube. The surface of the mercury, instead of being concave (or hollow), is convex (or bulging); that part nearest the side of the tube being lower than that in the centre. This is well seen in a barometer, especially if its tube be small. In a tube of 1-10th of an inch in diameter, the mercury is kept down as much as 1-7th of an inch. It is consequently necessary, in calculating the height of mountains, from the place at which the mercury stands in the barometer*, to make a correction for this depression, by adding to the observed height a small amount which shall express the influence of the glass in lowering its surface. This want of adhesion between the mercury and the glass produces another result, which may in time greatly injure the action of the barometer. The perfection of this action requires that the space between the mercury and the top of the tube should be completely void of air; but, owing to the want of complete contact between the glass of the tube and the mercury contained in it, and surrounding its lower end, the air gradually steals in, dipping down below the end of the tube, and slowly rising up through the column. This is prevented, in the best barometers, by welding a ring of platinum to the bottom of the tube; for, as mercury *wets* platinum, no air can pass between them. The effect of plunging into water a capillary tube, whose interior surface has been covered with a thin coat of oil, so that the water will not wet it, is exactly the same as that of plunging a glass tube into mercury; the fluid being depressed in the former case, as in the latter.

* See Treatise on PNEUMATICS.

32. Hitherto the effects of heterogeneous attraction have been considered only in reference to its most common form,—the action of solids and liquids on each other. But they are by no means confined to this. Solids may have a considerable attraction for each other; but this is best seen when one of them has been changed from the fluid state, whilst in contact with the other, so that the two surfaces are exactly adapted to each other. Upon this principle depends the action of cements of all kinds, —mortar for stones, glue for wood, resinous and other substances for glass. It is in part through the same adhesion, that solid surfaces cannot be slid over one another without a certain amount of resistance, even when the surfaces are very smooth. Other things being equal, however, it is generally found that the loss of force by this rubbing, or *friction*, is less when the two surfaces are composed of different substances, than when they are of the same; thus the resistance of iron moving upon iron is nearly double that of iron moving upon copper. However injurious this friction may become in the operation of machinery, it is of great importance in our ordinary actions; of this we shall become well aware, if we compare the ease with which we walk on a stone pavement, on which we can plant our feet firmly, with the difficulty we have in walking across ice, on which, from the absence of friction, our feet are continually slipping. If it were not for friction, too, we should slide from our chairs, when our bodies slope in any direction; and the pens with which we write would slip through our fingers. When *both* surfaces are made very even, the friction is very much increased, so that it is difficult to slide one over the other. This is particularly the case with two surfaces of the same kind, such as plate-glass. After the plates have received their final polish, it is usual to place them in an upright position in the warehouse, like books in the shelves of a library. In this position they not unfrequently acquire, in the course of time, an adhesion which renders it very difficult and sometimes impossible to separate them. Instances have been recorded, in which three or four pieces were thus absolutely incorporated with each other, so that they might be treated in all respects as one piece, and even cut

with the diamond as one, adhering almost as perfectly as if they had been melted. When an exceedingly great mechanical force was applied in the proper direction, they were caused to slip one on another; and when at last they came asunder, it was found that they had not separated at their common surfaces, but that the thickness of the glass had yielded, so that the surface of one carried off with it a thin layer of another. In this and similar cases, such as the fixture of a ground-glass stopper in the neck of a bottle, the particles of the two surfaces are brought so close together, as to *cohere* instead of simply *adhering*.

33. Solid bodies not unfrequently exercise the same kind of attraction over gaseous particles, as over those of a solid or liquid substance. This is shown by the floating on water of bodies which are really heavier than itself, but which are buoyed up by a layer of air that adheres to them. Thus with a little care we may lay a fine sewing-needle on the surface of water, in such a manner that it shall not sink; but if the needle be too thick, its weight will bear a larger proportion to the quantity of air that surrounds it, so that it cannot be made to float. It seems that iron surfaces have a peculiar adhesion to air, as the following experiment shows. If we take some iron filings, and sift them upon the surface of water in a tall glass jar, they will float upon the top, until a stratum (or layer) of considerable thickness has thus been laid upon the water. At length this stratum will break up into masses which will sink; and it will then be seen that they had been previously buoyed up by the adhesion of particles of air, which their accumulated weight will even carry to the bottom with them. If we sift powdered magnesia upon water in the same manner, its particles will sink almost immediately; for as they do not possess the same property of causing air to adhere to them, they become wetted by the close contact of the liquid; and being heavier than water, there is nothing whatever to buoy them up. There is a considerable adhesion, also, between glass and air, which is shown in the making of barometers. If we fill a clean glass tube with mercury, we shall find a quantity of small glass bubbles adhering pertinaciously

to its sides; many of these may be swept away, by causing a large bubble to pass several times from top to bottom by inclining the tube towards each end alternately; but it will still remain coated throughout with a film, which can only be removed by boiling the mercury in it. Even this, however, does not produce that close contact between the mercury and the glass, which is necessary for the complete exclusion of the air; since, as already stated (§ 31), the air will creep in, if not prevented by giving to the glass tube a termination of platinum, which is wetted by the mercury.

34. This attraction of solid bodies for gases produces several important results in the economy of Nature. There are many insects, which, although they breathe air, are inhabitants of the water; and they are enabled to surround themselves with a film of air, by its adhesion to their hairy bodies, which they can carry down with them for use at a considerable distance beneath the surface. In the same manner, the diving spider* carries down successive quantities, by which it gradually fills its delicate little bell with a quantity sufficient for its supply during the whole winter; the amount of adherent air is so considerable, that the spider cannot descend by its own weight, but is obliged to creep, with considerable muscular exertion, down any stems or leaves that may conduct it from the surface of the water to its destination. There is another most important practical result, that arises from the attraction exercised over gases by many porous substances, which will absorb and retain quantities of gaseous matter equal to many times their own bulk. Thus newly-burned charcoal will absorb 90 times its bulk of ammonia (the pungent gas contained in spirits of hartshorn), and 35 times its bulk of carbonic acid (the foul air of wells, caverns, &c., also produced by the breathing of animals, the burning of charcoal, &c.). It will also take in watery vapour; the weight of the charcoal being in some cases increased nearly one-fifth by a week's exposure to air. Other porous substances possess the same property, though usually in a less degree; and it is by the exercise of this attraction by our soil, for the ammonia and car-

* See Treatise on Zoology.

bonic acid of the atmosphere, that a large proportion of the nourishment obtained by plants is derived*. So large a quantity of common air is sometimes condensed by powdered charcoal, that a great amount of heat is given out by it, according to principles which will be explained in the treatise on Heat; and in one instance which has come under the author's knowledge, a cask of animal charcoal in powder had actually become red-hot in the interior, from no other cause.

35. Various animal and vegetable substances have a considerable power of attracting watery vapour; and of readily yielding it again. The changes in their size and form which are produced by such alterations, make them present tolerably good indications of the amount of dampness in the atmosphere. Thus every lady knows that it is more difficult to keep her hair in curl in damp weather than in dry. The catgut strings of a harp shorten with damp, and will not unfrequently break if they be not slackened. Various instruments have been devised for measuring the quantity of dampness in the air on this principle. Thus, a piece of sponge (prepared by steeping it in pearl-ash and water) will change its weight by the absorption or the loss of water, according to the dampness or the dryness of the atmosphere; and if it be suspended from a balance, the alterations in its weight may be indicated by a scale to which the arm of the balance is made to point. Another instrument, for a long time the most accurate in use, was constructed to measure the alterations in length

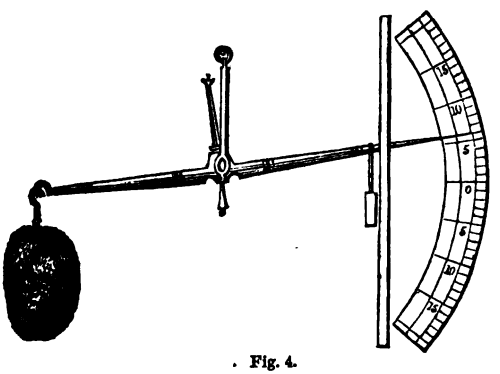


Fig. 4.

* See Treatise on Vegetable Physiology, Chapter vi.

produced by the dryness and dampness of the atmosphere, in a human hair. The hair (prepared in the same manner as the sponge) is held at each end by a pair of small nippers, of which the lower one is attached to the frame of the instrument, whilst the upper one hangs from a thread or fine wire that winds round the axis on which the index is placed. The hair is kept on the stretch by means of a small weight suspended from the axis in the contrary direction. When the atmosphere is dry, the hair has a tendency to twist and contract itself in length, so that the axis is turned round by its strain; and when the air is damp, the hair lengthens, so that the index is turned in a contrary direction by the weight, as far as it is permitted. The index moves over a scale, on which are marked the points of extreme dryness and dampness; which are ascertained by placing the instrument first in air rendered as dry as possible, and then in air loaded with moisture. The indications given by such an instrument (which is termed a Hygrometer or measurer of moisture) are much less correct, however, than those of an instrument invented by Professor Daniell, which enables us to ascertain the exact amount of moisture contained in the air.*

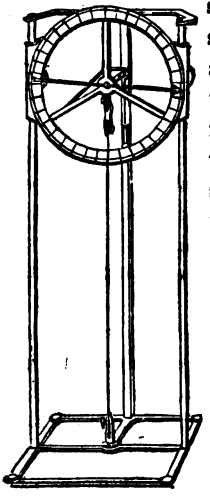


Fig. 5.

36. Liquids, also, have an attraction for gases; and this is shown every time that we pour water from one vessel to another. Large quantities of air, strongly attached to the surface, are carried down by the falling mass below the surface of the liquid in the lower vessel; but as they become elastic from the additional compression they undergo, they detach themselves from the particles of water to which they adhered, and rise again in a stream of bubbles. This process takes place on a large scale in natural falls of water; the foam and dashing spray of which are produced

* This will be described in the Treatise on Heat; as a knowledge of the laws of that agent is necessary to the comprehension of its action.

by the escape of the air, rebounding from a considerable depth to which it has been carried by the downward impulse, and in its turn carrying with it a quantity of watery particles. There are waterfalls at which advantage has been taken of this perpetual supply, by receiving the air, whilst in a state of compression, into vessels, from which it is conveyed to blast-furnaces for the smelting of metals. Were it not for this tendency of water to absorb air, no living being could continue to exist below its surface; for both aquatic plants and animals derive the air which (like plants and animals living on land) they perpetually require, from that contained in the liquid medium they inhabit. Thus gold-fish, kept in a small quantity of water, gradually exhaust it, by the action of their gills, of the air which it contains; and unless they receive a fresh supply in some way or other, they die. Where they are living in a pond which is continually exposing a large surface to the air, and the water of which, therefore, is continually absorbing a fresh supply, no further attention on this point is required. But when they are confined in a glass globe, such as is commonly employed for the purpose, it is necessary that the water should be frequently changed. The form of these vessels is the worst possible; since, the more nearly they are filled with water, the smaller is the surface it presents to the air; and the less, therefore, is the renewal of the air, as it is exhausted by the fish, through absorption from the atmosphere.

37. We may easily prove the presence of air in ordinary water, by placing some of the latter in a glass vessel under the receiver of the air-pump. As soon as the pressure is partly removed from the surface of the liquid, we shall see a large quantity of minute bubbles in the water; the greater part of these are attracted to the surface of the glass, to which they adhere; but in proportion as the pressure is removed, they increase in size, and at last rise to the surface and make their escape. If we apply the same means, however, to water which has been recently boiled, we shall obtain no disengagement of air-bubbles; but if we allow the same water to remain exposed to the air for a few hours, we shall find that it will contain

nearly as much air as if none had been expelled from it by the boiling. The process by which the air is driven off by heat is still more easily watched, but we cannot always readily distinguish between bubbles of air and of steam. If we place a glass vessel, containing spring water, over a lamp, we shall very soon see a number of minute bubbles, at first adhering to its sides and bottoms, and then rising to the surface; these are formed by the air which the water contained, and which is made to show itself thus, not by the removal of pressure from the surface of the water (as in the last case), but by the increase of its own elasticity. But as the water becomes hotter, other bubbles are formed, which consist of watery vapour; and by the time it is brought to the boiling point all the air is expelled. It is to the loss of the air which it contains, that the flatness of water that has been recently boiled is due.

38. Water is capable of dissolving much larger quantities of many gases, than it can of common air. Thus, it may be made to take up many times its own bulk of carbonic acid gas, provided a sufficient pressure be kept upon its surface. This is the case with brisk, fermented liquors, which have been bottled before the process of fermentation (by which this gas is given off in large quantities) is complete; the gas afterwards produced remains dissolved through the liquid, until the pressure is taken off by the opening of the bottle, when it comes forth with effervescence, as in the case of champagne, porter, cider, &c. In bottled soda-water, the carbonic acid is forced into the water by great pressure, and escapes in the same manner when that pressure is removed; but in the common effervescing draughts, the gas is given off at the time of the mixture of the acid and the soda, by the action of one upon the other. By heating any of these fluids, the whole carbonic acid they contain is set free; and it is thus shown that the attraction of the carbonic acid for the water is readily overcome.

39. But there are other gases for which water has a much greater attraction,—taking them up with great eagerness, and not easily parting with them. This is the case with ammonia, and with muriatic acid gas. That these naturally exist in the

state of gas or vapour, we may satisfy ourselves by a simple experiment. If we hold the open mouth of a bottle containing hartshorn near another containing spirit of salt, we shall see the air between them filled with white fumes, which are produced by the action of the vapour of one upon the vapour of the other, forming, in a very finely-divided state, the substance which is known as sal-ammoniac. Now hartshorn is little else than water which has been caused to absorb a large quantity of ammonia in the state of vapour; and spirit of salt is water which has been caused to take up muriatic acid gas (which is given off when we pour sulphuric acid—oil of vitriol—upon common table-salt) in the same manner. For a certain quantity of these gases, water has so strong an attraction, that when a tube or bottle filled with either of them is opened under water, the liquid will rush up the tube as if it were perfectly empty,* absorbing the gas as it ascends. From this quantity, no amount of heat, or withdrawal of pressure, can set it free; but when the liquid has been caused to take up a large quantity of the gas, a part of this readily passes off, when pressure is withdrawn (by the removal of the stopper or cork), in the same manner as carbonic acid from soda-water.

40. We have lastly to consider the heterogeneous attraction of the particles of gases and vapours for each other; which gives rise to several curious and important results. Aeriform bodies are capable of mixing with each other with perfect freedom; since they have no *cohesion* nearly sufficient to resist the force of *adhesion*. They are all, consequently, able to diffuse themselves, without any limit or obstacle, through each other's masses; and this in spite of the greatest differences in their weight. Thus, if we confine a portion of hydrogen (a gas which forms a large part of what we burn in gas-lights, lamps, candles, &c.) in a strong phial, and a portion of oxygen (a gas which forms one-fifth of the atmosphere) in another, and connect the two by a narrow tube three or four feet long, placing the hydrogen-phial at the highest end, we shall find, in a short time, that a complete mixture will have taken place between the two,

* See Treatise on PNEUMATICS.

—as shown by the explosive power which the gas in both phials will now have acquired, the hydrogen from mixture with the oxygen, and the oxygen from mixture with the hydrogen. In this case, the weights of equal bulks of the two gases are as 1 to 16; and the oxygen, which is the heavier, is thus drawn by the attraction up to the very highest point in the upper vessel; whilst the hydrogen, which is only one-sixteenth part of its weight, descends to the bottom of the lower phial: when properly considered, this change of place is as striking as if it took place with two liquids, having the densities of water and mercury. It is to this tendency in the particles of gases to mutual adhesion, that the equal admixture of those composing atmospheric air, is due. It has been found by experiment that there is no marked difference in the proportions of oxygen and nitrogen, in air taken from the most different situations, such as the tops of high mountains, the heart of the most crowded cities, the surface of the ocean within the polar circle and at the equator; yet there is about as much difference in their respective weights as there is between oil and water. Again, the tendency of the atmosphere to dissolve watery vapour is another instance of the same attraction; it is never free from moisture, and sometimes it is completely loaded with it.

41. The tendency of gases to mingle shows itself not only when they are left quite free to do so, but also when a porous partition exists between them. Thus, if we confine a portion of common air in a membranous bag, such as that formed by the stomach of a rabbit or other small animal, and suspend this in a vessel filled with carbonic acid gas, the latter will penetrate the bag, and will at last burst it. Here an endosmose and exosmose occur, as in the case of liquids; but the tendency of the carbonic acid to pass in is greater than that of the air to pass out, so that an increase in the contents of the bag necessarily takes place. By the slow action of the same forces, hydrogen gas, confined in a bell-jar which has a crack in its side, gradually makes its escape, and is replaced by a certain quantity of air. If a portion of common air be confined in a jar, by tying tightly over it a piece of sheet India-rubber, and this be placed in a larger jar

filled with hydrogen, the latter gas will pass through the India-rubber partition faster than the air passes from the other side; and thus the partition will be made to bulge out, and will at last burst. The amount of each gas that will tend thus to diffuse itself through any other, has been determined by a very simple experiment. If a tube, closed at one end by a plug of dry plaster-of-Paris, about half an inch long, be filled with any gas lighter than air, and be placed over water, there will be a gradual interchange between the outer air and the contained gas, through the porous plug. The quantity of air which enters will be less than that of the gas which passes out; and the water into which the tube dips at its lower end will consequently rise in it. When the tube is filled with hydrogen gas, the rise of the water is very rapid; and it will fill, at last, a large proportion of the tube. Now it has been ascertained that the quantity of air which will replace any particular gas, is in the opposite or inverse proportion to the square-root of the density of that gas, as compared with air. Thus common air is nearly 16 times heavier than hydrogen; and the square-root of 16 being 4, the quantity of air which will replace any given amount of hydrogen is *inversely* as nearly 4 to 1, or in ordinary language as 1 to nearly 4,—that is, something more than one-fourth. The remainder of the tube—nearly three-fourths of its length—becomes filled with water, which is drawn up to fill the vacuum or empty space thus left. If, on the other hand, the tube be filled with a gas heavier than air, the latter will enter more quickly than the former will pass out; and the water will be pushed down instead of being drawn up*.

42. Not only does this force of heterogeneous attraction tend to bring together particles of bodies which were previously in a separate state, but it may, when very powerful, remove them from the influence of a similar force which had caused them to adhere less powerfully to other substances. Thus between water

* There is not the same regularity of result when animal membranes are employed instead of plaster-of-Paris; thus carbonic acid will pass towards common air, or even coal-gas (which is much lighter), and will distend a bladder filled with it, even to bursting; although according to the rule just given, the air or coal-gas, being much the lighter, should pass towards the denser carbonic acid, as in the endosmose of fluids.

and spirit-of-wine there is considerable attraction; the two liquids may be mixed in any proportions, and they cannot easily be separated. Between spirit and resin of any kind, again, there is a strong attraction, so that the latter is dissolved in the former. But between resin and water there is very little attraction; the solid may have its surface just wetted with the liquid; but it cannot be dissolved in it in the least degree. Now if we add water to a solution of resin in spirit, the tendency of the spirit to adhere to the water, will be sufficiently strong to detach it from the resin, which was previously diffused through it by its attractive force; and the resin is thrown down or *precipitated* in the solid form. In the same manner, if we shake well-burned charcoal in water with which the gas named sulphuretted hydrogen (that which is principally concerned in giving the fœtid smell to rotten eggs, cess-pools, &c.,) is united, the charcoal will completely absorb the gas, detaching it from the liquid, and will deprive the latter of the very disagreeable qualities it previously possessed. In this manner, water which has in any way become foul may be completely purified; those of its impurities which cannot be removed by ordinary filtering, being completely withdrawn by charcoal.

43. It is often necessary to separate bodies which are more or less closely united by heterogeneous adhesion; and various plans are adopted, according to the nature of the substances. Thus if it were desired to purify water which is rendered turbid by finely-divided particles of earth, chalk, &c. *suspended* in it, these may be removed by simply filtering it through a solid substance whose pores are too small to allow these particles to pass. Or if it were desired to collect any substance precipitated from a solution (as the resin from the spirit, in the case just mentioned) the filter is here again employed,—the solid matter remaining upon it being here the product we seek, and not, as in the filtering of impure water, the liquid which has passed through. Another mode is very often employed to clear a fluid from minute particles that injure its transparency. This is to shake it up with some viscid substance that shall attach these particles, and remove them from the liquid. The white of eggs is much used for this

purpose, in clarifying wine, syrup, coffee, melted jelly, &c.; and in the old process of sugar-refining, bullocks' blood was employed in large quantity. But no filtering, or other such process, could separate a liquid and a solid substance *dissolved* in it, as water and salt; and here we call in the aid of heat, which causes the particles that were previously adherent to repel one another. Thus, we might separate salt and water by applying heat to the solution, which would drive off the water in the state of vapour, leaving the salt behind. This process is termed *evaporation*; and it will of course take place the faster, in proportion to the heat applied. If we desire to collect and preserve the liquid, the vapour is conducted into a vessel to which cold is applied, and is there condensed, or brought back to the liquid form. This process is called *distillation*; and it is practised on a very large scale in the separation of alcohol (spirit-of-wine) from water. The alcohol is caused to pass off in vapour, by the application of heat, much more readily than water, its boiling point being low; and the latter, therefore, remains behind until nearly all the spirit has been driven off. The same process is applied in concentrating sulphuric acid (oil-of-vitriol), which, as first made, contains a large quantity of water; but in this instance the water is the most readily evaporated, and the strong sulphuric acid remains behind.

Divisibility of Matter.

44. There appears no limit to the degree of minuteness to which various forms of matter may be subdivided. Various interesting examples of this kind are presented to us by Nature, being revealed for the most part by the aid of the microscope; and others are furnished by Art. The former class will be first considered.—We not unfrequently speak of things as being "as fine as a hair." Now the human hair varies in thickness from the 1-250th to the 1-600th part of an inch; and it is a massive cable in comparison with the fibres produced by other animals. The thread of the silk-worm is many times finer, being from the 1-1700th to the 1-2000th of an inch. This, however, is nothing to the slenderness of the spider's thread, which has been found

in some instances to be no more than 1-30,000th of an inch in diameter. It is difficult even to imagine quantities so small; and the best way to realize them to the mind is to consider that 30,000 such threads laid side by side would make a riband only one inch broad. But this is by no means the entire wonder of the spider's thread. Each of these slender cords is composed of four or six twisted together; and every one of these is formed by the pressing out of a viscid fluid through a sort of teat or spinneret at the hinder part of the animal, the point of which, instead of having a single orifice, is perforated by not less than 1000 very minute apertures. The thread of each spinneret must be formed, therefore, by the union of not less than 1000 fibrils; and four or six of these threads go to form the cord of 1-30,000th of an inch in diameter, of which the spider's web is constructed. It has been calculated that two drachms (120 grains) of such thread would reach from Edinburgh to London. The fibres yielded by the vegetable kingdom are not so fine as these; yet their minuteness is most astonishing. Every fibre of flax is composed of a bundle of other fibrils, which have been found to be about 1-2500th of an inch in diameter. Similar fibres, obtained from the pine-apple plant, have been ascertained to be no more than 1-5000th or even 1-7000th of an inch in diameter; these when twisted into threads, and woven, are said to form cloth of very beautiful silky texture. The glistening spiral fibrils which are drawn out when the leaf-stalk of a geranium or a strawberry is snapped by drawing the ends apart, are not more than 1-12,000th of an inch in diameter.

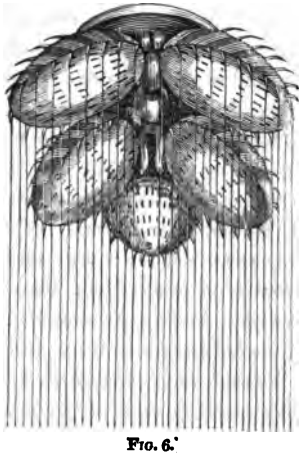


FIG. 6.
Spinnerets of the Spider.

45. Not only fibres, however, but rounded particles of extreme minuteness, are distinguishable by the microscope, in

the structures of animals and plants. Thus the red blood of the higher animals derives its colour from the presence of multitudes of flattened discs (resembling pieces of money in shape), which float in the fluid. These discs are round in the blood of quadrupeds; but are oval in that of birds, reptiles and fishes. They are largest in the blood of certain species of the frog tribe, in which their length is as much as 1-337th of an inch. In the common frog, however, it is no more than about 1-1000th of an inch. In man, their diameter varies from 1-3000th to 1-4000th of an inch; and it may be estimated that, in such a drop of blood as would hang from the point of a needle, there would be a million of such particles. In the musk-deer, their diameter is not more than 1-12,000th of an inch; these are the smallest blood-particles at present known.

46. Yet there are animalcules possessing a complex internal structure, having the power of imbibing and digesting food, of moving with great rapidity, and (as it would seem from their actions), of enjoying life during the brief span allotted to them, which are far smaller than the minutest particles of blood. The minutest kind of these are termed Monads; they present the appearance of little points of jelly, in active movement; and no distinct structure can be traced in them. When they are put into water, however, in which a little indigo or carmine has been rubbed, coloured spots are soon distinguishable in their bodies, arising from the filling of interior cavities with the minutest particles of the colouring substance. Now the whole monad is from 1-18,000th to 1-24,000th of an inch in diameter. In the smaller ones as many as four, and in the larger ones as many as six, of these spots could be seen, not occupying above half the diameter of the animal; the diameter of each spot, therefore, could not be more than 1-144,000th part of an inch. From the roundness of these spots there must be several particles in each; if we only assume three, we obtain proof that there must be particles of the colouring substance diffused through the water of no more than 1-432,000th part of an inch in diameter. Further, in the larger animals of similar structure, it is seen that the spots are separated by membraneous partitions of not greater

thickness than one-twentieth of the diameter of the spot; and this would make the thickness of the partitions no more than 1-2,880,000th of an inch, in the monads of smaller size. Again, in the larger species, the active movements may be distinctly seen to be due to the vibration of delicate hair-like filaments, termed *cilia;* though these cilia cannot be distinguished in the smallest monads, yet, as the movements are evidently the same, they must doubtless exist in them; and they cannot have a greater diameter than 1-450,000th of an inch. If the same calculations were extended to the young animals, or to species too small to be discerned, except under the most favourable circumstances, the minuteness of the particles of whose existence we should then have evidence would be found to be still more inconceivable.

47. Again, there is found at Bilin in Germany, a deposit of siliceous (flinty) character, which occupies a surface of great extent (probably the site of an ancient lake,) and forms slaty layers of fourteen feet in thickness. This bed supplies the *tripoli* used by artisans in metal for polishing their work, and also the fine sand employed to form moulds for casting small articles in Berlin iron*. For these purposes its consumption in Berlin alone is not less than from 50 to 60 cwt. yearly. It is almost entirely composed of the sheaths or coverings of a kind of animalcule, which has the power of separating flinty matter from the water in which it dwells, and of producing out of this a sort of case analogous to the shell of a crab or lobster. The length of one of these is about the 1-3500th of an inch; and it is hence calculated, that about 23 *millions* of them are contained in a cubic line of the sand, and 41,000 *millions* in a cubic inch. As a cubic inch weighs 220 grains, about 187 millions would be contained in a grain weight of this sand.

48. The minuteness of these is yet surpassed by that of the animalcules of the iron-ochre, a yellowish-brown substance found in certain marshes. These are only about 1-12,000th of an inch in diameter; so that a cubic line would thus contain 1000

* It is partly on the fineness of this sand, that the beauty and finish of these articles depends.

millions of them, and a cubic inch nearly *two million millions*. Yet these animalcules must have each had a fabric composed of a number of parts, whose size would be small in comparison to that of its whole body. There seems, therefore, no limit whatever to the subdivision of material particles in the natural growth of animal bodies.

49. The Vegetable world has not been found to afford any examples of such wonderful minuteness. There is one instance, however, which it may be well to mention, on account of its frequent occurrence. The common puff-ball fungus, when it bursts, sends out a kind of smoke, or very fine dust. This consists of particles of extreme minuteness, each of which is capable of producing a new plant; their diameter is about 1-20,000th or 1-30,000th of an inch; and we can thus readily comprehend what it is otherwise very difficult to understand,—the universal diffusion of the germs of the fungi, so that they are ready to spring up wherever there is a soil adapted to them, as we see in the case of the various kinds of mould, blight, &c.*

50. The thickness of transparent films may be sometimes determined by the colours † they transmit. In this way, it has been ascertained that the wings of certain insects are not more than the 1-100,000th of an inch in thickness; that is, a hundred thousand such wings laid on one another would form a pile not more than an inch in height. A thinness as excessive as this may be given to glass, by blowing it into bubbles (like those of soap and water), until they burst. In the same way it was determined by Newton, that the part of a soap-bubble in which colours are seen is less than 1-25,000th of an inch thick; and that, in the situation of the black spot which is seen just before the bubble bursts, it is no more than the 4-*millionth* of an inch in thickness. There are various other modes of proving, by calculation, the extreme divisibility of the particles of matter in a fluid form; and some of these give results surpassing any that have been mentioned. Thus, a grain of copper, being dissolved in nitric acid (*aqua-fortis*), will give a blue colour to the fluid; and if this be

* See VEGETABLE PHYSIOLOGY, § 61—64.

† See *Colours* in Treatise on OPTICS.

mixed with three pints of water, the whole will be sensibly coloured. Now three pints contain 104 cubic inches; and as a line of an inch in length may be divided into at least 100 parts distinguishable by the naked eye, a cube of an inch each way may be divided into a million such parts; hence, every millionth of a cubic inch of such a solution will contain no more than 1-104 *millionth* of a grain of metallic copper. The presence of copper in it might be proved by chemical tests, which would separate it in the metallic state. In the same manner it may be calculated that, if a grain of gold be dissolved in a mixture of nitric and muriatic acids, and a piece of ivory or white satin be dipped in it, the film of metal with which this is covered (which may be brought back to the metallic state by placing the substance in hydrogen gas), is no more than 1-10 *millionth* part in thickness. Chemistry furnishes us with a great abundance of such examples.

51. Even in ordinary mechanical processes, the division of certain metals, especially gold, silver, and platinum, is carried to an extent which we should by no means have guessed, but which may be positively determined by a little simple calculation. Thus, an ounce of gold would form a cube of something less than half an inch each way (5-12ths); so that, when placed on a table, it would cover less than a quarter of a square inch, and would stand not quite half an inch high. Now, by continual hammering, the gold-beater extends this cube until it covers a surface of 146 square feet, or 21,024 square inches. It is thus extended to more than 121,000 times its original surface; and it will consequently be only 1-121,000th part of its original thickness, or about 1-290,000th of an inch. Fifteen hundred such leaves placed upon one another would not equal in thickness a single leaf of ordinary writing-paper.—In the ordinary process of gilding, the gold is mixed up with quicksilver, forming a soft amalgam; this is spread over the surface to be gilded; and the mercury being then driven off by heat, a very thin film of gold is left. Now, though it can be easily proved that this film is no more than 1-233,000th of an inch in thickness, it forms a complete surface, in which there are no pores or deficiencies; as is

shown by its power of protecting the brass, silver, &c. that it may cover, from the action of aqua-fortis poured upon it.

52. In the manufacture of embroidery, it is necessary to obtain very fine gilt silver threads. To accomplish this, a cylindrical bar of silver, weighing 360 ounces, is covered with about 2 ounces of gold. This gilt bar is then drawn into wire so fine, that 3400 feet of it weigh less than an ounce. The wire is then flattened by passing it between rollers under a severe pressure; by which process its length is increased, so that about 4000 feet weigh an ounce. Hence, one foot will weigh 1-4000th of an ounce. Now the original proportion of the gold to the silver was 2 parts to 360, or 1 to 180; so that the quantity of gold on a foot of the wire is only the 1-180th of 1-4000th of an ounce, or 1-720,000th. Now if we divide this foot of wire into 12 parts of an inch long, and then again divide an inch into 100 equal parts, every one of which will be visible to the naked eye, we shall obtain particles on which a quantity of gold exists equal only to the 1-1200th part of 1-720,000th of an ounce, or 1-864 *millionth*. But we might proceed even further; for we might magnify one of these particles in a microscope 500 times each way, or 250,000 times in area; so that we should be able to distinguish clearly portions of its surface, each of which shall seem equal to the magnified particle, and yet be really no more than 1-250,000th part of it. Hence the quantity of gold (which is seen by the microscope to be uniformly distributed over the whole) covering such a portion of the surface, will weigh only 1-250,000th part of 1-864 *millionth* of an ounce, or 1-216 *million-millionth* of an ounce,—an amount of which it is utterly impossible to form a true conception. So completely does the film of gold upon the wire retain all its properties, that, if a small piece be cut off and placed in nitric acid, this fluid will dissolve the silver within the coating, by attacking its unprotected ends, and will leave the gold, forming a tube.

Indestructibility of Matter.

53. Few errors are more common, than the idea that there can be a waste or destruction of matter, or at least of some kinds

of it. Such an idea would seem to be abundantly confirmed by daily experience. We see the wood or the coal of our fires, the wax or tallow of our candles, the oil of our lamps, gradually burning away; the materials seem entirely to vanish, and to leave little or no trace behind them. But they only change their form. In all cases of ordinary combustion, two elements, carbon* and hydrogen—of which most of our combustible substances are chiefly composed—unite with the oxygen of the atmosphere, and give off light and heat in their union; and as the result of this union, it may be shown that water, and a gas termed carbonic acid, are produced. Precisely the same change is effected by the breathing of animals; which process is a kind of slow combustion, serving to keep up the heat of the body. It is because carbonic acid gas (which is the gas that is often found at the bottom of wells and pits that have been closed for some time, and in brewers' vats) is a poison to animals, that the continuance of a number of persons in an ill-ventilated room makes its air unwholesome; the injurious change being further aided by the combustion of lamps or candles.

54. Both of the substances which are thus given off to the atmosphere are again taken from it by Plants; and are by them made subservient to the purposes of the growth of Animals. Vegetables have a wonderful power of *decomposing* the carbonic acid of the air;—that is, of separating it into its two elements, of which they take the carbon, giving back the oxygen to the atmosphere. In this manner the carbon is again reduced to the solid state; and it may either enter into the formation of wood, or may become part of the wax and oil, &c. which many plants produce in abundance. By the combustion of wood, either in its ordinary state or that altered form which we know as coal, the carbon is again given off to the atmosphere, again to be withdrawn from it by the agency of another generation of plants; and the same occurs with regard to vegetable oil and wax. But Vegetables also employ the carbon which they withdraw from the air, as one of the materials of the substances which they produce

* Carbon is nearly the same with very pure charcoal. Hydrogen forms a considerable part of coal-gas.

for the benefit of the Animal creation; and these substances, taken in as food, partly enter into the construction of the solid framework of their bodies, but chiefly serve as the fuel for the slow combustion, just now mentioned as adapted to keep up the temperature of the system. Thus, of the carbon which is taken from the atmosphere by plants, a large part is restored by the breathing of animals, or by the uses to which man puts the materials he obtains from them; and the remainder is given back by the decay of the vegetable and animal fabrics not so employed. This decay is but another change of the same kind with those already mentioned. There is no more absolute loss of material in this case than in the others. The elements of the substances which we observe to disappear by slow or rapid degrees, do but change their state by entering into new combinations; and they are separated from these in turn, that they may become the elements of new fabrics, which again restore them to the atmosphere by their own death. What has been said of the carbonic acid given off in combustion may be said of the water also; for this unites with the vapour that rises from the surface of the sea, the lakes, and the rivers of our globe, to descend in fertilizing showers on the ground, that would otherwise be parched and arid; and being taken up by the roots of plants, it supplies the elements which, combined with the carbon obtained by the atmosphere, make up the great bulk of the vegetable fabric.

55. Thus we see that, in the most familiar of all cases,—in which the idea of the *destruction* of the substance most strongly occurs to our minds,—there is really no loss of material, but only an alteration of its condition; and that this constantly-occurring alteration is a part of the grand scheme of Providence. For if there were not thus a constant renewal,—by the process of combustion, decay, and animal respiration,—of the carbonic acid in the air, it would be entirely withdrawn, in process of time, by the growth of plants, and vegetation must cease for want of a supply of materials. Or if, on the other hand, these processes were to continue to impart carbonic acid to the air, and there were no corresponding withdrawal of it by the growth of

plants, the atmosphere would in time become unfit for the maintenance of the lives of animals.*

56. There are many instances in which there is a gradual disappearance of solid material, by the wearing away of the surface of a mass; and in which, therefore, it would seem as if there was an actual loss of substance. Thus, according to the old proverb, "a continual dropping will wear away the hardest rock." There are many relics, preserved with religious veneration, which, although composed of hard stone, exhibit deep impressions produced by the kisses of the devotees, who have been anxious thus to signify their faith and love. Thus, the friction, even of the softest substances, leaves its mark upon the hardest, when the act is often repeated. That of hard substances upon each other operates much more rapidly; and a *wear* of their surfaces takes place, especially if they are not rendered smooth by oil or grease interposed between them; as in the case of a hard road, over which many carriages pass. The *wear* of our clothes into holes is another example of the gradual disappearance of material, from the same cause. In all these instances, and many more, there is no *loss* of particles, but only a *separation* of some of those on the surface from the rest; thus, at every contact of another substance, some few are carried off; and the loss is the more rapid in proportion as the friction is greater. The particles thus separated form the greatest part of the *dust* which is continually floating about in the air, and is deposited on bodies at rest.

* See, on the subject of this and the two preceding paragraphs, the Treatise on VEGETABLE PHYSIOLOGY, chapters vi. and viii.

CHAPTER II.

PROPERTIES OF MATTER NOT UNIVERSAL.—ELASTICITY, DUCTILITY, MALLEABILITY.—STRENGTH OF MATERIALS.

57. THE property by which certain forms of matter admit of being compressed, and return to their original condition after the compressing force is removed, has been hitherto described only as it is possessed by aeriform bodies (§ 8). These, indeed, are the only bodies in nature which are *perfectly elastic*,—that is, which exert a force in returning to their previous state, equal to that which compressed or displaced them. This property is easily shown to exist in air or in other gases, by means of a tube bent at the bottom like the letter U, but having one of the legs greatly prolonged and open, whilst the short one is closed at the top. If mercury be poured into the long leg, it will not rise in the short leg to the same height; since there is a certain quantity of air in it which has no means of escape. If more and more mercury be poured into the long leg of the tube, its increasing weight will compress the air pent up in the short leg; so that, if the column of mercury stand 30 inches higher on one side than on the other, the air will be compressed into half its previous dimensions. Now, if, under these circumstances, the tube be laid down sideways upon a horizontal table, the whole compressing force of the mercury will be taken off, since the column then rests entirely on the sides of the tube; and it will be found that the air will then occupy precisely the same space within the tube, as it did when the quantity of mercury in the tube was such as to occupy only the bend of the U, and when the height of the column was the same on each side, so that no pressure was being exerted by it on the air. By such a simple apparatus as this, the law of the

constant inverse proportion between the bulk of elastic fluids, and the amount of pressure on them, is determined. For, not only may one of the legs of the tube be so much prolonged, as to admit a column of mercury of 2, 3, or even 4 times 30 inches in height, so as to add 2, 3, or 4 times the usual pressure,—but, by inverting the tube, the pressure may be diminished in the same proportion: and it will be found that the same law still holds good; the space occupied by the gas increasing just as the pressure is diminished, and being diminished to exactly the same amount as before, as soon as the original pressure is restored. This is called the law of Marriotte, from the name of its discoverer.

58. The elasticity of liquids can only be shown to exist, by their return to their original bulk after undergoing compression. This they do as perfectly as aeriform bodies; but, as already stated, the amount of compressibility is so small, that their elasticity is but little called into operation.

59. Nearly all solid substances possess some degree of elasticity. There are none whose particles are altogether incapable of separation or displacement; and, in general, when the displacement does not extend beyond a certain distance, each particle tends to return to the place it before occupied in the mass of which it forms a part, and with a force exactly proportional to the distance through which it has been displaced. So far, then, the elasticity is perfect, or nearly so. But if the displacement be carried beyond a certain distance, there is no tendency in the particles to regain their former positions; and they remain passively in the new positions they have been made to take up; or take up some other positions different from those which they had at first. It is in the *limit* to the operation of elasticity that we find the greatest difference among various bodies. Thus, a ball of steel or of ivory shall be as elastic up to a certain point as a ball of India-rubber; this may be proved by letting the three drop down upon a hard surface, from the same height, and then marking the heights to which they rebound. But the particles of the steel or ivory could not be displaced from each other, as can those of the India-rubber, without a total separation; and thus, whilst the elasticity of the latter

extends to almost any degree, that of the former is very limited. The elasticity of metals is most perfect when they are drawn out into wires; and it may be shown in several modes. If a wire be tightly stretched between two fixed points, and it be then pulled ever so little aside from the straight line, it must undergo a certain increase in length, the amount of which can be calculated from the angle to which the wire is drawn; and if this is done by means of a scale-pan suspended from the middle of the wire, in which known weights are placed, it may be shown that the strain exerted on the wire is exactly proportional to its increase in length.

60. Another proof of the perfect elasticity of metal wires, up to a certain point, is to be found in the results of their *torsion* or twisting. Let a wire be suspended by a fixed point at its upper end, and carry at the bottom a light cross bar, the movement of which may be exactly observed by a circular scale. Now if the place of this bar, when it is hanging at perfect rest, be accurately noted, and it be then made to turn, by a force applied to it, in such a manner as to twist the wire a whole revolution or a portion of it, the elasticity of the wire will cause the bar to resume its former position, after it has oscillated forwards and backwards for some time. Moreover, if a certain force move the bar through a given angle, twice the force will cause it to move through twice the angle,—the torsion of the wire being thus in direct proportion to the force applied, and the elasticity produced by that torsion having a tendency to untwist the wire to its original condition, as soon as the force is withdrawn. Indeed, it will at first cause it to twist in the opposite direction, just as a pendulum swings up on one side as far as it descended on the other; and though it may be some time before the bar entirely settles itself, it will at last come to the line in which it originally pointed.

61. This *elasticity of torsion*, as it is termed, may be shown to exist, not only in wires of steel, or in thin rods of wood, but in those metals which seem most destitute of the property. Thus, if we take a *leaden* wire, 1-15th of an inch in diameter, and 10 feet long, and fix one end of it firmly to the ceiling,

whilst the other carries the cross-wire or index,—when the latter has been twisted round twice and let go, it will be observed to make almost four revolutions in the contrary direction, two being produced by the untwisting of the wire, and two by its twisting in the opposite way, in consequence of the continuance of the movement it had acquired. The index will then be observed to move round towards the side from which it commenced its revolutions, and will nearly attain the point from which it started; and it will continue to make a series of oscillations, backwards and forwards, each less in amount than the last, until it finally comes to rest in the position it had before it was at first twisted. Thus the elasticity is equally perfect in this leaden wire, up to the point yet mentioned, as it could be in any case; since in no instance can the oscillations continue long, on account of the resistance of the air. And, moreover, it would be found in a lead wire, as in any other, that the forces with which the cross-bar or index, when twisted through different angles, tends to return to its first position, are accurately proportional to the angles through which it has been moved.

62. But if we twist the wire four times instead of two, it will not unwind itself completely; for when the index has come to a state of rest, it will be found not to have returned to its first position, but to be short of that position by nearly two revolutions. Thus, then, the particles of the lead could be displaced to such an extent, as to allow a wire of this length and thickness to be twisted twice round, without the occurrence of any such change as should prevent their return to their original places. But the additional displacement, requisite to give the wire four turns, is more than its particles can bear; and they remain, therefore, permanently displaced, the wire having taken what is technically termed a *set*. Examined in this manner, there are probably very few bodies which will not manifest elasticity. Thus a thin cylinder of pipe-clay (which is generally considered as destitute of elasticity as almost any substance can be) shows the existence of an elasticity as perfect as can be found in the best tempered steel. It is in the *limits* of this elasticity that the chief difference lies; for a steel wire similarly circumstanced might be twisted

a great many times, before its particles receive such a *set* as to prevent it from completely untwisting again.

63. From the facility with which threads or wires may thus be twisted by slight forces, and the regularity of the proportion between the amount of force and the degree of torsion in each case, the change in the place of an index suspended by such a wire has been employed to measure degrees of force too minute to be estimated in ordinary ways; and particularly forces of attraction,—such as that of all bodies amongst each other (as in Cavendish's experiment, § 100), or the peculiar attractions produced by electric or magnetic influences. The instrument constructed for this purpose is termed the *torsion balance;* in its simplest form it consists merely of a stand having a clip at its top, for tightly holding the wire, and a circle below, divided like a dial-plate, for marking the oscillations of the needle or index suspended by the wire. It is found advantageous, however, to enclose the whole in a glass cylinder, that the oscillations may not be influenced by currents of air, or other such disturbing causes.

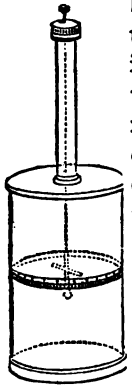


FIG. 7. Torsion Balance.

64. It is remarkable that, even when the relative positions of the particles have been so much altered as to take a permanent *set*, the elasticity of the whole mass remains exactly the same as before. Thus, when a wire has been permanently lengthened by a great strain, it shows itself to be as perfectly elastic as before, by recovering from the effects of smaller degrees of extension. Suppose, for instance, that a wire of 3 feet in length, stretched tightly between two pins, requires a weight of 1 lb. to be suspended from the middle of its length, to draw it a quarter of an inch out of the straight line; and that, when this weight is removed, it returns completely to its former state and position;—its elasticity is, therefore, so far, perfect. But let us further suppose, that a weight of 20 lbs. suspended from the same point, draws it so far out of the straight line as permanently to increase its length a quarter of an inch; the wire will

then remain slack when the weight is removed, since it has not the power of recovering its previous condition. Now, if the wire be again tightened to the same degree as before, it will be found that its elasticity remains precisely the same in this new *set* or position of its particles, as it was previously; for that, by hanging a weight of 1 lb. from the middle of its length, it will be drawn a quarter of an inch out of the straight line, and will recover its state and position when the weight is removed. Experiments on the torsion of wires give just the same results. Thus it was found by Coulomb, that, after he had given a *set* to lead wire by twisting it *four* times round (the limit of its elasticity being *twice*, § 62), the wire was as elastic in its new condition as before; requiring the same force to give it a further twist, and recovering itself as completely when that force was withdrawn.

65. *Ductility.* Now the making of a wire from a bar of metal, or, as it is termed, the *drawing* of wire, is nothing else than giving a new *set* to the particles composing the bar; that is, they are thrown into new positions in regard to each other, but have the same relation towards the particles which now surround them, as they had for those in the neighbourhood of which they previously were. Different metals possess the property of submitting to this new arrangement without giving way—which is termed *ductility*—in different degrees. Gold is the most ductile, next silver and platinum, and then iron, copper, zinc, tin, and lead. Some remarkable instances have already been given (§ 52) of the extreme fineness of the metallic threads which human art can produce; and it is unnecessary, therefore, now to do more than refer to them. This change of form produces a very curious and unexpected result. Although the particles of the wire are really less close together after the operation of *drawing* than they were before, yet they hold together more firmly; so that the *tenacity* of the wire, or its power of sustaining a powerful strain without breaking, is greater. The increase in the distance of the particles from each other resulting from extension, is shown by the increased bulk of the wire, which is of course accompanied by diminished density; and it is very

remarkable that in this condition it should have more tenacity than before. The cohesion of lead may thus be *tripled* in drawing; and that of iron is increased in a degree so remarkable, as to make thin iron wire the most tenacious of all materials,—that is, the one which requires the greatest strain to tear it asunder, when its size is taken into account. Thus, a bar 1 inch square of the best *cast* iron, may be extended by a weight of $9\frac{3}{4}$ tons per square inch; a bar of the same size, of the best *wrought* iron, will sustain a weight of 30 tons; a bundle of wires 1-10th of an inch in diameter, of such a size as to have the same quantity of material (which, as the wires do not lie in perfectly close contact, must be more than an inch square) will sustain a weight of from 36 to 43 tons; and if the wire be drawn more finely, so as to have a diameter of only 1-20th or 1-30th of an inch, a bundle containing the same quantity of material will sustain a weight of from 60 to 90 tons. For the sake of comparison it may be mentioned that a mass of hemp fibres glued together will sustain a weight of 41 tons per square inch; whilst copper wire will not sustain more than $27\frac{1}{3}$ tons, silver wire only 17 tons, gold wire only 14 tons, and lead wire only $1\frac{1}{10}$ tons per square inch.

66. Hence cables made of fine iron wire twisted together will sustain a far greater weight than chains having the same weight; and they are now coming extensively into use. Many of the suspension-bridges on the Continent are constructed with wire ropes, and there is one at Fribourg, which is 700 feet between the points of support, being 100 feet wider than the Menai Bridge. The diminished weight of iron wire which is required to support an equal strain with the more massive chains, is, however, attended with certain disadvantages, so far as the construction of bridges is concerned. For in consequence of their lightness, wire bridges are much more liable to be thrown into vibration by winds, the passing of vehicles, and other causes; and there is no doubt that the continuance of such vibrations has a tendency to diminish the tenacity of the iron (§ 22). It is probably from this circumstance that the bridge at Fribourg is becoming unsafe. Wire ropes are now much employed in this country in mines and coal-pits; and there is one of $6\frac{1}{2}$ miles

in length, in continual operation on the London and Blackwall Railway, the trains on which are drawn by means of a stationary engine. It is found, however, that after they have been in use for some time, they are liable to break, without the employment of any extra force, although not perceptibly worn; and this change is probably due to the same cause as that which acts upon solid bars of iron.

67. There is a limit, however, to the tenacity of particles that have undergone a displacement; not merely because a certain force completely overcomes it, but because a far less force will produce such a permanent change in the internal structure of the material, that it is thenceforwards greatly weakened. Thus if a cannon be fired with a charge of powder, producing a *strain* above that which the elastic force of certain parts of its material can bear, a permanent alteration of its internal structure will take place, so that a second discharge will burst it. Yet the first force may have been not much more than a quarter of that which would have been necessary to burst the cannon at once. It has been stated that a cannon of large dimensions, so overstrained by an excessive charge, may be broken in pieces by a single blow from a sledge-hammer. On the same principle, a wire may be broken by frequently bending it backwards and forwards. At each bending there is a change in the place of certain of the particles, which separate from each other; and this separation is made to extend gradually across the wire, by repeatedly bending it.

68. *Malleability.*—It is not alone by the *extension* of the material, as in wire-drawing, that the new *set* may be given to its particles, without any diminution of their elasticity or tenacity; for the same result may be produced by force applied in other ways. Thus, masses of metal may be pressed out between powerful rollers into bars, rods, or plates; or they may be extended into thin leaves by means of the hammer; or they may be made to assume, by the same means, that great variety of forms on which their utility to man so much depends. The property of receiving a new *set* by the blow of a hammer, or by *impact*, as it is properly termed, is called *malleability*. As there

is a great difference in the ductility of different metals, so also there is in their malleability. In general, the most ductile metals are also the most malleable; but there is a remarkable exception in the case of platinum and iron. Platinum and iron are more *ductile* than copper, zinc, tin, and lead; but copper and tin are more *malleable* than platinum; and even lead and zinc are more malleable than iron,—that is, they can be extended into thinner leaves, without breaking. There are certain metals which are only malleable in particular states; thus common zinc (which is somewhat impure) is brittle at low temperatures, but acquires a considerable degree of malleability at a heat a little above that of boiling water, and in this condition it is rolled into sheets, which are much used for various purposes; yet if heated to 400°, it becomes so brittle that it may be reduced to powder in a mortar. In general, the malleability of metals is increased by heat; and this is most the case with iron and platinum, which are highly malleable at a white heat, but are very little so when cold.

69. The capability of receiving impressions from blows, which the malleable metals possess, is continually made use of in various processes of the arts and manufactures, besides those which have been already alluded to. Thus, the *stamping* of coins, and the embossing of figures on surfaces of various kinds, are accomplished in this mode. The impression is made by means of a *die*, in which it is *sunk;* just as the raised impression which the wax is to present is sunk in the seal. The die, which is made of the hardest steel, is forced down upon the blank coin by means of a powerful screw; and the metal of the coin, being comparatively soft, is driven with great force into the cavities of the die, takes there a set, and retains the impression. Very similar to this, is the process by which thin plates of metal are made to take the projecting figures with which plated goods are ornamented. The die, in which is cut a hollow corresponding to the figure, is laid under the plate; and a heavy weight, with a piece of lead at the bottom of it, is allowed to drop repeatedly upon the plate; the force which it acquires in falling, drives the soft substance of

the lead, and with it the thin plate of metal, into the cavities of the die; so that, after repeated blows, the surfaces are brought into complete contact with each other, and a perfect impression is produced.

70. The mode in which these dies are themselves produced is only a variation of the same process. Numerous copies of the same die are required; since, in order to coin with sufficient rapidity, several presses are at work; and moreover the dies are continually breaking or cracking, and need to be replaced by others. The artist, who originally makes the figure which the coin is to bear, does not *sink* it, but carves the surface of a piece of soft steel into a projecting impression, exactly like that which is to be made on the surface of the coin. This steel block is then hardened (§ 73); and it is forced by a powerful press against another piece of steel which has been made as soft as possible; in this manner a sunk impression is made on the latter, exactly corresponding to the projection which was borne by the block. The same impression may be made, without injuring the first block, on a considerable number of pieces of soft steel; and when these are hardened, they become so many dies, from which coins may be struck. A very similar process is employed in repeating a small engraving over a large extent of surface; which it is often very advantageous to do in calico-printing, and also in producing highly-finished borders. The pattern is engraved on a piece of soft steel, which is then hardened; and a roller of soft steel is then passed along this hardened plate, under an extremely heavy pressure. This pressure causes the comparatively soft metal of the roller to be pressed into all the lines of the engraving; so that a perfect impression *in relief* (or projecting from the surface), is thus obtained from it. As in the case of dies, many such duplicates may thus be obtained from the first plate; and every one of them may be made to produce a considerable surface of engraving. For the steel roller being hardened, and being made to pass over another plate of soft steel, the latter is indented with all the lines and markings which the roller had received from the

first plate; and as the roller may be made to turn not only once, but any number of times, the impression may be repeated over as large a surface as may be desired.

71. The facility with which impressions are thus communicated to metals greatly depends upon their being softened as much as possible, by the process which is termed *annealing*. This consists in heating the substance to a high temperature, and then allowing it to cool very gradually. On the other hand, if the substance, when so heated, be cooled rapidly, as by plunging it into cold water, it is rendered extremely hard, but at the same time very brittle. The difference is well seen in glass. When a glass vessel is first blown, it cools rapidly and irregularly; and the varying hardness of its different parts gives to it such a degree of brittleness, that the slightest shock, or a small change of temperature, would break it. In order to prevent this, it is placed in a long furnace, of which the heat is very great at one end, whilst it gradually diminishes towards the other. Through this furnace it is slowly drawn, the time which it remains there being usually from two to four days; and it is thus cooled so gradually and equably, that its particles can take the most uniform position with regard to each other, so as to be affected alike by any shock or change of temperature. An unannealed glass flask may be made strong enough to resist the blow of a pistol-bullet dropped into it; yet the fall of a small angular substance that can give it the slightest scratch, will make it burst into minute fragments; and this may not occur until some minutes after the scratch.

72. The same kind of effect is shown by the pieces of glass which are known under the name of Prince Rupert's drops; these are made by dropping melted glass into water, which of course cools them suddenly; and they have a long oval form, tapering to a point at one end. The large part or body is so hard that it will bear a smart stroke; but if a small portion be broken off from the small end or tail, the whole immediately flies into minute particles with a loud snap. The exact explanation of this curious circumstance is not yet known; but it appears due to the following cause. When the melted drop falls

into water, the outside is, of course, hardened first, and the inside is cooled more slowly. The outside, being first cooled, contracts upon the interior (§ 9), and thus binds its particles together in a condition unnatural to them. On the other hand, the particles of the interior, being more slowly cooled, retain their elasticity, and are prepared to set themselves free, when anything impairs the firmness of the coating in which they are bound together. Many of these drops burst in the water, in the act of cooling; the elastic force of the interior being sufficient to overcome the compression of the exterior. But in those which do not, a very slight disturbance of the outside coat, especially at the tail-end, is enough to give the superiority to the interior, so as to occasion the sudden separation of its particles. It would seem as if the *suddenness* of this disturbance were an essential condition of its effect; for the tail may be slowly ground away on a wheel, without the breaking of the drop. By careful annealing, a very high degree of elasticity may be given to glass; and it has lately been introduced as the material of the balance-springs of chronometers, for which it seems to have some advantages above steel.

73. The effects of change of temperature, in varying the degree of hardness and elasticity, are most important in regard to steel; since to this influence we owe the power of applying steel to so great a variety of useful purposes. The greatest hardness is given to steel, by heating it to a white heat, and then suddenly cooling it, by plunging it into mercury. Although water is commonly employed for this purpose, the hardness which the steel receives from being plunged into it is not so great as it derives from being immersed in mercury; probably because, owing to the less power which water possesses of conducting heat, the cooling of the steel is less rapid in it. In its state of greatest hardness, steel is scarcely fit for any purpose in the arts and manufactures; since it is so brittle that its points or edges are broken by a very slight resistance. But by heating it to a lower temperature, and then gradually cooling it, this extreme hardness may be reduced, and a greater amount of elasticity substituted for it. Thus, if it were heated to a red

heat, and allowed to cool very gradually, it would be rendered soft again. If heated to a less degree, and cooled gradually, it would retain more of its hardness, and would be less elastic; and various degrees of these properties may be communicated to it at will, by varying the degree of heat to which it is raised, after having been completely hardened at first.

74. This process is termed *letting down* or *tempering;* and the workman is guided in the effects he wishes to produce, by the changes of colour which the surface of the steel exhibits (if first brightened) at different temperatures. The first tint which it shows, when gradually heated, is a light straw colour; this deepens as the heat increases, and presents a shade of red; the red increases, so that the colour first becomes orange, and then a reddish purple—a blue tint making its appearance; the blue increases with the heat, and the colour changes first into a violet blue, next to a bright blue, then into a full blue, and lastly into a deep blue verging on black, after which the next change is into the glow of red heat. These colours guide the workman as to the point at which he shall stop for the purpose he designs. Thus the hardest steel is used for little else than the making of dies for coining; and, as already explained, these are hardened after the impression is made in them. The steel of the hardest files is but slightly let down. The first shade of yellow indicates that the second heating has been carried sufficiently far for lancets, and other small surgeons' instruments, in which the keenest possible edge is required; the straw-colour is the guide to the tempering of razors and penknives; the first red tint for scissors, shears, and chisels, in which greater tenacity is required; the violet for table-knives, in which a certain amount of elasticity is more desirable than the hardness which would give a very fine but brittle edge; the full blue for watch-springs and small fine saws; and the deeper blue for large saws and coach-springs. When heated beyond this point, steel is scarcely harder than iron, and it loses its elasticity; this is well known to every one who has heated a piece of watch-spring, or the point of a penknife, in a lamp or candle. In many manufactories of steel instruments, the *temper* is given by immersing the articles in a bath of mer-

cury or oil, the heat of which is regulated by a thermometer. As it is known what degree of heat corresponds with each tint on the surface of steel, the operation can be performed with more certainty and convenience in this way, than by heating each article separately over a charcoal fire, as was formerly done; for the workman has only to heat the bath and its contents up to the required point, and allow it to cool gradually; by which any number of articles, that are to receive the same temper, are equably heated and gradually cooled.

75. Most other metals are acted on by heat and cold in somewhat the same manner, though in a less degree. Copper, however, forms a remarkable exception; for its properties are just the reverse of those of steel and glass. When it is cooled *slowly*, it becomes hard and brittle; but when cooled *rapidly*, soft and malleable. This curious property is possessed in a yet more remarkable degree by an alloy, composed of 4 parts of copper and 1 of tin, which has been long used by the Chinese (under the name of tam-tam) in the construction of gongs and other musical instruments. The mode of rendering it malleable has been known in Europe of late years only.

76. *Strength of Materials.*—Closely connected with these subjects are the inquiries into the strength of the different materials used for constructing buildings, ships, machines, &c., which are of such great practical importance. Many details in regard to them, however, would be out of place in a work of this description; but the general principles, which should be the guides in the selection and arrangement of materials, may be advantageously explained.

77. We have already considered the property of cohesion, as it acts in producing *tenacity*, or resistance to an *extending* force; but we must now advert to the power which different bodies possess, in various degrees, of resisting a *compressing* or crushing force. This does not at all correspond with the amount of tenacity; for in metals the degree of resistance to a compressing force is much greater than the tenacity; whilst in wood it is much less. A cube of that kind of cast-iron which is known as gun-metal, a quarter of an inch each way, requires a weight of

10 tons to crush it; from which it may be calculated that, as each side of the cube is 1-16th of a square inch, the compressing force required to crush a cube of an inch every way must be 160 tons. No other material, on which experiments have been made, exhibits a power of resistance equal to this. The ordinary kinds of iron, however, have a much less resisting power; but still it is many times greater than the tenacity. Thus, the same cast-iron, of which a bar an inch square will resist an extending force of from 6 to 8 tons, will resist a compressing force of from 37 to 48 tons. The power of resistance varies much, however, in proportion to the *length* of the portion experimented on. The cube is the form which best resists crushing in every direction; but if a square pillar be compressed in the same manner, by a force applied to its ends, it yields to a much inferior crushing force. Thus, whilst a cube of cast-iron 1-4th of an inch every way, will sustain a force of 72 tons per square inch, a pillar an inch high, and 1-4th of an inch square (such as would be made by piling four of the cubes together), will only resist 45 tons per square inch. In all cases where a certain height is passed, the rupture takes place by the *sliding* of one part upon another, and not by the breaking up of the whole mass, which occurs when it is completely crushed. In stone, the power of resisting pressure is greater than its tenacity or power of resisting extension; Aberdeen granite is the strongest yet experimented on, as it bears 5 tons per square inch (or about 1-8th of the pressure which good cast-iron will sustain), whilst compact limestone bears 4 tons, Portland stone only $1\frac{1}{2}$ ton, and brick from $1\frac{3}{4}$ to $\frac{3}{4}$ of a ton. But wood offers a far less resistance to a crushing force acting in the direction of its fibre or grain, than it does to an extending force. Experiments on this subject are somewhat uncertain; but it seems to be the result of those which have been made, that a bar of oak an inch square, which will bear an extension of 5 tons (nearly that of ordinary cast-iron), will not bear a compression of $1\frac{7}{10}$ ton; and that elm and deal, which will bear an extending force of 6 tons, will not bear a compressing force of more than from $\frac{1}{2}$ to $\frac{4}{5}$ of a ton. These results vary, however, according to the length of the piece to which

the pressure is applied; as the parts of a piece of wood have even more tendency to separate by sliding over each other, than have those of a piece of iron.

78. Hence we see that, for resisting *extension*, wood would be much preferable to iron, provided it were equally durable, on account of the far inferior weight of a beam required to possess a certain tenacity. Thus, deal has only 1-15th the weight of *cast*-iron, though it has considerably more than half the tenacity; and 16 bars of it would weigh only the same as one bar of *wrought*-iron, although they would have together more than the tenacity of three. Now it is evident that a structure may be composed of parts which are in themselves of great strength; and yet it may be so heavy, that it may not be able to bear its own weight. Several iron roofs have fallen in from this cause; accidents of this kind, at the Brunswick Theatre and the Conservatory at Brighton, attracted much attention at the time of their occurrence. It would be impossible to construct such roofs and bridges of iron, as have been built of timber; since the downward pressure of the fabric goes on increasing in greater proportion than its power of resistance, when it passes a certain size; so that no additional strength given to its parts can make up for this. The danger of decay in timber is of course a great obstacle to its employment in bridge-building; but if it should prove that this is prevented by the Kyanizing* process now employed, wood will probably come into more general use for this purpose. It has been much employed in America, where timber is plentiful and cheap; and also in Germany. The longest bridge of one arch in the world is probably that over the river Schuylkill at Philadelphia, the span of which is 340 feet. There are other timber bridges in the United States, of 250, 200, and 194 feet; and some fine bridges of similar dimensions have been recently constructed in Germany. The longest bridge of one arch ever built was probably that constructed in 1778, over the river Limmat, near the Abbey of Wettengen; it was above 390 feet in span. This wonderful bridge was destroyed

* This process, so named after its inventor, consists in steeping the wood for some time in a solution of corrosive sublimate.

in the campaign of 1799. The widest arch that has yet been constructed of iron is the one at Bishop's Wearmouth, near Sunderland, the span of which is 250 feet; the span of the centre arch of Southwark bridge is 240 feet. These very nearly approach the limit at which iron can be advantageously applied.

79. In the actual *practice* of construction, it is not at all safe to calculate upon the amount of pressure or strain which certain materials have been found by experiment to be capable of sustaining; and this for several reasons. It has been already stated that a strain of a less violent kind than that which is necessary to produce rupture by extension, will often make such a change in the internal arrangement of the particles, as very greatly to weaken the resistance of the body; and the same is true of compressing force. Thus one half the pressure which will crush stone renders it liable to chip, so that a slight blow will break it; and the tendency of overloaded stone to yield increases with time. It is not considered safe, therefore, to load stone with more than 1-6th of the pressure that crushes it. Moreover, allowance must always be made for the probability that, from some accidental cause, some one part will be weaker than the rest; and if this give way, the whole structure may be destroyed. Again, the structure may be liable to an occasional or accidental increase of pressure, which, if it were already loaded too nearly to its limit, would produce its downfall; and allowance must therefore be made for this: as also for the probability that some slight giving way, or *settlement*, of the parts that have to sustain the greatest pressure (as the walls or foundations in the case of a roof, or the abutments of a bridge), may diminish the stability of the whole. For the same reasons, fire-arms of any description should always be *proved*, before they are taken into use, by being subjected to a much greater force than that which they are intended to sustain in being discharged.

80. The strength of a fabric depends, however, not only upon its materials, but upon the mode in which they are arranged; as may be easily shown. Let us suppose a beam of wood, resting on a solid base at each end, to have a weight placed in its middle,

sufficiently great to bend it downwards. Now it is obvious that, in thus bending, the upper side must be compressed, whilst the lower side is extended; and it is further obvious that there must be a portion of the thickness of the beam which is neither compressed nor extended,—all *below* it being *extended,* and all *above* it being *compressed.* The part of the beam where it is neither extended nor compressed, is termed its *neutral* line; and this will obviously afford no resistance to its bend, so that it might be removed without weakening the beam. On the other hand, the amount to which each portion above or below the neutral line is compressed or extended by a certain bend of the beam, is greater in proportion to its distance from it; hence the most distant portions, or those forming the upper and lower edges of the beam, exert the greatest resistance.

81. This principle is illustrated by the well-known fact, that an ordinary plank of timber will bear a much greater weight, when placed with its edge beneath it, than when the weight bears on its broad side. Hence the joists on which the floor of a house rests, and which have to sustain its downward pressure, are set with their edges upwards. Suppose a joist to be 10 inches broad and 2 inches thick;—then if the weight rest on its broad side, it would support just the same pressure that would be sustained by 5 separate bars, each 2 inches square. Supposing the neutral line to run through the middle of its thickness (which it does very nearly), then the parts where the greatest resistance is required are nowhere more than 1 inch from that line. But when placed so that the weight shall bear upon one of its edges, the joist will sustain a much greater pressure than that which would be borne by 5 separate joists of 2 inches square; for supposing 5 such bars to be laid one upon the other, and firmly fastened together, so as to represent one beam;—then the middle one of the five would have just the same strength that it would possess if separate from the rest, since its surfaces would be each at the distance of an inch from the neutral line of the whole mass; whilst the bars next above and below would have one of *their* surfaces at the distance of 1 inch, but the other at 3 inches from this neutral line; and the bars forming the upper and lower

edges of this compound beam would have their two surfaces at 3 and 5 inches distance from it;—hence the resistance to strain or pressure, which the middle bar would exercise, being the same as if it were alone, each of the bars next above and below it would exercise about twice the resistance, and each of those forming the edges of the beam about four times the resistance; so that the whole beam would bear a pressure equal to that which about (4+4+2+2+1) 13 separate bars would sustain.

82. We see, further, that as the power of resistance is greater in proportion to the distance of each part from the centre or neutral line of the beam, the beam will be strongest when the greatest part of its material is disposed at the greatest distance from that line. Thus let the central bar, in the compound beam just referred to, be removed, the loss will only be that of the support given by a single bar; but let the same amount of material be employed in increasing the thickness of the edges, it will then have more than four times the effect; and thus the more we withdraw from the centre and add to the edges, the stronger will the beam become, provided that the parts of it are still sufficiently connected together.

83. In this and other similar cases, the beam is adapted to sustain a pressure in two directions only; but it is easily seen that it is the same principle, which gives the great advantage in strength to *hollow* cylinders (such as those which form the stems of grasses, or the bones of most animals,) over *solid* cylinders containing the same quantity of matter. For the central axis of such a cylinder will be nearly its neutral line; so that, in whatever direction force be applied, it will meet with an equal resistance; and this resistance is greater, the further removed the resisting points are from the centre. By the cylindrical form of such stems, their parts are all firmly bound together, so that no one can yield without the rest being also affected; and thus they have all the advantage that results from the distance of their walls from the centre, without any of the weakening that would be perceptible in a beam, of which the upper and lower parts should be separated by a considerable space, in consequence of the pressure on the upper part causing it to yield without affecting

the lower. Supposing that the wall of the hollow cylinder were made thicker on one side than on the other;—the resisting power would be greatest in the direction of this increase; and it will be more increased by making this addition on the outside than in the interior. This is exactly the principle which is followed in the construction of bones: where the strain is likely to be equal in all directions, they are uniformly thick all round the central hollow; and their diameter is such as to give a strength equal to about three times that, which the same material would possess if disposed in a solid cylinder: but we frequently find a ridge running along the bone in one direction, which serves for the attachment of muscles, and also gives the bone increased power of resistance in the direction of the strain which these muscles will exert upon it. Of such a bone, the accompanying figure represents a cross section; and its power of resisting a force applied above or below is greater than its power of resisting a force applied sideways, in proportion as the distance *a b* exceeds the distance *b c*. But had this ridge occupied part of the hollow interior of the bone, it would have been comparatively useless.

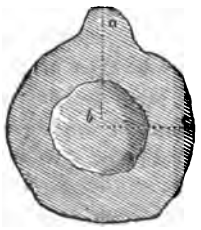


FIG. 8.

84. From what has been said of the relative actions of the upper and lower portions of a wooden beam, on which a weight is placed, in resisting pressure and extension respectively, it follows that, if we cut the beam half through from above downwards, with a thin saw which shall not remove any of its material, we do not diminish its power of resistance in that direction; for the action of this part of the beam, being that of resisting compression, may be performed as well if it were composed of several pieces lying close together, as if the resistance were made by one continuous piece. The contrary is, of course, the case in regard to a similar cut made on the under side; for the whole efficacy of that part of the beam, which entirely consists of its tenacity, is thus destroyed. It has been found by experiment, that a beam may be cut from its upper side to about 5-8ths of its depth, without its strength being impaired; this depth, there-

fore, may be regarded as the exact position of the neutral portion of the beam; all above it being compressed, and all below it extended, by pressure on its upper surface. It has also been found that, if the saw-cut be filled up with harder wood, which resists compression better than that which it replaces, the strength of the beam is increased rather than diminished.

85. The arrangement of a certain quantity of material in such a mode as to give the greatest strength, is a problem of even more importance in the construction of iron beams than in regard to those of timber; for we cannot, in the latter, conveniently take away material from one part, and add it in another where it may be more efficacious; and as the increase in weight produced by what may be comparatively superfluous is small, it is not worth while to remove it. In the casting of iron beams, the form is entirely at the pleasure of the maker; and thus, according to the kind and direction of the strain which the beam is to receive, he may, by the knowledge of the comparative powers of tenacity and of resistance to compression which his material possesses, adapt the form of his beam so as to give the greatest strength with the least material; and thus there is not only a saving of material, but of weight, which is a matter of equal importance. This has been lately proved by the experiments of Mr. Hodgkinson of Manchester, the results of which are of great importance. The neutral line is not, in an ordinary iron beam, as it is in a wooden, a little below the middle; for, as the metals resist compression much better than extension, a much smaller quantity of material will be required above the neutral line than below it. Hence the most advantageous form of beam is one which, when cut across, shows this figure. It may be described as consisting below of a flat portion, or flanch, from which rises an upright rib; and at the top of this there is a flanch much smaller than the first. An iron beam thus shaped, will

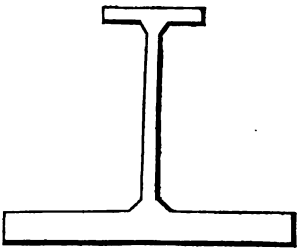


FIG. 9.

have several advantages over one shaped like an ordinary timber joist.

86. In the first place, the greatest part of its material is disposed at a distance from the neutral point; and therefore it will afford a much greater resistance than if it were nearer the centre, as it would be if the flanches were smaller and the rib thicker. But if these flanches were equal, we should not have a beam of the greatest strength which the material can produce; for such a beam would break, by the extension of its lower side, under a far less degree of pressure than would be required to break it by compression of the upper. It is obvious, therefore, that a large part of the upper flanch might be removed without the least injury to the strength of the beam; and if the same amount be added to the lower flanch, its resistance will be greatly increased. The strongest form is obtained when the arrangement is such, that at the instant the beam is about to break by extension on one side, it is about to break by compression on the other. So long as this is *not* the case, the strongest form is not attained; since, on the side which is least disposed to break, there is superfluous material, which may be advantageously transferred to the other. It appears from Mr. Hodgkinson's experiments, that a beam of cast iron, shaped as in the figure,—in which the section of the upper flanch (or surface of compression) is to the section of the lower flanch (or surface of extension) as 1 to 6,—has nearly twice the strength of a beam containing the same amount of material, but having two flanches of equal size. The large iron beams or *girders*, which are much used in cotton-factories and similar buildings, are now usually cast upon this principle; and it is found that, to gain the same strength, one-fourth less iron is now required, than was employed in the best forms of girders before Mr. Hodgkinson's experiments.

87. There is another principle, however, to be attended to, in devising the best shape for a cast-iron beam. The strength of a beam of uniform shape from end to end, estimated by the load it will carry without breaking, is different in different parts of its length. A load which will break it, if placed upon its

middle point, may be very well borne by it near its extremities. It may be shown by mathematical investigation, that the effect of a load is exactly proportional to the product of its distances from the two ends of the beam. Thus, let a beam be 12 feet long; a load placed at the distance of 1 foot from one end will be 11 feet from the other; if it be at 2 feet from one end, it will be at 10 feet from the other; if at 3 feet from one end, it will be at 9 feet from the other; if at 4 feet from one end, it will be at 8 feet from the other; if at 5 feet from one end, it will be at 7 feet from the other; and if in the middle, it will be at 6 feet from each end. Hence the relative pressures of the load in these different positions will be expressed by the numbers 11, 20, 27, 32, 35, and 36; and in order that the beam may be equally able to bear a load in every part, it should have its flanches made much broader at the middle than at the two ends, so that the quantity of material at each part may bear an exact proportion to the resistance it has to afford. The form of the flanch which fulfils this condition, is given in the subjoined figure. Its border on each side is a parabola; which curve will

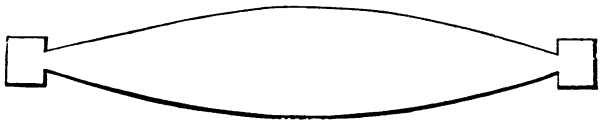


Fig. 10.

make its breadth at the several distances just mentioned, exactly proportional to the series of numbers which represent the pressure at each point. Both the upper and lower flanches should have this form; the lower one being made six times as large as the upper.

CHAPTER III.

ATTRACTION OF GRAVITATION.

88. We have hitherto spoken only of the attraction which exists at very minute distances among the particles of matter, causing those of the same mass to cohere, and causing those of different substances to adhere. We have now to consider the attraction which exists among the masses themselves, and which gives rise to a force that is constantly and universally operating throughout nature. We are best acquainted with this force as it acts on the surface of the earth, causing all bodies which are not supported to fall to the ground; and also causing them to exert that downward pressure, when they are not in motion, which we term their *weight*. But it is the very same force, acting on a scale which becomes sublime from its stupendous greatness, which keeps the moon from leaving its regular course round the earth, which causes the earth to maintain its regular distance from the sun whilst moving onwards through space, and which acts in like manner, not only upon all the other planets and their satellites belonging to the solar system, but upon those bodies placed at an almost infinite distance from this globe of ours, which we commonly term the fixed stars.

89. The concise statement of the mode in which this attraction acts, constitutes what is known as the *law of gravitation*; and this is expressed in the following terms. *All masses of matter attract one another, with forces directly proportional to the quantity of matter contained in each, and inversely proportional to the squares of their distances from each other.* This rule is very easily comprehended and remembered, when it has been once sufficiently explained; and a few illustrations of it will be

given for that purpose. We will suppose that there were two masses of matter, A and B, of equal bulk, and placed at any distance from each other; and that these masses were not influenced by any other force than that of their mutual attraction. This, then, would draw them towards each other; and as the two bodies are equal in size, they will produce equal effects upon each other, so that they will move towards each other with equal quickness, until they meet in a point midway between their original situations. But supposing their bulks to be unequal, as if A were 3 and B 2, then the attraction which A has for B will be expressed by the number 3, whilst that of B for A will be only 2; and this unequal attraction will show itself in the respective distances which each will travel towards the point of meeting; for B will be drawn by A's force through a distance of 3, whilst A will be moved by B's force only 2. Or if A were 100 and B only 1, then the attraction of A for B would be a hundred times that of B for A; and B would move towards A a hundred feet, yards, or miles, whilst A only moved one towards B. The first part of the law of gravitation,—that the attraction acts with a force directly proportional to the quantity of matter contained in the bodies, is not, therefore, by any means difficult of comprehension; nor is the second part more so.

90. If two masses of matter be very near each other, their mutual attraction is much stronger than if they were at a distance; and it diminishes in a certain proportion as the distance increases. This diminution follows a regular law,—being exactly proportionate to the square of the distance. Thus, supposing A and B are at the distance of 1 foot, and we represent their mutual attraction by the number 1, the attraction between A and B, when removed to a distance of 2 feet, will be only 1-4th of the previous amount; for, since 4 is the square of 2, the distance having been doubled, the attraction is diminished in the proportion of 4 to 1. Or, supposing them to be separated to 10 times their original distance, the force of attraction would be diminished to 1-100th. The term "inverse proportion" is a simple method of expressing, that one quantity increases as another diminishes, or diminishes as another increases. And

thus it may be said, that the force of attraction diminishes as the square of the distance increases; which is only a less concise mode of expressing the general fact set forth in the second part of the law of gravitation, just stated.

91. A little consideration will make it evident why not only the attraction of gravitation, but likewise all other influences—such as light, sound, electricity, &c.—emanating in like manner from a centre, should diminish in their intensity in proportion as the squares of the distances increase. Suppose a candle to be in the place A, and to send its light through a square hole, B P O *v* at the distance A B from the candle: then that light will cover the

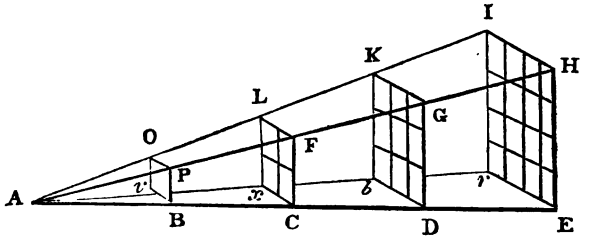


Fig. 11.

space C F L *x* upon a paper held at the distance of A C, or twice A B; and will spread itself over D G K *b* at the distance of A D or three times A B; and over E H I *r* at the distance A E or four times A B. Now it is evident that the space C P L *x* is four times B P O *v*; that D G K *b* is nine times the same amount; and that E H I *r* is sixteen times. Thus the distances being as 1, 2, 3, 4, the spaces covered by the same amount of light issuing from A, are, at these distances respectively, 1, 4, 9, 16,—the squares of the preceding numbers. But it is evident that if the amount of light which passes through the opening 1 at the distance 1, be spread over the surface 4 at the distance 2, it will be only one-fourth as strong or intense; when spread over the surface 9 at the distance 3, it can only be one-ninth as intense; and when spread over the surface 16 at the distance 4, it will be only one-sixteenth as intense. Therefore, the light diminishes in intensity as the squares of the distances increase

and it is easy to see, that the same explanation will apply to the attraction of gravitation, or to any other influence emanating like it from a particular centre.

92. We are most familiar with this force in its action between the earth and bodies upon or near its surface. The bulk of the earth is so great, in respect to that of any mass of matter which comes within its immediate influence, that, although they really have a mutual attraction for each other, the effect of the earth's attraction in producing the motion of the body towards it, is alone visible to us (§. 238). It will be shown, however, that the earth itself is influenced by the sun, very much in the same manner that any small body on the earth's surface is influenced by it;—that is, if left completely to itself, it would fall towards the sun, whilst the movement of the sun towards it would be extremely trifling, so much greater is his bulk than that of the earth. In all instances of attraction exercised between spherical bodies, that attraction operates to draw their *centres* together; and thus, the earth being a sphere (or nearly so) the small bodies on its surface are attracted towards its centre. Let us suppose that the centre of the earth were hollow,—then a body placed exactly *in* the central point would remain there without support: it would, in fact, have no *weight* or downward pressure; for it would be equally attracted on all sides by the hollow sphere around it, and would remain in the same point so long as it might be allowed to do so. But if drawn out of its position, it would immediately tend to return towards it, and would thus exercise pressure, or have a degree of weight; since, when removed from the centre, the attraction of the different parts of the hollow sphere around it would no longer be equal, and it would be drawn towards the direction in which it is acted upon by the greatest mass.

93. All bodies which fall towards the surface of the earth, move (if influenced by the force of gravitation alone) in lines which would meet in its centre; and therefore two bodies, let fall at a short distance from each other, do not really descend in parallel lines, as they appear to do. This will be made evident in the accompanying diagram. Let A B C be a part of the earth's

circumference, and *a* and *b* be the points from which the bodies commence their descent; then, by the attraction of these towards the centre O they will fall in the lines *a* A and *b* B, which are not parallel, but, if prolonged, would meet in the centre; and a body let fall at the distance of a quarter of the earth's circumference, as from *c*, will fall in the line *c* C, which is precisely at right angles to *a* A. But the smaller the space A B in proportion to the distance of the point O, the more nearly parallel will be the lines *a* A and *b* B; and, in fact, the spaces between bodies on which we can make such observations are so small in proportion to the distance of the earth's centre, that we may, for all practical purposes, speak of the earth's attraction as exercised in parallel lines over bodies upon or near its surface. (§. 168.)

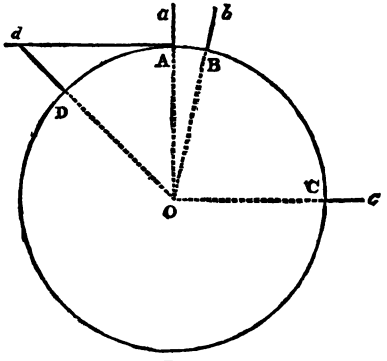


FIG. 12.

94. It is by the equal attraction of the earth for all the particles of a liquid, and the parallel direction of the lines of this attraction, that the surfaces of small collections of liquid are always *level;* but when the surface of the liquid is sufficiently large, we see by its curvature that the attraction is *not* exercised in lines exactly parallel. Thus, in Fig. 12, let A *d* represent a straight or level surface of liquid; it is obvious that the attraction of the earth for different parts of it would not be the same; since at A it is exercised at the distance O A; whilst at *d* it operates through the much greater distance O *d*. As the attraction for the different parts of the liquid, therefore, is unequal, and as the particles of the liquid are free to move upon each other, they cannot retain their perfect level, but will change their position until all of them are at an equal distance from the earth's centre, so that the earth's attraction for all of them will

be equal. When this is the case, the curve of the liquid will follow that of its surface A D; and this we know, in fact, to be the nature of the surface of any large collection of water—such as a lake—still more evidently that of the sea; although in a small surface, such as that of a vessel of water, or a pond, the curvature is too small to be seen. It is on account of this curvature, that the masts of vessels which are approaching the shore are first seen, and their hulls last; and that the hulls first disappear as the vessel is receding, the masts being seen for some time afterwards. The curvature of the earth's surface must be allowed for in cutting a canal; for if the bottom of the canal were cut to an exact level, the water would be deep in one part and shallow at the other, owing to the curvature of its surface. In order that the water may be of the same depth everywhere, it is necessary that the bottom of the canal have the same curvature; and for this, it is necessary to depart from the dead level about eight inches for every mile.

95. It has been already stated, that the attraction of gravitation diminishes as the square of the distance increases; and consequently the downward pressure of a body—that is, its weight—would be less if it were removed to a height above the earth's surface, sufficient to make the difference perceptible. This can scarcely be proved by ascent in a balloon, or up a mountain, as a height of 3 or 4 miles is not sufficient; but it is clearly shown to be the case, by experiments made on different parts of the earth's surface, whose distance from the centre varies considerably. The earth is not perfectly spherical in form, but is flattened at the poles, like an orange; and the distance of its surface from the centre is only 3950 miles at the poles, whilst it is 3963 at the equator. This difference of 13 miles is quite sufficient to produce a marked effect upon the attraction of bodies towards the earth. It has been found by accurate experiments, that the motion of a pendulum, which is entirely dependent on the force of gravity, is much slower over the equator, where the attraction is less, than it is in temperate regions, or near the poles, where the attraction is greater, in consequence of the shorter distance from the centre. And from experiments of this kind it

may be calculated that *at* the poles, a body which would be drawn downwards (or towards the earth's centre) at the equator, with a force of 590 lbs., will be drawn down with a force of 591 lbs.

96. It is obvious that, if we balance a body in a pair of scales, with weights equivalent to 590 lbs., the scales will remain balanced in whatever part of the earth's surface we try the experiment; because the variation in the earth's attraction, and in the downward pressure it produces, would be the same for that which each scale contains. But if the experiment could be tried with a spring-dial weighing machine, we should find that the same weight, which would cause the hand to point to 590 lbs. at the equator, would cause it to point to 591 lbs. at the poles; since the downward pressure of the body is increased without any change being made in the resisting power of the spring. The same fact may be illustrated in another way. Let us conceive a weight E suspended at the equator by a string, which should pass over a pulley A, and be conducted over the other pulleys, such as B, C, along the earth's surface, till the other end hang down at the pole, where a weight P is suspended by it. Now if the weights E and P were such as would balance each other when placed in an ordinary pair of scales, they will not pull equally upon the string under these circumstances; for the weight E will be more powerfully drawn by the earth's attraction at the poles, than the weight E will be at the equator; and, in fact, a mass of 590 cubic inches at P will balance a mass of 591 cubic inches at E. The difference is greatly increased by the action of centrifugal force, as will be explained hereafter (§. 220).

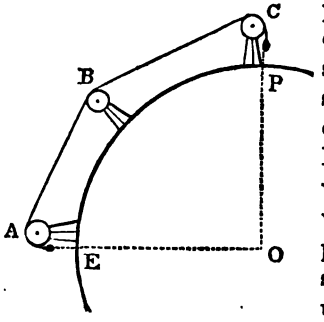


FIG. 13.

97. The law of attraction might seem to be contradicted by the fact, that there are bodies which have a tendency to rise from the surface of the earth instead of descending towards it; thus we see the smoke, on a fine calm day, gently

ascending to a considerable distance from the earth's surface; and a balloon rises with force enough to carry up two or three human bodies of full weight. This, however, does not prove that the earth has no attraction for these bodies, but merely that it has more attraction for another body, the atmosphere. The pressure or weight of the atmosphere over any part of the earth is continually varying, being sometimes 15 lbs. on every square inch, and sometimes only 14 lbs.; this variation produces the different heights of the barometer, on which we rely as indications of the weather. Now when the weight of the atmosphere is great, which it is in fair weather, its lower part is more pressed together, and thus becomes denser; and in this state it is attracted by the earth with greater force than smoke is, so that the latter rises through it, for precisely the same reason that a cork will rise from the bottom of a vessel of water to its surface. But when the density of the atmosphere is less, which it usually is in rainy weather, the smoke is as heavy as the air, and thus has no tendency to rise through it. A balloon when filled with hydrogen gas, or even with heated air, weighs far less (that is, presses far less towards the earth) than the same bulk of cold air; and thus it will not only rise through it, but will even carry up any heavy bodies which do not make the whole weight equal to that of the same bulk of air. Its ascent is limited, however, by the gradual decrease in the weight of the air, arising from its diminished density; for any measure of air on the top of a mountain, weighs much less than the same bulk of air at a level surface; as the latter, being more closely pressed together, contains a much greater number of particles. The balloon will rise, therefore, until the weight of the same bulk of air is so much diminished, as to be no greater than its own; and it will then rise no more, the attraction of the earth for the balloon, and for the mass of air which must be displaced for the balloon to rise, being equal.

98. Hence we see that there are no bodies which can be properly regarded as having no tendency to approach the earth; all the instances in which bodies are disposed to rise from its surface, being due to the inferior attraction which the earth has for them, when compared with that which it has for the same bulk of

the fluid through which they rise. Thus a balloon filled with hydrogen gas rises through the atmosphere; but a balloon filled with common air would rise through heavier gases; these gases would rise through water; a vessel filled with water would rise through strong sulphuric acid (oil of vitriol); and a vessel filled with this would rise through mercury, which is the heaviest fluid we know. It will be shown hereafter (§. 253) that the different rates at which different bodies fall to the ground, are due to the *resistance* of the air against their surfaces; and that, if the air be removed, *heavy* and *light* bodies (as they are commonly termed) will fall in the same time.

99. The powerful attraction which the earth, in consequence of its vast bulk, has for all bodies upon its surface, prevents us from perceiving, under ordinary circumstances at least, the attraction which they have for each other; and yet this exists in a degree exactly proportionate to their respective bulks. If two balls of lead were placed at a little distance on a smooth surface, they would have exactly the same tendency to move towards each other, as if they were not attracted by the earth; but this tendency is prevented by the friction which would be produced if they were to begin to roll, and which their mutual attraction is not sufficient to overcome. Or if they were suspended by strings from the ceiling of a room, they could not approach each other without moving out of the line in which the earth's attraction causes them to hang; and this attraction their mutual attraction is not powerful enough to overcome. But when two bodies are floating on a liquid, there is no obstacle but the resistance of the liquid to prevent their mutual attraction from bringing them together; and accordingly we see two floating bodies attracted into contact with each other, when they have been brought sufficiently near for their mutual attraction to overcome that resistance. It has been determined by accurate observation, that a plumb-line suspended in the neighbourhood of a lofty mountain, does not hang in a direction quite perpendicular, but is drawn a little to one side by the attraction of the mountain. But as the greatest mountain upon the earth's surface is little more than the *sixty-millionth* part of its bulk, its attraction for the lead ball must

evidently be very trifling, compared with that which the earth has for the same solid; and consequently the deviation of the plummet will be *very* small.

100. The attraction of solid bodies for each other may be shown, however, by balancing a small mass in such a manner that it may be moved by the slightest influence; and then bringing another mass into its neighbourhood. Thus, if we suspend two equal balls of lead from the opposite extremities of a slender bar of wood, and suspend this at its centre by a very fine wire, the only force required to move the balls will be that which suffices to produce a slight twisting of the wire that suspends the rod (§. 63). Now if a large mass of lead be brought into the neighbourhood of each ball, (the rod having been previously hanging at rest,) its attraction will cause the rod to turn round, until the small balls have come into the same line with the large masses. If the masses be now moved a little further, the balls will follow them; twisting the wire, from which the rod is suspended, still more. Now, as the force which is required to produce any amount of alteration in the position of the rod can be ascertained in another way, the actual amount of the attraction which the masses exercise over the balls may be determined; and this may be compared with the attractive influence which the earth has over them. From the knowledge of these facts, the *quantity of matter* in the earth may be compared with that in the masses of lead; for the weight of the earth is just as much greater than that of the masses of lead, as the force with which it attracts the balls exceeds that with which the masses attract them,—proper allowance being made for their difference in distance. When the actual weight of the earth is known, we may estimate its density as compared with water; since we may easily calculate the weight of a globe of water of equal size. And from the weight and density of the earth, that of other planets and of the sun may be ascertained.

101. This experiment is known as that of Cavendish, by whom it was first devised. In order to perform it accurately, very great care is necessary to prevent various sources of error. Thus, the rod and balls must be inclosed in a case, so contrived

as to prevent their motions from being affected by currents of air; and the whole apparatus must be inclosed in a room which has a uniform temperature, and in which there are no other openings than those absolutely necessary for making the observations. The necessity for these precautions is evident from the fact mentioned by Mr. Baily (under whose direction the experiment has lately been repeated), that the slightest change of temperature on one side of the case in which the rod and balls were suspended, would produce an immediate effect upon them. The average result of 2004 experiments lately made, is that the weight of the earth is about $5\frac{2}{3}$ times as great as that of a globe of water of the same bulk*. Thus, we may regard the apparatus of Cavendish as a scale in which the earth, sun, moon, and planets have been weighed.

* Philosophical Magazine, August 1842.

CHAPTER IV.

CENTRE OF GRAVITY.—STABILITY OF SOLID MASSES.

102. It has been shown that, by the attraction of the earth for bodies near its surface, all their particles are drawn downwards, in lines which, at short distances from each other, may be regarded as having exactly the same direction. We have also seen that, in liquids, this attraction is exerted over the different particles *separately;* so that they arrange themselves in conformity with it. But in solid bodies, the particles are so coherent, that the combined attraction of the earth for them all is diffused over the whole mass. Now it is very desirable that we should be able, in some easy and simple manner, to discover the whole effect of this force;—or, which is the same thing, to find out whether any single force, acting from any one part of the body, would produce the same result. It is in this manner only that we can determine (except by a very tedious process) a great variety of questions, in which the influence of gravity, or the earth's attraction, upon bodies of various forms and sizes is concerned. That such a method *may* be followed, will be evident from a few simple considerations; by which also we shall be led to the mode of arriving at it.

103. Let us first consider the attraction of the earth for a body, whose particles are arranged in a simple regular line, A C B. The earth's attraction for all those particles separately will draw them downwards in the directions A *a*, B *b*, C *c*, and in other lines between these and parallel to them. If we

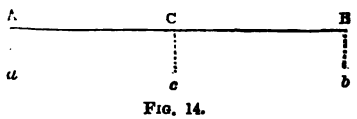


Fig. 14.

conceive the particles to be divided into two equal parts at the point C, then it is obvious that the earth's attraction for those on the side A C will be equal to its attraction for those on the side B C; and that thus the whole line will remain at rest, if it is suspended at the point C; like the beam of a balance in whose scales there are equal weights. It is also evident that we may represent the whole of the forces acting on A B, by a single force acting directly downwards from C,—that is, in the line C *c*. But if the line be supported at any other point, it will not remain at rest; because the weight of its two sides will then be unequal. Next, let us consider the case of a solid body of regular form, such as the parallelogram A B C D, whose downward pressure is exerted in the directions A C, E F, and B D; this body we can readily divide into two equal halves by the line E F. The weights, or downward pressures on each side of this line, will then be equal; so that, if we support the body by a prop placed under the point F, or hang it from the point E, it will have no tendency to change its position; whilst, if it be supported on either side of E F, the weights of the two divisions must be unequal, and the body will sink, therefore, on the heavier side. We learn from this, that the force which represents the combined attraction of the earth for all the particles of the body, acts in the line E F.

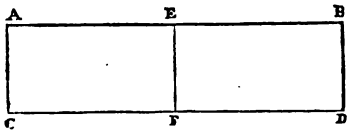


FIG. 15.

104. But we wish to ascertain the exact point in that line, round which the weight of *all* sides of the body is balanced. Let the position of the body be changed, so that its downward pressure comes to be in the direction of the lines A B, G H, and C D; then, in the same manner, we see that it will remain at rest if supported in the line G H, and in no other; so that the point O, which is common to both the lines E F and G H, is one

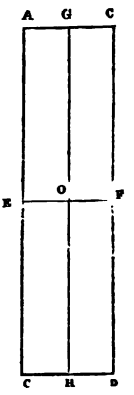


FIG. 16.

round which the weights are balanced, whether the body is supported in the direction E F or G H. It would not be difficult to show, that in whatever other direction such a body is supported, the direction of its pressure will always be in a line passing through O, in consequence of the division of its weight into two equal parts acting on each side of O; and thus the point O is one round which the weight of *all* sides of the body is equally distributed; so that, if this point be supported, the body will remain at rest.

105. The case appears different in regard to a body of irregular form, such as that represented in Fig. 17; but it is in reality precisely the same. Let A B, be a solid body placed within the influence of the earth's attraction, which will be exercised upon its particles in the downward direction. It is easy to understand that, in spite of its irregularity of form, we might divide it, by a line C D, into two parts whose weights should be equal; so that if a prop were placed under D, or the body were suspended at C, the two parts, C A and C F, would balance each other, and the body would remain at rest in its present position. This will only be the case, however, when the body is so placed that C D is perpendicular; so that the pressure of the body at C or D may be directly downwards. For supposing the body to be so placed that D C were somewhat inclined, and D E were the perpendicular drawn upwards from the point of support; then D C, being out of the perpendicular, would no longer represent the combined action of the weights on each side of it; and, as D E does not equally divide the body, it would sink on the heavier side E B. But we may find many

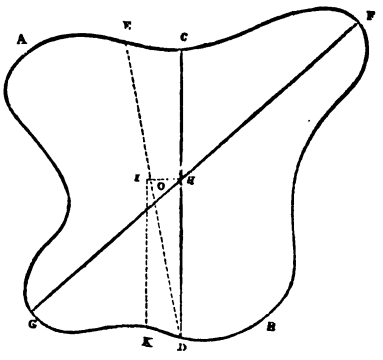


FIG. 17.

other lines, on each side of which the weights are equal,—for instance, F G; and when the body is so supported that F G is perpendicular, it will remain at rest, as in the former case. Hence the point H, where C D and F G cross each other, is that round which the weights of the several particles, composing the two sides of the body in each case, are equally balanced; and it can be shown that, in *any* other position of the body, the line which divides it into two portions of equal weight will pass through the same point H. Hence the weight of the body is equally distributed in all directions about this point; so that, if *it* be supported, the whole body will remain at rest in any position in which it may be. This point is called the *centre of gravity.*

106. It will be evident, from what has been stated, that the centre of gravity of any body may be found by a very simple process. For if we suspend the body freely by any point, it will take such a position that its centre of gravity will be somewhere in a line drawn perpendicularly downwards from that point; since into that line the centre of gravity has just the same tendency to come, as a weight at the end of a line has to cause that line to hang perpendicularly. The simplest mode of finding the place of such a line, is to let drop a plumb-line from the point from which the body is suspended; and to mark the direction of the line upon the surface of the body. If the body be suspended from another point, and the same thing be again done, the second line will be found to cross the first; and the point of intersection is the centre of gravity.

107. In all our reasoning respecting the attraction of the earth for bodies of any form whatever, we may regard its whole influence as being exercised towards this point of the body,—its centre of gravity; and it is obvious that the tendency of this point will always be, to assume the lowest position that the mode in which the body is supported will admit of. Thus, if the body A B (Fig. 17) be suspended from C, the centre of gravity H will hang immediately below it; since in any other position it must be higher. If the body be supported upon a prop placed beneath D, and the centre of gravity be in the perpendicular

D C, the body will be at rest; because its weight, concentrated in the point H, presses directly downwards upon the point of support. But suppose the body to be a little thrown to one side, so that the centre of gravity is moved out of the point H into the position I; then, since its pressure will still be exerted directly downwards, or in the line I K, the body cannot remain in such a position, since there is no support beneath K; and it will move round, until the centre of gravity hangs in the perpendicular line *below* D. Where the point of support is very small, there will be considerable difficulty in placing the body so as to remain at rest upon it; since the slightest change of the centre of gravity from that position in which it is directly above the point of support, will cause it to upset. Thus we may endeavour for a long time to cause a pencil to stand upright upon its point, without being able to succeed; because the slightest shake or breath of air will disturb it, even if for a moment we should have succeeded in poising it. A taller and heavier body, such as a walking-stick, is more easily poised; since its centre of gravity will not be so easily disturbed by slight causes. It is much easier to keep such a body poised upon the hand, than to balance it upon a fixed point; because, if we see it inclining towards one side or the other, by slightly moving the point of support in the same direction, we again bring it under the centre of gravity.

108. When the point of support is extended into a surface or base, the body is more stable; because the centre of gravity must be made to move through a much larger space, to cause the perpendicular let fall from it (which, expressing the direction of the earth's attraction for the whole body, is called the *line of direction*) to fall beyond the point of support. Thus the solid A B C D (Fig. 18) stands

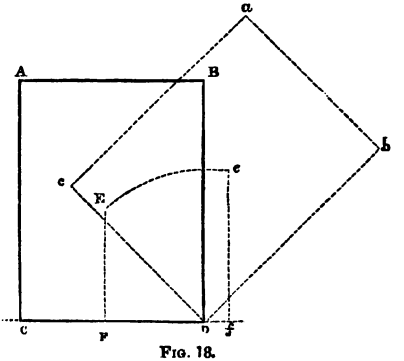


FIG. 18.

securely upon the base C D, because it must be moved into the position *a b c* D, before the centre of gravity E is thrown into such a position *e*, that its perpendicular *e f* falls beyond the point D, so as to give it a tendency to upset; any less degree of disturbance, that does not move the centre of gravity so far as *e*, will not upset the body; since it will tend to return to its original position, so long as the line of direction falls anywhere upon C D. If the body were so inclined that the perpendicular from *e* were to fall exactly upon the corner D on which it would then be resting, it might remain poised there, but would be upset to one side or the other, by the slightest touch. It is a general rule, then, that so long as the line of direction (or perpendicular let fall from the centre of gravity) falls within the base or surface of support, so long will the body stand at rest, because the centre of gravity will be supported; but that, if the line of direction fall without the base, the body must fall, because its centre of gravity is not supported. Thus, let A B C D (Fig. 19) be a building which has very much sunk in at one side, and let E be its centre of gravity; the building will yet stand, because the line of direction falls within the base. But suppose that an addition were made to the top of such a building, so as to carry it up to F G; it would then necessarily fall, since by the change in the place of the centre of gravity from E to *e*, the line of direction is now made to fall outside the base.

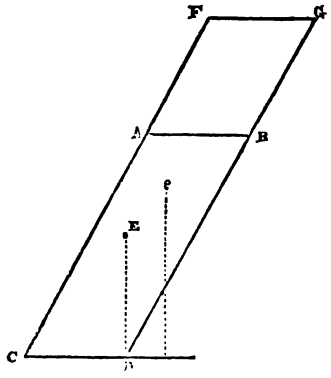


Fig. 19.

109. It is obvious that the broader the base in proportion to the height, the more steadiness will the body have; since the further must its centre of gravity be moved by any disturbing force, for the line of direction to be carried beyond it. Thus, a thick book will stand more firmly than a thin one; and any

book will be overturned with much less difficulty when standing on either of its edges, than when lying on its side. A body may be high and yet it may be very steady, on account of the lowness of its centre of gravity resulting from its form; thus the centre of gravity of a pyramid, twice as high as it is broad at the base, will be at only one-fourth of its whole height from the ground; for the part below this line contains as much matter as the lofty portion above; and it will therefore be as steady as a cube having the same base, and only half the height of the pyramid.

110. It is not necessary that the body should rest on the whole surface of its base; thus a table standing upon four legs is just as steady as if it stood upon a basement covering the whole space between the legs, because its centre of gravity must, in either case, move through the same space before the table can be upset. The difference in the degree of resistance afforded by bases of different forms to forces tending to displace the body, is well illustrated by the various degrees of steadiness with which we stand erect, in different positions of the feet. The base on which we may really be considered as resting, is included by lines joining the points of the toes and the backs of the heels; since, as in the case of the table, the line of direction must fall beyond that space, before an upset can occur. If a man stand with one foot exactly in front of the other, so that the toes and heels of both shall be in the same straight line, he will have a long base, but a very narrow one; so that, although he might not be easily pushed forwards or backwards, he would be very readily overset by a force acting on one side. On the other hand, if he were to stand with his heels touching one another, and the toes of each foot directed outwards,—so that the two feet should again be in the same line, but this line run from side to side instead of forwards and backwards,—the base would be wide, but would be so short from back to front, that the body would be very easily upset by a push either in front or behind. If a person stand upright against a wall, touching it with his heels, and then turn his toes out until the whole line of each foot is against the wall, he will not be able to remain upright;

for if the back be not allowed to project behind the base, the front of the body will project in front of it, so as to throw the centre of gravity so far forwards, that the line of direction will fall in front of the feet; hence, unless one foot is planted a little forwards, or the toes be turned towards the front, the body must fall. The position of the feet in which the body will most firmly resist forces coming in any direction, is that in which they are planted at about a foot distant from one another, with the toes pointed slightly outwards; but if it be desired to resist a force acting from the front, one of the feet should be advanced before the other, so as to extend the base in that direction; whilst if the force comes from the side, the feet may be more widely separated, so as to increase the breadth of the base. It is from the habit of walking on the decks of ships whilst these are continually rolling from side to side, that sailors acquire their peculiar gait; for they are well aware by experience, that the wider the space between their feet, the greater resistance they will be enabled to make to the force, by which they would otherwise be upset to one side.

111. Many familiar illustrations might be given of the fact, that the steadiness of different bodies having the same base depends upon the height of their centre of gravity. Thus, on a highway, level in the middle, but sloping towards the sides, let there be a coach on the level, and a waggon on each slope at the side, one loaded with hay and the other with stones; their centres of gravity *a*, *a*, *a*, and lines of direction *ab*, *ab*, *ab*, will be as shown in the figures. It is obvious that the hay-waggon must upset, because the line of direction falls without the base; the coach is very secure, because the line of direction falls in the very middle of its base; and the stone-cart, though the centre of pressure is low

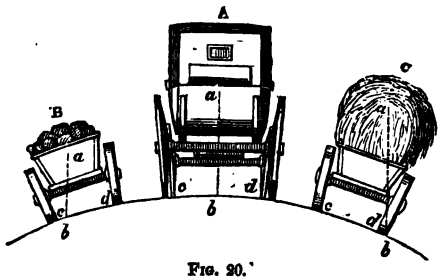


FIG. 20.

down, is not very secure, because the line of direction falls very near the outside of the base. For the same reason that the hay-cart here represented must upset, a stage-coach without inside passengers, but heavily loaded with passengers and luggage on the roof, is very insecure; because its centre of gravity is high, so that the line of direction is thrown beyond the base with a less inclination of the road; and it will be rendered much more secure by causing some of the outside passengers to take the inside places, which will lower the centre of gravity considerably.

112. The attitudes of men and animals are the result of the instinctive efforts, which cause them so to alter the position of different parts of their bodies, as to bring the centre of gravity immediately above the point of support. These efforts result involuntarily from the feeling of loss of balance and of tendency to fall, which affects us disagreeably whenever the position of the body has been so changed as to render it insecure. Thus a porter bearing a heavy load on his back so inclines himself forward, as to bring the common centre of gravity of his body and load within the space bounded by his feet. If he stood upright, even though the weight of his load were small in proportion to that of his body, the line of direction must come to the ground behind his heels, and he must fall backwards. On the other hand, a woman carrying a heavy basket in front, by a strap passing over her shoulders, throws the upper part of her person back, in order to prevent the centre of gravity from being brought so far forwards, that the line of direction shall fall in front of the toes. In stooping to place a weight upon the ground or to lift it up, we necessarily throw forwards the head and shoulders; but to compensate this, the rest of the body is bent backwards beyond the line of the heels; and the greater the weight in the hands, the more curved will the body be. But as in stooping, even with this compensation, the centre of gravity is necessarily brought much forwards, the body is more easily overthrown than it is in the upright position; and a person standing with his back to a wall, will be unable to pick up a piece of money placed between his feet, and will infallibly fall forwards if he attempt to do so.

113. For the same reason, persons with a great accumulation of fat in the front of their bodies, walk with the spine erect, and the shoulders even thrown backwards, that the centre of gravity may be kept above the middle of the base. A man carrying a bucket of water in one hand, inclines his head and shoulders to the other side; and may even derive still further assistance in maintaining his balance, by throwing out the other arm sideways to its full extent. But if he carry an equally heavy bucket in each hand, the position of the centre of gravity is not changed towards one side or the other, and he stands firmly when quite upright. Or when a load is carried *on* the head, the centre of pressure being still in the middle line of the body, there is no tendency to bend it over towards either side. When a person stands upon one foot, his base is so small that he is easily pushed down; and he finds it difficult to keep the different parts of his body so completely at rest, as to maintain his balance for any length of time. If he find himself falling towards one side, he stretches out his hand or his leg towards the other in such a manner, as to bring the centre of gravity again over the base, and thus to restore his balance. A rope-dancer is supported upon a much narrower base, and has consequently much more difficulty in keeping the body balanced; but he is provided with a long pole loaded with lead at each end, and this he moves to either side, according as he finds that he has a tendency to fall towards the other. It will be easily understood, when the principle of the lever is comprehended, that the weights at the end of the pole will act much more readily and advantageously in restoring the balance, than if they were attached to the sides of the body, or were simply carried in the hands.

114. The fact that a body stands more firmly, the wider its points of support lie apart from one another, enables us to understand the advantage which is often gained by arranging the same materials in a hollow rather than in a solid structure. Thus, let us suppose a table to be resting upon a square column placed beneath its centre; if this column be not enlarged at the base, the table will be very easily upset; but if it were separated into four pieces which should spread out at the bottom, whilst they

still meet at the top, the table would be rendered secure in proportion to the distance of its feet from each other. It is just on this principle that a lofty column may be made to stand much more securely, by arranging the materials for its pedestal in a large hollow base, rather than in a solid one of much smaller dimensions; as is well exemplified in the accompanying figure of Lord Nelson's monument on the Denes at Yarmouth.

FIG. 21.

115. It is not always that the centre of gravity of a body is in some part of its substance. Thus, in a ring, the centre of gravity is the centre of the circle, and not in any part of the ring itself; but this point has the characteristic property of the centre of gravity,—the tendency to settle itself at the lowest possible point. For from whatever point we hang the ring, its centre will always be in the perpendicular line from that point; and we can only poise the ring upon a support when its centre is perpendicularly above the point of support. If we could imagine the central point to be connected with the different parts of the ring, by threads so fine, that their weight need not be taken into account, the ring would be balanced in any position upon a point supporting the central knot in which the threads meet.

116. A body which remains in a state of rest, in consequence of its centre of gravity being supported, is said to be in *equilibrium*—a term derived from the Latin, and denoting equality of weight; because in that position, the weight or pressure which tends to upset it, is balanced by that which causes it to maintain

its place. But its equilibrium may be such that it is easily disturbed,—a greater or less amount of force so changing the place of its centre of gravity, that the line of direction shall fall without the base, so that it must turn over. This equilibrium is said to be *unstable*. On the other hand, a body may be so placed that it may be disturbed by a very small amount of force; and yet it shall always have a tendency to recover its state of equilibrium, because its centre of gravity, when the disturbing cause is withdrawn, constantly has a tendency to return to the same situation. Its equilibrium is then said to be *stable*. Now the conditions of the two are very easily understood. A body is in stable equilibrium when its centre of gravity is in such a position, that it cannot be made lower by any change in the place of the body; and consequently it will always tend to return to its original place, when it has been disturbed in such a manner as to raise the centre of gravity. But a body is in unstable equilibrium, when its position may be changed, though with some difficulty, into another in which its centre of gravity is lower. Thus, a book placed upright on either of its edges, is in unstable equilibrium; because we can push it over upon its side, thus causing it to take a position in which its centre of gravity shall be much nearer the surface which supports it; and when in this last position, its equilibrium is stable, because there is no other in which its centre of gravity can be still further lowered.

117. A globe, whose centre of gravity is its centre, can be made to rest in any position upon a level surface; because no change of position can bring the centre of gravity nearer to the point of support: but it can be displaced by the slightest force, because, being always in equilibrium, it has no more tendency to remain in one position than in another. On the other hand, an egg placed upon either of its ends will not stand; because the centre of gravity is brought much lower by the rolling-over of the egg until it lies upon some part of its middle, which therefore is the position it will seek, and in which alone it will rest. The equilibrium of a body which may be readily disturbed,—as that of a sphere, a cylinder, or an egg-shaped body,—without any more stable position being gained, is said to be *neutral*. When

any such body is rolled upon a level surface, its centre of gravity moves forwards at a rate exactly the same with that, at which the point of support is changed; and the line of direction let fall perpendicularly from the former always drops upon the latter. But if the surface be not plane, the body will roll down it; because the point of support is *not* below the line of direction; and the centre of gravity, not being supported, has a tendency to descend.

118. If the globe or cylinder, however, be not of the same kind of substance throughout, but be heavier in one part than in another, it will not remain at rest in any position, but will always seek to take that, in which its centre of gravity—now thrown out of its former position—may find its lowest point. In this manner, bodies may be made to seem to run *up* an inclined plane. Thus, suppose a solid wheel of wood to have a mass of lead imbedded in one part of its edge; that part will tend to take the lowest place, since the centre of gravity is much nearer to it than to any other; and in order that it may do so, the wheel will even roll up a hill, the centre of gravity being really lowered whilst the centre of the wheel rises. In the same manner, if two equal cones be fixed together by their bases, and this double cone be laid upon two edges inclined towards each other like those of the letter V, it will have a tendency to run from the narrow end towards the broad one; since, in doing so, the centre of gravity of the double cone, being in the middle of their united bases, will be lowered by the change of the points of support from the large part of the cone towards the small, as it runs from the part where the edges approach one another to that at which they are separated. And if these edges be somewhat raised at the wide end, the cone will still roll towards it, appearing to run up hill, but really descending.

119. It is often necessary to ascertain the position of the common centre of gravity of two or more bodies connected together; and this is not difficult, when we know the centres of gravity of the several bodies themselves. For let A and B be two globes connected by a rod (which we shall suppose, for the sake of simplifying the explanation) to have no weight in itself:

the centres of these globes are their centres of gravity; and, as we may regard all their weight as acting from those points, the same reasoning, which enabled us to understand that the forces acting upon the different parts of any one body may balance each other round a certain point, leads to the belief that a point may exist in which we may regard the actions of A and B as jointly and equally exercised. This point is evidently somewhere in the line A B, which joins their centres; and it is easily determined (as will be shown when the principle of the lever is explained) by dividing the line A B into two such parts, that the distance of each body from the point C shall be proportional to the weight of the other. Thus, suppose A to weigh 6 lbs. and B 1 lb., then A's distance from C must be to B's distance as 1 to 6; for the weight 6 at the distance 1 will have an action equal to that of the weight 1 at the distance 6. Hence, as the weights A and B bear directly downwards, and as they balance one another at C, it is evident that a support placed at C will sustain them both at rest, and will be pressed upon with the weight of both combined. It would be in this manner that we should ascertain the centre of gravity of a compound body, such as that of a man with a loaded basket on his back, when that of each of the parts,—the body and the basket,—is known; for the common centre will be in a line joining the two single centres, and will be found by dividing it in the proportion of the two weights, as in the last instance.

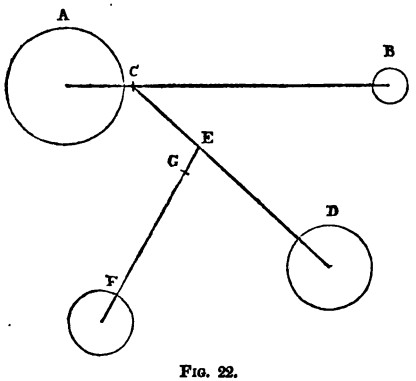


FIG. 22.

120. Supposing that a third body, D, were connected with A and B, by a rod proceeding from the point C; then the common centre of gravity of all three bodies will be in the line C D,

since the weights of A and B may be regarded as concentrated at C, and act as if a single body of their total weight were placed there; and it will be determined in the same manner as before. In like manner, if another body F be connected with the system, by a rod uniting it with the rest at their common centre of gravity, E, the centre of gravity of the four will be in the line between E and F; and will be at such a distance from F, that its weight multiplied by the distance F G, shall be equal to the combined weights of the other bodies acting at the distance G E. In this manner, any number of bodies may be connected with the system; or, in a system already existing, we may ascertain the common centre of gravity by a similar process. It will be hereafter shown that the moon does not, strictly speaking, move round the earth; but that, as the moon attracts the earth whilst the earth attracts the moon, both of them move round their common centre of gravity; and that, in like manner, the sun and the whole solar system have a common centre of gravity, round which they all perform certain motions. As the bulk of the sun so enormously exceeds that of all the planets and their moons put together, the centre of gravity of the whole system (even supposing that all the planets were arranged in one direction,) would be within the mass of the sun; and as the place of the planets is constantly varying, this centre is continually shifting its position, so that the sun is drawn a little in one direction or another, according to the situation of the planets which have most influence over it. But these changes have all a tendency to counteract each other; so that the place of the sun would be found to be the same at any period, as it was when the planets last had the same position with respect to him and to each other.

CHAPTER V.

STABILITY OF STRUCTURES COMPOSED OF MOVABLE PARTS.—FRAME-WORK.—ARCH.

121. We have hitherto considered the stability of bodies of determined forms; and have examined the circumstances in which they will stand or fall. We have now to inquire into the conditions that are requisite for the stability of bodies, of which the form is liable to change; and to examine how those parts may be best connected together, so as to render them stable. It is upon such principles only that arches can be securely built, or that any kind of framing which is to support a heavy pressure can be constructed.

122. The two most important classes of bodies of variable form are:

I. Those which are composed of rods or cords, the parts of which are connected together at their angles, but movable about them. Of this kind are all sorts of frame-work; whether supported on a solid base, like that of a roof; or suspended from fixed points, as in the case of the suspension-bridge; or intended to resist pressure in every direction, as that of a ship.

II. Those which are composed of solid bodies in contact, whose surfaces are not held together in any other way than by their mutual pressures, or by imperfect adhesion. Of this kind are all buildings in which the arch is employed, especially bridges, domes, &c.

123. The principle which should govern the construction of frame-work, in order to insure its stability, is a very simple one. If we take three strips of wood, and pin them together at their ends in such a manner as to form a triangle, we shall

find that the shape of that triangle cannot be altered by pressure in any direction. But this is not true of any other figure. For let four rods be pinned together in the same manner, so as to make a four-sided or quadrilateral figure of any shape, the form of this figure may be altered by pressure in any direction. If the opposite sides were originally parallel, they will always remain so; but their inclination to the other sides may be made to vary in any degree, if the joints are movable. Still greater changes may be made in figures having a larger number of sides. Hence we see that, in the construction of framing of all kinds, this simple principle is to be followed;—the parts of the framing are to be so united together, as to form a series of triangles, so that, as every one of these is immovable, the whole shall be also. Hence a framing so made will be able to resist any amount of pressure consistent with the strength of its materials. A very familiar illustration of this principle may be seen in the construction of the common gate. If made of upright and horizontal pieces only, it would have been liable to change its form, provided the framing were at all movable at the points where the pieces unite; and the addition of any number of pieces in either of these directions would only have rendered it stronger by increasing the number of joints. But a single piece fixed *diagonally* will prevent any alteration in shape from taking place, even though the joints are by no means firm; for this converts the framing, from a set of movable parallelograms, into a series of triangles, every one of which is immovable. The shape of the gate cannot be altered without altering the length of the diagonal; and this change is prevented by the additional piece.

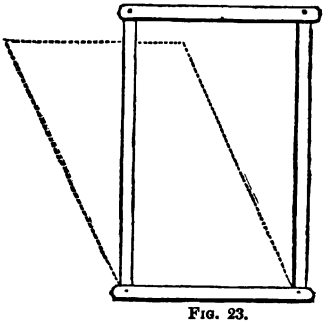


FIG. 23.

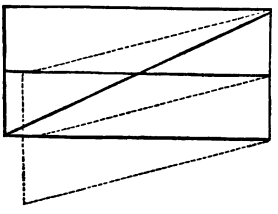


FIG. 24.

124. This principle is very extensively adopted in framing of all kinds. Thus in frames for wooden houses, such as are now frequently constructed in this country to be sent out to newly-settled colonies, or in those of oil-cloth factories (which are usually constructed on the same plan, as great size is required for them, without the strength which is necessary in mills), these diagonal pieces may always be seen, crossing the parallelograms which would be left between the upright and horizontal timbers. Although such parallel framing rests on a solid and extensive base, and would be able to resist a vast amount of pressure acting directly downwards, it would not be able to stand against a force acting sideways, as that of the wind; and however great the mass of timber employed, the building would have no security against such a change of form as that represented in Fig. 24, without the employment of diagonal braces. This principle has only been applied to ship-building, within a comparatively recent period; it was introduced by Sir R. Seppings, to whom we owe several other important improvements in Naval Architecture.

125. Frame-work may be very securely made upon the same plan, even when the centre of pressure is not directly supported. Thus, the large frames that form the *centering* on which stone arches are to be built, are constructed upon this principle, when the arch is of great size. Various plans may be employed for this purpose; but in all of them the frame-work is so arranged, as to form a series of triangles. In the accompanying figure is represented one mode of so constructing the centering of a large arch, that it shall not require any support but that given by the piers on which the arch is to rest. Up to the time when the arch is completed, its whole weight rests upon the centering; and as any alteration

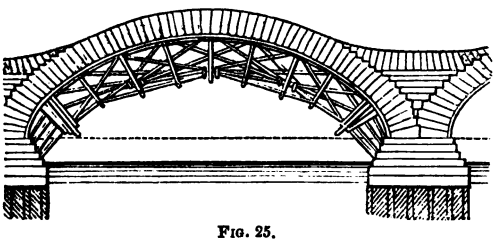


FIG. 25.

in the form of the latter will displace the stones of which the arch is to be composed, and as every such displacement will greatly weaken the arch, it is of the utmost importance that the whole should be a perfectly immovable structure. The same plan of framing is adopted in the construction of wooden and iron bridges and roofs. The accompanying figure shows the kind of framing adopted in a timber-bridge erected over the Thames at Walton. A much simpler plan may be adopted, however, when the main beams of timber are so connected as to have the properties of the arch; for the triangular framing is then required only to keep these in their proper position. It is on this plan that the largest timber-bridges have been constructed, such as that which crosses the Schuylkill, at Philadelphia (§. 78).

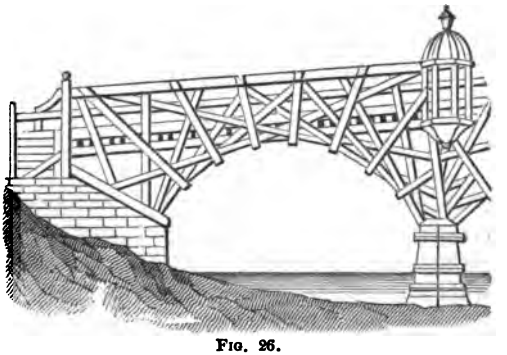


FIG. 26.

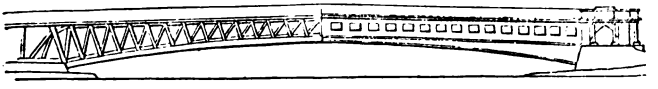


FIG. 27.

Such an arch may be made so flat as to have no inconvenient rise in the centre; that of the Schuylkill bridge is only 20 feet in a length of 340 feet.

126. The same kind of frame-work is employed in smal domes, in which neither masonry nor brick-work is required; and as every part is connected not only with those above and below it, but with those on each side, the pressure is so distributed over the whole, that the top need not be closed in, but may be left open for the purpose of affording light (Fig. 28). There is a tendency, however, at the lower part, as in the arch (§. 137), to

the separation of the points of support; since the pressure will cause them to fall asunder, if they are not properly kept together.

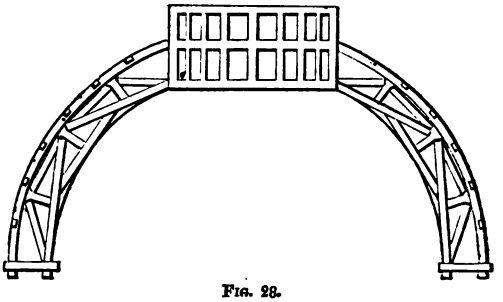


Fig. 28.

In order to give security enough, therefore, especially in the construction of large domes, it is found requisite either greatly to increase the thickness of the lower portion, or to bind it round with a strong hoop or chain. The former plan is that which

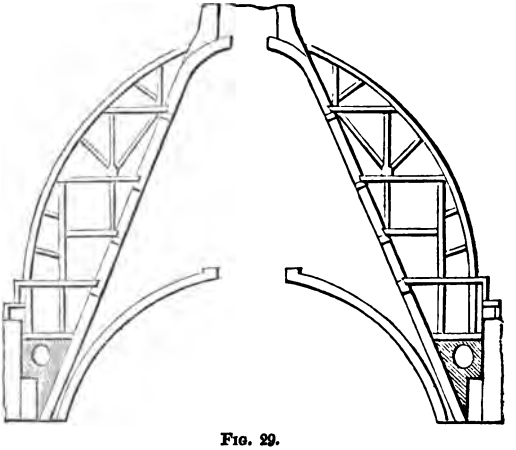


Fig. 29.

was employed in the ancient dome of the Pantheon at Rome; the latter in St. Peter's at Rome,* and St. Paul's in London.

* Owing to this dome having been too hastily built, various parts of it subsided in an unequal degree; and the iron bands placed around it were thus broken. New ones of much larger size have been since placed round the dome; but there are even now some large cracks in it.

The construction of the dome of St. Paul's, however, differs from that of most others: the external dome is of timber, the framing of which rests upon a conical tower of brick; and upon the top of this also, the lantern, with its ball and cross, is supported. It is between this brick cone and the external dome, that the visitor ascends to the top; and the framing which is represented in the preceding figure may be thus seen by any one. There is an interior dome below the cone, having little connection with it; but its outward pressure, as well as the weight of the external dome, has a tendency to force out the bottom of the brick cone; and this is prevented by a strong chain, which encircles it near its base.

127. When a system of rods or bars, freely jointed together, is hung from fixed points, it will naturally assume the position in which its own equilibrium is stable,—that, namely, to which it will return if disturbed. Thus we see that a chain composed of a number of short links, or a simple cord, supported at both ends, hangs in a regular curve, which is termed the *catenary* (from the Latin *catena*, a chain); and that, if disturbed, it will return to this curve again. The mode in which this curve is occasioned by the downward force acting equally upon the parts of the chain or cord, can only be shown by a complex mathematical process; and it is sufficient, therefore, here to state, that the property of the curve is such, that the tension or strain is the least at its middle point, and the greatest near the points of suspension, where it is equal to the whole weight of the cord. Hence, in order to be of equal strength throughout, the chain or cord should be made thicker near the points of suspension than elsewhere.

128. A good illustration of the operation of the catenary curve, is seen when a ship is at anchor. The cable or chain which connects the ship and the anchor takes this curve; and the force which acts upon the ship is not only the same, as if the resistance which the anchor gives were directly applied at the spot from which the cable issues, but is increased by as much of the weight of the cable itself as is not sustained by the water. Hence there is a great advantage in the use of iron chain-cables over hempen

cords; for the latter, being nearly of the same weight with the water, are buoyed up by it, so that their action on the ship is very small; and if it be impelled by the wind or waves, it drags directly upon the anchor. But, from the much greater density of the iron chain, it is but little supported by the water; and nearly its whole weight is exerted in drawing the ship nearer to the anchor, so that the chain usually hangs in a deep curve. Hence when the ship is driven by the wind or waves in such a manner as to put an increased strain upon the cable, the large resistance produced by the weight of the chain has to be gradually overcome, before the ship can pull directly at the anchor. The advantage, therefore, in substituting iron for hempen cables, consists in this,—that the latter have little tendency to prevent those sudden jerks, which occur when the ship is *brought up* by the tightening of the cable, and which not only strain the ship, but endanger the firmness of the anchor; whilst the former gradually check its motion by their own weight, so that it often ceases before the chain is tightened so much as to permit such a strain. Another illustration may be found in the action of a horse in towing a barge; the cord which connects them is a catenary; and the effective force by which the barge is moved forward, is that which the horse exerts, with the weight of the cord added to it.

129. The properties of the catenary have acquired much additional importance of late years, from the general use of that curve in the construction of suspension-bridges. These consist of massive chains, raised near each end on firm supports, and strongly fixed into the ground at their extremities. At equal distances from each other, along the whole length of these chains, there are fixed to them vertical rods, to which the roadway is hung. The weight of the bridge itself is so great in proportion to that of any load which would be likely to pass over it, that the curve which depends on the former, will not be altered in any perceivable degree by the latter; and thus the structure may, for all practical purposes, be regarded as perfectly stable (that is, up to the limit of the tenacity of its materials), although it is really capable of being disturbed by a very slight force. There

is one kind of force, however, which would seriously affect any suspension-bridge; this is, the heavy and measured tread of a large body of men. When it is remembered that their steps are so many *impacts* all acting together, and that the impact even of a light body produces greater disturbance than the simple pressure of a heavy one (§. 206), the danger of serious consequences, from the passage of a comparatively small number of soldiers over a suspension-bridge, is easily understood to be much greater than that which would result from the crowding of the whole surface of the bridge with persons standing still. But such a curve may be easily disturbed by a force acting from below, or on the sides; and in this manner suspension-bridges may be greatly injured by the wind, if it should be violent, or if it should blow for some time in the same direction. The Menai bridge is sometimes disturbed by the wind in such a manner, that the road-way is lifted at some part into an elevation, which travels like a wave from one end to the other. The length of this bridge, between the points of suspension, is 560 feet; and the height of the roadway above the water is 100 feet. The total weight suspended by the chains (including their own) is about 489 tons. This bridge will be surpassed by the one in progress of erection over the Avon at Clifton. Its length between the points of suspension will be about 700 feet; and its height above the river about 180 feet. The total length of this bridge, however, will not equal that of the Menai bridge, which is carried on arches for some distance.

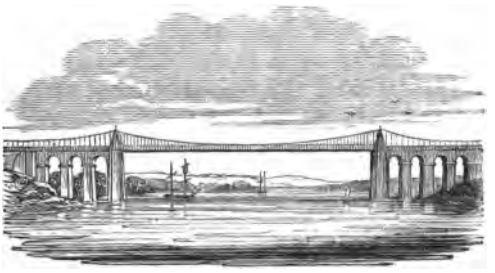


FIG. 30.

130. The principle of the suspension-bridge has been long employed in India and in South America. Many remarkable rope-bridges are described by the celebrated traveller Humboldt,

as establishing communications across the mountain torrents of the Andes. These are of the simplest possible construction; being made of ropes (twisted from the fibres of the *Agave Americana* or American Aloe) stretched across from point to point as tightly as possible; and upon these ropes is laid a flooring or roadway of bamboos. There is some danger incurred in passing these bridges, in consequence of their tendency to swing to one side or the other, according as the weight is distributed upon them; this may be partly prevented by ropes attached to the middle of the bridge, and stretching diagonally towards the shore. It is by means of a bridge of ropes of extraordinary length, which may be traversed by loaded mules, that the South Americans have succeeded (within these few years) in establishing a permanent communication between the towns of Quito and Lima; after having failed in constructing a stone bridge over a torrent that descends between them. In India similar bridges have been lately constructed of canes, which grow to a length of above 200 feet, and serve as admirable ropes; their durability, however, is yet to be tested.

131. It has been recently proposed to adapt the principle of the suspension-bridge to the *roofing* of very large buildings; and there is much to be said in favour of the plan.

132. *Arch.* The stability of the arch depends upon the proper arrangement of the pressures on its several parts, so that they may all resist and counteract each other. The subject is of so complex a nature, however, that a full account of it would be quite unsuitable to the present work; and all which it seems desirable to embrace here, are some of the general principles which have been obtained by mathematical investigation. To understand these, it is necessary to be acquainted with the parts of which an arch consists. In the accompanying figure,

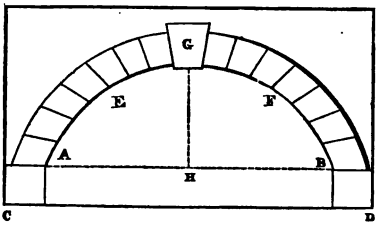


FIG. 31.

A C and B D, are the supports or *piers* of the arch, which is said to *spring* from A and B; A E and G F are the *flanks;* and G the crown. The wedge-shaped stones forming the arch are termed *voussoirs;* the highest of these at G is the *key-stone;* the lower line of the arch is called the *intrados*, and the upper the *extrados*. The line A B is the *span* of the arch; and G H is its *height*.

133. The first question to be considered in the theory of the arch, is, how it may be so built, that all its parts shall remain in equilibrium, even though unconnected by cement, and not held together by the friction of their surfaces. It is not difficult to understand the general principle on which this must be done. Let us first suppose a number of flat stones or of books to be piled one upon another, and a heavy weight to be pressing vertically on the highest; this pile will be perfectly secure, so long as no force acts upon it in any other direction; since the weight of its several parts, and the pressure acting on the highest, unite to force each of them directly downwards on the one below,—that is, in a line exactly perpendicular to the surfaces in contact, so that neither will have any tendency to slide upon that below it. But if the surface of one or more of the stones be inclined, the perpendicular force, together with the weight of the stones themselves, will give this stone a tendency to slide upon the one below it; and the pile will thus yield, if the steepness of that surface be sufficient to cause the pressure to overcome the friction, produced by the sliding of one upon the other. Or let the pressure upon the highest stone *not* be in a direction exactly downwards; it will then give the stones of the pile a tendency either to slip or to turn upon one another, according to the direction of the force; but this tendency will be partly neutralized by the weight of the stones themselves, which will tend to keep them in their places.

134. We have seen that, when the direction of the pressure produced by the earth's attraction is such that it falls within the base or surface on which the body is resting, the body will be stable; and the same is true of any other pressure. Thus, if we lay a box upon the table, and press it with the hand in such a direction, that the line of pressure still falls within the base

on which it rests, this new pressure will not tend to upset it, but will only cause it to slide along the table, if the surfaces are smooth enough. But if the direction of the force be such that it does *not* pass through the base or surface of support, then the box will have a tendency to turn over, unless kept down by its own weight. In order to ascertain the combined effect of these two forces, we have only to find their *resultant*, according to the principles to be explained in the next chapter (§. 163); if this resultant falls within the base or surface of pressure, the box retains its position; but if not, it is overturned. It is further to be remembered that, the nearer the line of pressure falls towards the *centre* of the base or surface of pressure, the more secure will be the body from being upset.

135. The same principles apply to pressure in *any* direction. Thus we might place a pile of books, or of flat stones, in a slanting or even a horizontal position, and retain them there by pressure applied against their surfaces; provided this pressure be sufficient to overcome the effect of their weight. The most advantageous direction of such pressure would be that, in which the resultant of this and of the downward pressure (or weight of the pile,) should be perpendicular to the surfaces in contact.

136. This principle embodies the whole theory of the simple arch; and it is only in the application of it to practice that a difficulty exists. It may be shown that, in an arch shaped like that in Fig. 32, the *line of pressure* (which is the *resultant* of the different forces acting on the arch) will follow the curve S R R′ S′. Thus at the top, the pressure will be sideways and partly downwards; and will be nearly perpendicular to the surfaces of the key-stone and the voussoirs on each side of it, so that they have no tendency to become displaced. The line of pressure then bends downwards, but still falls upon some part of the surface of each voussoir, until it comes to R, where it touches

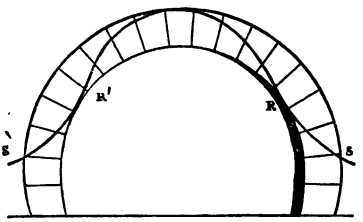


FIG. 32.

the intrados, and falls therefore on the edge of the surface of support. At this point, then, the voussoirs have no stability, and may be easily displaced by pressure applied there. From R, the line again bends outwards towards S; and at S it cuts the extrados, so as to fall completely without the surface of pressure. If such an arch were constructed of uncemented stones, therefore, it would fall by the turning outwards of the voussoirs at S and S′; the voussoirs between R and S would continue to rest upon them; but those above R and R′ would fall inwards. If this were prevented by additional pressure at S and S′, there would be no tendency to a falling-in at the crown; since with whatever force the two parts above R and R′ tend to fall inwards, the two are balanced against each other. By increasing the weight of the stones at the lower part of the arch, and by giving a proper direction to their surfaces, it is possible so to construct an arch, that it shall retain its form without cement between the voussoirs, and without any assistance from the friction of their surfaces; but such an arch, though the forces produced by its own pressure would be in equilibrium, would not be able to resist a force applied to it in any particular direction, such as that produced by a load passing over it. The shifting of the surfaces of the voussoirs would be resisted, however, by their friction; and this increases with the load that is laid upon them. It is further resisted in practice by the cement interposed between them; which, if it held them perfectly together, would render the whole as one mass.

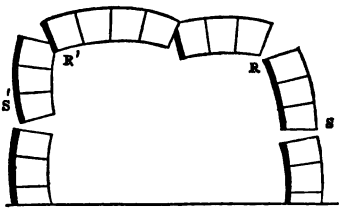


FIG. 33.

137. It is obvious that, the flatter the arch, the greater will be its outward pressure. This may be easily shown by a simple experiment. Let a thin lath of wood be bent into a curve, and placed upright like an arch, with its ends kept at a certain distance from each other by two heavy weights; it will then be found that, if the weights do not move, the lath will sustain a

very heavy pressure on the top of its curve. If the curve be *high*, the principal part of this pressure goes *downwards* through the ends of the lath, to the surface on which they rest; and the pressure against the weights is not considerable. But if the arch be nearly flat, the greater part of the pressure is exerted against the weights at the side, which may be moved from their places by a comparatively small pressure on the crown of the arch. Hence we see the necessity of great firmness in the *abutments* against which an arch rests at its sides; and this is the case whether the arch be of stone, wood, or iron. Where a bridge is composed of several arches, the outward pressure of each of the middle arches is counterbalanced by the antagonist pressure of those on either side of it; so that the pressure upon the piers on which they rest is downwards only,—the lateral pressure or *thrust* being sustained by the abutments at the extremities. Hence the piers must be so built as to support a considerable weight; and in rivers in which there is a rapid current, or in which masses of floating ice are liable to be carried down by the stream, these piers must have great solidity, in order to maintain the stability of the bridge. In general they are built with their greatest length in the direction of the stream; and with the surfaces facing the current pointed like a wedge, in order to present as little resistance as possible. Great care should always be taken in the construction of a bridge or any other arch, to procure abutments which shall remain perfectly immovable; since a very slight yielding on their parts will greatly alter the form of the arch, and may destroy its stability. In general it is so managed, that the abutment shall be part of the solid bank of the stream; but it is occasionally necessary to substitute for this, either in part or entirely, masses of solid masonry.

138. In the building of a bridge, it is requisite to consider not merely the weight of the arch itself, and the load which may have to pass over it, but also the weight of the materials laid upon the arch for the purpose of rendering the roadway as little inclined as possible. To a certain extent, this weight will be advantageous, in preventing the flanks of the arch from turning outwards (as in Fig. 33); and the greatest thickness is at the

point where the pressure may be advantageously exerted. But if the flanks be overloaded, they will be pressed inwards, and the crown of the arch will be forced up. This occurrence has more than once taken place. It happened to a bridge formerly built over the Taff in Glamorganshire, the arch of which was purposely made high, in order to avoid its destruction by the masses of timber &c., brought down by the stream when swollen by floods, which had caused the fall of a previous bridge. The persevering architect (who was an ordinary working mason) then devised a most ingenious bridge, a view of which is here given. He made the arch high as before; but lightened the weight bearing on the flanks, by leaving large open spaces in the stone-work. The span of this curious bridge, which notwithstanding its flimsy appearance, is perfectly secure, is rather more than 140 feet, and its height is 35 feet.

FIG. 34.

139. Arches may be constructed with stones of irregular form, or even with bricks; in these, however, the cement performs an essential part, since there is no provision for stability in the portions of the arch itself. The more completely the cement unites the whole into a solid mass, the more firm will be the structure. Many very large bridges and viaducts of brick have been erected in the several lines of railway recently constructed; and in many of these, the passage beneath the arch is not directly across its length, but is more or less inclined towards it, so as sometimes to form but a small angle with it. Such are termed *skew* arches.

CHAPTER VI

OF INERTIA AS A PROPERTY OF MATTER.—LAWS OF MOTION, COMPOSITION AND RESOLUTION OF FORCES, EFFECTS OF IMPACT.

140. THERE are few errors more common, than that of supposing that the movement of a body, once set in motion, ceases because its force is worn out or spent. The state of motion is as natural to a body as that of rest; and no change can be made in its state, whether the setting it in motion when at rest, or the bringing it to rest when in motion, without the application of a force. This is taught us by common experience. Thus if we put a carriage into motion by a push behind it, and a person standing in front checks its movement by pushing in the opposite direction,—the force the latter will be obliged to exert in order to check it, will be just the same as that which the former employed to put it in motion. Or suppose we cause a wheel to revolve quickly on its axle by the impulse of the hand, and presently apply the hand to check it, the hand will be struck with a force equal to that which it applied to the wheel. Or if a carriage be put in motion by horses, and they be made to stop it suddenly; as much exertion is required from them to do this, as to *back* it when at rest. A body in motion, then, has no more power of bringing itself to rest, than a body at rest has of setting itself in motion.

141. The idea that matter has a greater tendency to the state of rest than to that of motion, results from the fact, that all the motions of which we see the commencement, gradually become weaker and weaker, and at last cease altogether. Thus, when a pendulum is set swinging, the space through which it moves gradually becomes less and less, and at last it stops, if it be not

connected with any machinery to give it continual impulses. Or we set a top spinning; and this revolves gradually more and more slowly, and at last ceases. Or we roll a ball over a smooth road, and this too has but a limited movement. But in all these, as in other instances of bodies moving on the surface of our earth, there are causes which are continually acting against the movement, and are therefore constantly diminishing its force. These causes are *friction*, and the *resistance of the air*.

142. By friction is meant the loss of force which is produced by the rubbing together of two surfaces, of which one is moving over the other; it is influenced in its amount by the roughness or smoothness of the surfaces, the pressure that brings them together, the rapidity of movement, &c. Thus, if two pieces of rough board be rubbed together, the friction will be considerable, the projections of one being caught in the hollows of the other; on the other hand, if two pieces of ice be rubbed together, the amount of friction will be comparatively small. Again, if one of the surfaces of the rough boards were fixed, and the other were moved over it under a heavy load, the force necessary to make the one slide over the other would be very much increased. Or, lastly, if the motion of the two surfaces against each other be very rapid, the friction is greatly increased in amount; as is shown by the quantity of heat which is then produced, which may be even sufficient to set the wood on fire. The precise laws regulating the amount of friction, and the influence of different kinds of surface upon it, will be considered hereafter. (Chap. XII.) Now, as all bodies upon this globe have a tendency to press downwards towards its centre, it is evident that none can remain without a support of some kind; and that there must be friction between the bodies and their supports, whenever either are moved without the other. The only case that appears an exception to this, is the motion of bodies through water; but this, too, is accompanied with a considerable amount of friction. When a heavy body is moving through the air, it is continually being drawn downwards by the earth's attraction, and cannot, therefore, be said to be moving on a support.

143. The resistance of the air is another cause by which all

motion on the earth's surface is retarded." Of this we may easily become sensible by waving the hand rapidly through the air, with the palm or back of the hand opposed to the direction in which we strike; we then *feel* the resistance of the air, which is much diminished when we move the hand edgeways, so that a much smaller surface is opposed to it. We feel it still more strongly, when we hold an open umbrella against the wind; or when we draw one, with its concave side forwards, through the calm air of a room. This resistance, being a constantly-acting force, must continually retard the velocity of any body which is moving through the air, and must gradually check it. Its influence is well seen by causing two wheels, of the same dimensions and weight, and supported in the same manner,—but one having vanes opposed to the direction in which the wheel revolves, whilst those of the latter are in the same plane with it,—to be set in action by the same force. The wheel with the vanes opposed to the air will be soonest brought to a stand by its resistance; but if the two be placed in a vessel exhausted of its air, and be then set in action as before, the times during which they will continue to revolve will be equal. The greatest amount of the air's resistance is exerted, therefore, against bodies with the largest surface: and it does not operate at all against a perfectly circular body, such as a top, which is made to revolve upon one point without moving onwards. The resistance of the air is peculiarly seen in the slowness it produces in the fall of bodies which expose to it a large surface in proportion to their weight, —such as leaves, feathers, shreds of paper, &c. These we are consequently in the habit of calling *light* bodies; yet the earth's attraction, if it were not resisted by the air, would bring them down to its surface as fast as the heaviest masses (§. 253).

144. That the motion of a body, when not retarded by these or any other opposing causes, will continue for an indefinite period, there is the most satisfactory evidence, derived from those cases in which they are reduced to their smallest possible amount. Every reduction in the amount of friction, and of the air's resistance, is attended with a proportional increase in the continuance of the motion. Thus, when the body is made to

bear upon a single hard point, which has itself but a slow revolution, the friction is greatly diminished; and if it be quite circular, and keep spinning like a top upon the same point, the *resistance* of the air will not operate against it,—though there will be a certain degree of *friction* between the surface of the body and the air in contact with it, which would still cause the revolution to cease sooner in the air than in a vacuum. The heavier the body thus put in motion, the longer it will continue; since its force will be in direct proportion to its weight (its velocity remaining the same), whilst the retarding causes do not increase in the same proportion. Mr. Roberts of Manchester is said to have constructed a body, presenting as little resistance as possible to the air, and truly balanced upon a fine point; so that, when it is put in motion upon this point, it will not lose the force of its motion, but will continue to spin for 43 minutes. If this were made to revolve in a vacuum, its revolution would continue for several hours. A pendulum, suspended in such a manner as to have as little friction as possible, has been known to continue its vibrations for more than a day.

145. In order to have absolute proof, however, of the permanence of motion, when there is no retarding force applied to check it, we must look towards a class of bodies, which are not influenced in their movements either by friction or the resistance of air. These we shall find among the heavenly bodies: they are the planets and their satellites,—the comets,—and, as there is now good reason to believe, many of the (so-called) fixed stars. Their movements have continued for ages, without the slightest tendency to diminution;—they commenced, when they were first impelled by the hand of the Creator, to traverse the paths they still describe; and these they will continue to traverse, with the same undiminished force, so long as it shall be His will to permit them. All force is but the direct or indirect operation of the Creator's power; and it would be totally wrong to say that He does not continue to exert this, in sustaining the movements of the planets, as in providing for the well-being of the living inhabitants of their surface. Without His energy nothing could continue to exist; for all the properties of matter are the result of

His ordinations. When we say, therefore, that it is the property of any mass of matter to attract another,—or that it is the property of matter to continue in the state of rest or motion in which it may be, so long as no new force acts upon it,—we mean no more than that the Creator is continually acting in these methods, which, for wise and good purposes He has prescribed to Himself. As the sacred poet has beautifully sung:—

"His piercing eye at once surveys
Where thousand suns and systems blaze,
And where the sparrow falls;
While seraphs tune their harps on high,
His ear attends the softest cry,
When human misery calls."

146. The Divine energy is exerted as much in sustaining the commonest movements on the surface of the globe, as it is in upholding the planets in their majestic orbits; and when we say that these continue to revolve with the force impressed upon them at the creation of the system, nothing else is meant than that they have received no new impulse, but that their revolutions have continued in the same perfectly uniform and equable manner, because they are not affected by any forces, either retarding or accelerating. Such proof would not be satisfactory, if it could not be shown (as it will hereafter be, §. 173), that the movements of the planets are perfectly analogous to the movements of bodies on the surface of the earth, being produced by the combined effects of two causes, the tendency to move forwards in a straight line, and the attraction of the sun; and that we have a right, therefore, to extend that principle to one class of bodies, which can only show itself in full action in the other.

First Law of Motion.

147. We are now, therefore, prepared to understand and to admit the first of the three laws of motion, which was stated by Newton in the following terms:—"*Every body must persevere in its state of rest, or of uniform motion in a straight line, unless it be compelled to change that state by forces impressed upon it.*" That bodies have a tendency to remain in a state of rest, unless they

are acted on by some force external to themselves,*—or, in other words, that they have not the power of *spontaneously* putting themselves in motion,—is a fact with which every one is familiar. But it has now been shown, that bodies have in themselves no greater tendency or power to come to a state of rest whilst in motion, than to put themselves in motion when at rest; since that gradual diminution of their movement, which we commonly witness, is due to the influence of forces external to them. Hence we are led to understand, that it is one of the fundamental properties of matter, that it tends always to remain in the same state, whatever that state may be. This property is termed *inertia*. The meaning of the term, in the Latin language (from which it is taken), is strictly *inactivity;* and the term was applied, when it was imagined that the state of rest is that which is natural to the body, and that the state of motion is forced,—an idea which was not altogether dropped until Newton proved its falsity. The English word which most nearly expresses the meaning that philosophers now attach to the term inertia, is *passiveness;* by which we understand the tendency of matter to continue in its previous state, whatever that may be, and the power of being influenced by any force brought to bear upon it. As the term *inertia* is in constant use, however, it will be much better to employ it here; and it cannot be misunderstood, if due attention be given to a few examples of its application.

148. When a railway-train is first put in motion by its locomotive engine, its movement is very slow; its speed is gradually increased, however; and at last it is raised to the full amount that the engine is capable of producing. When it is necessary to stop the train, the action of the engine is checked whilst the train is still at a long distance from its stopping-place; and the movement of the train gradually becomes slower, until it is at last brought to rest. Now let us examine the nature of this simple and familiar operation. The loaded railway-train is a heavy mass of matter, which, by its *inertia*, would remain at rest for any length of time, unless some power were applied for the purpose of moving it. But the engine is attached, the steam

* Of course we are here speaking of inanimate bodies only.

put on, its power applied, and the train begins to move very slowly. The engine pulls with exactly the same amount of force at first that it does afterwards; and therefore the first slow movement of the train is a measure of the amount of that force. But the motion of the train gradually becomes faster; why is this? It is due to the spontaneous continuance of the motion first communicated to the train; and to the continued addition of new force exerted by the engine. Suppose that, after having put the train into very slow movement, the engine-power were shut off; the motion of the train at the same rate would continue for some time, in consequence of its inertia. During every instant, therefore, the train is carried on by its own tendency to keep moving, at the rate which it had acquired the instant before; and it is also constantly receiving a constant addition of new force from the engine. Its rate of motion, therefore, gradually increases; and the train continues to move faster and faster, until its full speed is attained.

149. Thus, then, we see that the *inertia* of the train is the force which has to be overcome by the action of the engine, in setting the train in motion; and that it is the same inertia which tends to keep the train at the rate of motion, which shall have been communicated to it at any particular period. This inertia, then, resists the action of any new force, when the train is in motion, just as much as when the train is at rest; so that the same engine-power will be required, to increase the velocity of the train from 15 to 20 miles per hour, as to put the train in motion at first, at the rate of 5 miles per hour.* If it were not for the inertia, or tendency to permanence of motion, there would be no increase in the speed of the train; for the engine could only drag it along at a very slow rate, if the power which it applied at one minute were quite lost at the next.

150. We might be led to imagine from this reasoning, that the speed of a train ought to be continually increasing without

* This is not strictly correct in practice; but the variation is caused by friction and the resistance of the air: since there is more difference between the friction and resistance which oppose the change from a state of rest to a speed of five miles per hour, than there is between the amounts that would operate at a speed of fifteen and twenty miles. (See §. 386.)

limit; since the previous force is constantly acting, and a new force being continually applied. But this is not the case; for there is a certain amount of speed which cannot be increased, unless the engine-power be augmented. The reason is this. By no means the whole of the engine-power is expended in overcoming the inertia of the train; for a large part of it goes to counterbalance the opposing effects of friction, and the resistance of the air. Now these opposing forces continue to increase with the velocity acquired; and at last they become so great, as to equal the power of the engine; so that, when a certain speed has been attained, the steam-power is entirely expended in keeping up that speed by overcoming these obstacles, and there is consequently no force left to produce an increase in it. But suppose an additional power to be applied;—there would then be a further increase to the speed; but friction and the resistance of the air would also increase in a very large proportion; so that these will soon counterbalance the engine-power, and any further increase, therefore, will be prevented.

151. The operation of these causes in preventing the increase of speed beyond a certain point, is well seen in some experiments which were made a few years since on the Liverpool and Manchester Railway, for the purpose of determining their effects. There is an incline on that railway, in which the road rises 1 foot for every 89 feet of its length. When a loaded carriage is let to run down this incline by its own force, its motion is at first slow, but it continues to increase at a rapid rate, under the continued influence of the earth's attraction, just as it would under the application of engine-power. If this increase continued uniformly, the carriage would have an immense velocity by the time it reached the bottom of the incline, (§. 260); but it is found by experiment that it does not continue to increase beyond a certain point,—about 35 miles per hour; since at that point the friction and resistance of the air are so great, as to counterbalance the continued influence of gravity, so that the carriage moves on only at the rate it had previously acquired.

152. The *inertia* of the train is further seen, to great advantage, when the steam is shut off, for the purpose of bringing it to

a stand. If a heavily loaded train, moving at the rate of 40 miles an hour, were thus left to itself, it would probably not come to rest until it had run two or three miles along the railway; the gradual diminution of its motion being due to the retarding causes just mentioned, which are now unbalanced by any forward impulse. But if this were the only mode of stopping the train, there would be great loss of time; since it would be necessary to shut off the steam two or three miles from the stopping-place, and to let the velocity of the train be gradually diminished during the whole of that distance. This difficulty is easily got over, however, by the application of increased friction to the wheels of the carriages, which causes their motion to be much more rapidly retarded; so that the full velocity may be kept up, and the steam-power continually applied, until the train has arrived at a distance of perhaps only half a mile from the station. This increased friction is applied by means of a screw turned by a handle; which, by means of a system of levers, brings blocks of wood to bear against the wheels of each carriage. The power thus gained gives a very complete control over the movement of the train; so that, by a more or less forcible application of the friction-blocks, the train may be brought to a stand more or less rapidly, and may be made to stop at any desired point.

153. The preceding example has been fully entered into; because it affords a very good illustration of the principle of inertia. There are many others, equally familiar, which need now be only slightly noticed. The flying of the dust out of a carpet on one side, when it is struck on the other, is a very good illustration of the tendency to permanence in motion once communicated; the blow gives motion to the carpet and to the dust which it contains; but whilst the former can yield but little, the latter can move freely, and is thus driven out of the carpet. When a man rides in a carriage or on horseback, he feels, at first suddenly starting, a tendency to fall backwards; this is produced by his *inertia* (which *then* tends to keep him at rest), at the moment that the forward motion is being communicated to him. But if, when in rapid motion, the carriage or the horse is suddenly checked, the man feels a tendency to be thrown forwards,

which he can scarcely resist if he be not prepared for it. A ludicrous instance of this occurred some years ago, shortly after the general peace. A troop of yeomanry which was raised about that time, was supplied with horses from one of the cavalry regiments then disbanded; and these horses were well trained to obey the word of command. On the first day of exercise, the whole line was advancing at a brisk trot, when the word "halt" was given, the horses immediately obeyed it, and stood still; whilst the riders, who had been accustomed to use some little exertion in pulling in their steeds when they desired them to stand still, were thrown, almost to a man, over their horses' heads,—fortunately without any serious injury, the ground being soft. Here the *inertia* acted by its tendency to keep their bodies in that state of motion, in which they were at the moment of the horses' halting.

154. If a man stand upright in a boat, as it approaches the shore, he will be in great danger of falling forwards, if the boat should suddenly strike the ground; for, its motion being checked, his own inertia will tend to carry forwards his body, especially the upper part of it which is most distant from the point of support. The best way of avoiding such an accident, is to stand facing the shore, with one foot planted a good deal in front of the other, so as to resist the forward tendency of the body at the moment of the boat's grounding. Again, when a man jumps from a carriage in rapid motion, he is likely to receive severe injury; since the motion he possessed, in the act of quitting the carriage, will tend to throw him prostrate as soon as he reaches the ground,—his feet being there arrested, whilst the tendency to movement still continues to act on the upper part of his body. His only way of avoiding this, is to jump with his face in the direction of the movement, and to commence running forwards immediately that he touches the ground. The best way of doing this, is to descend from the back of the carriage, rather than from its side; since the force of the leap backwards will partly counterbalance the forward movement. It is from the same action of *inertia* in producing a tendency to continued movement, that a race-horse, whatever efforts he make to check himself, cannot

come to a stand-still until he has long past the winning-post ;— or, again, that a man can leap further, when he has been able to take a previous run sufficiently long to get up his full speed.

155. The mode commonly adopted by workmen, of fixing tools into handles, or handles into tools, is founded on the same principle ; although *they* know nothing but that it is successful in practice. Thus, in fixing a handle into the hole in the head of the hammer, the latter is put loosely upon its top, and the lower end of the handle is then struck smartly upon the work-bench ; this has the effect of suddenly arresting the handle, whilst both it and the hammer-head are in rapid movement ; and the latter is thrown upon the former, and fixed firmly upon it, by the action of its inertia. The same happens, if a chisel or file is to be fixed into a handle ; for, if the lower end of the handle be struck in the same manner, the chisel will tend to bury *its* lower end in its top, by the continuance of its own movement. On the other hand, if we desire to loosen the wedge that holds in the iron of the common plane, we strike the back of the plane rather smartly with the hammer ; the body of the plane is thus thrown suddenly forwards, but the wedge has a tendency, by its own inertia, to be left behind ; and as its direction is partly backwards, it springs from its place.

156. These illustrations are quite sufficient to bring home to the mind the general principle, that motion once communicated to a body has a tendency to continue ; and that it can only be increased or diminished, by a force applied to the body in the direction of its movement or opposed to it. Hence we see the truth of that part of the law, which states that bodies will persevere in a state of *uniform* motion, unless affected by some external force. By *uniform* motion is meant a movement continued always at the same rate,—that is, the same amount of space passed over in the same time. If a body in a state of uniform motion, have a new force applied to it *for a moment*, it will have its motion increased by the same amount which the force would have produced had the body been at rest ; but its motion will then continue at a uniform rate, though faster than before. Every successive impulse it may receive will act thus

upon it. We may conceive, for example, a heavy, smooth iron ball to be rolling along a surface of ice (in which case its friction will be very small); and to pass, in its course, a number of persons provided with bats, each of whom gives it a stroke equal to that by which it was at first impelled. The rate of its motion, therefore, will be progressively increased to twice, three times, four times, &c. the rate with which it started, in proportion to the number of additional impulses it has received; but its rate of movement between each impulse will be uniform, and its increase will be sudden. An increasing motion of this kind is said to be *accelerated*. Now when the new force is *continually* operating, instead of being applied at intervals, it will be continually increasing the velocity with which the body moves; and this increase will be equal in equal times, since the force applied is equal. Such a regular increase would take place in the application of steam-power to propel a railway-train, as already explained (§. 148), or in the motion of a carriage down an inclined plane (§. 151); were it not for the obstacles occasioned by friction and the resistance of the air. It is best seen in the descent of falling bodies, in which there is no friction but that of the air (Chap. VIII). Motion, whose rate is thus being constantly and regularly increased, is said to be *uniformly accelerated*.

157. On the other hand, an opposing force will produce results exactly analogous, in *retarding* the velocity of a body which is being carried on by its inertia in a uniform motion. For if a body in rapid movement meet with an obstacle which is not powerful enough to check its motion altogether, and which does not alter its course, it will continue with a motion diminished in a degree proportional to the force thus acting against it. Thus suppose, as in the former case, the ball rolling on the ice with a certain velocity, produced by the combined impulses of a certain number of bats; and let it then pass by another set of bats, of which every one gives it the same amount of impulse in a direction contrary to that of its movement:—then (leaving the loss by friction and resistance of the air out of consideration) just so many of these retarding impulses would

be required, to check its motion and bring it altogether to a stand, as there were propelling impulses to produce that motion. This may perhaps be made plainer by the following arrangement of figures—

Accelerating *Retarding.*

A 1—2—3—4—5—6——6′—5′—4′—3′—2′—1′B.

Suppose the ball to be impelled at A with a force that carries it onward towards B, with a velocity of 1 yard per second; it continues to move with that uniform velocity, until it receives another equal impulse at 2, which causes it to move onwards with a uniform speed of 2 yards per second; at 3, 4, 5, and 6 it receives successive impulses, each equal to the first; and its velocity is thus increased to 6 yards per second, with which it moves at an uniform rate from 6 to 6′. At 6′, however, it receives an impulse to the same amount, but in a direction that would carry it towards A; this destroys the effect of its forward impulse at 6, and reduces its velocity to 5 yards per second. At 5′ it receives another backward impulse, which reduces its velocity to 4 yards; and the same takes place at 4′, 3′, and 2′; so that after passing 2′, it has a velocity of only 1 yard per second. This is destroyed by the last backward impulse it receives at 1′; which neutralizes the effect of the forward impulse it had received at 1, and brings the body to rest at B. Now if the retarding force be acting constantly, instead of at intervals, it will produce a *uniformly retarded* motion: this is the kind of movement, which the retarding influences of friction and the resistance of the air would produce, in a railway-train left to itself by shutting off the steam when it is in rapid movement, or in a wheel rapidly whirled round on its axle; and it is the same which happens, under the retarding influence of the earth's attraction, when a stone is flung directly upwards into the air.

158. The continued motion of a body left to itself after being propelled, will not only be uniform in its rate (provided that it be not acted on by any new accelerating or retarding forces), but it will go on *in the same straight line.* This seems to require no

explanation, since the fact is familiar to every one. In the rolling of a ball along the ground, the sliding of a boy upon the ice, the trundling of a hoop, and many other such common actions, we see examples of this principle. If the tendency to continued movement *in a straight line*, indeed, were not a part of this simple law of motion, all that has been said in regard to accelerated motion would be worthless and untrue; since, if the body should have a tendency to go off in any other direction, the new impulses would not act upon it in the same line with the old, and great irregularity would be the result. The effects of a new force applied in a different direction to that in which a body is moving, —or of two forces acting on a body at the same time, in different directions,—will come to be considered under the next head; and it will be there seen, that all movements which are not in a straight line (like those of the planets in their orbits) are owing to the combined action of two or more forces.

159. The continued uniform motion of a body in a straight line is, in fact, merely the result of the tendency of its motion to permanence, not only in degree but also in direction. No one can doubt this tendency, who has ever noticed a loaded carriage ascending the first part of a hill, with scarcely any effort on the part of the horses, solely by the force it had acquired during its preceding descent down another hill; or who has noticed the swinging of the pendulum as far in the upward direction, as it has just descended; or who has seen, in the "Russian Mountains," as they are termed, a carriage passing up and down over a long series of double inclined planes, by the force it had acquired in descending the first; or who has witnessed that still more remarkable manifestation of acquired force which is shown in the action of the "Centrifugal Railway" (§. 222). In the *coursing* of a hare by a greyhound, the movements of the hare seem to be instinctively directed by the same principle; for the *doubling* of the hare—that is, her sudden change of direction to the one almost opposite to that at which she had been running —enables her to gain upon the greyhound; since his comparatively heavy body, in rapid motion, is carried by its inertia much beyond the spot at which the hare had doubled, before he can

can change his direction in pursuit of the hare, who has gained ground in the mean time. There would be no difficulty in multiplying illustrations of this first law of motion; since there is scarcely an action that takes place around us, in which it does not operate.

Second Law of Motion.—Composition of Forces.

160. The second of the laws of motion expresses certain general facts relative to the action of new forces upon bodies already in motion, or in the act of being set in motion. "*Every change of motion must be proportional to the impressed force, and its direction will depend upon that of the straight line in which the force is impressed.*" Thus, if a ball be rolling along the ground with a force that would carry it to a point A, and it be struck on one side with a blow which, had it been at rest, would have sent it to B, it will not go either to A or B, but to a point between them. This principle is very readily understood; but the precise rule which governs the motion of a body, thus acted on by two or more forces at the same time, requires some explanation.

161. We will first suppose two forces to be pulling the same body in directions exactly contrary (as if we were to fasten two horses of equal power, one to the front, and the other to the back of a carriage); it is evident that the body will not be moved in either direction. But if one force were greater than the other, it would be pulled in the direction of that one, with an amount of force equal to the difference between the two. Thus if one of the forces, A, be pulling with a power of 16 lbs., and the other, B, be pulling against it with a power of 15 lbs., the body will be moved in the direction of A, with a force of 1 pound. On the other hand, if they were both pulling in the same direction, the body would be moved towards them with a force of 31 lbs. Now it is obvious that, if they be made to pull neither exactly against each other, nor in the same direction, they will neutralize each other in a greater or less degree, and the body will not be moved in the direction of either. The law by which the direction and amount of the body's motion is determined under such

circumstances, is termed that of the *composition of forces*: because it enables us to ascertain the combined action of any two forces; acting in any direction. It is simply this.

162. If a body at A be acted on at the same moment, by one force which would have driven it to B, and by another which would have carried it to C, it moves in the direction A D, the diagonal of a parallelogram, of which A B and A C form two sides; and its amount is determined by the length of the diagonal, compared with that of either of the sides. Thus if the force acting in the direction A B would have carried the ball over a length of 5 parts, and the force acting in the direction A C, would have carried it over a length of 3 parts, the diagonal A D contains 6 such parts, and shows that the body would have moved with a force sufficient to carry it to that distance. Hence, if we desire to know the direction taken by a body, acted on at the same time by two forces of different amounts and in different directions, we draw the two lines, A B and A C, expressing the direction and amount of them; we then draw C D parallel to A B, and B D parallel to A D, so as to complete the parallelogram; and by drawing the diagonal B D, we obtain the direction and amount of the force, which results from the union of the two. This force is called the *resultant*. It is not difficult to understand the reason of this. The change of place which the body will undergo becomes ultimately the same, whether the two forces act upon it at the same moment, or one of them does not act until after the other has finished. Thus, if the force A B had carried the body to B, and a force equivalent to A C were *then* to act upon it, it would be carried by this to D; since B D is equal to A C, and in the same direction with it. If, therefore, the two forces act at the same time, the body will be carried by them to D, but along the direct line A D.

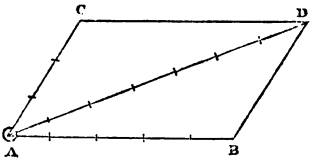


FIG. 35.

163. Not only may we thus express the combined action of *two* forces by a single line; but that of any number of forces may be determined in a similar manner. For suppose a body at O to

be pulled by five different forces in the directions O A, O B, O C, O D, and O E, we might determine the combined result of all, by first finding, according to the method just described, the *resultant* of any two, say O A and O B, which will be a line between them. This resultant may next be combined with a third force, O C; and *their* resultant with O D; so that the resultant of these last, being combined with O E, would give a line that would represent the whole of the acting forces, and would express their combined direction and amount. But this may be ascertained in a shorter mode, by following out the idea that the result will be the same, if the forces act one after the other and not at one moment. For if we draw a line from A, parallel and equal to O B, it will bring us to the point *b*, at which the body would arrive by the combined action of O A and O B. From *b* we

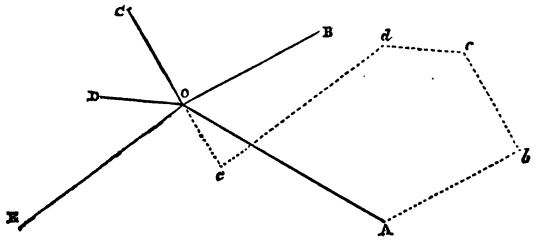


FIG. 36.

draw a similar line in the direction of O C and equal to it; this line brings us to *c*, the point at which the body would arrive by the additional action of O C; and by drawing the similar lines *c d* and *d e*, in the direction of O D and O E respectively, and equal to them, we shall find the point *e*, at which the body will arrive by the combined action of all the five forces. By joining O *e*, we shall obtain the direction in which the body will really move, under the combined influence of all of them acting together; and the amount of this general resulting force is determined, as before, by comparing the length of this line, with the lengths of the lines that represent the several forces.

164. A number of familiar examples might be mentioned, to

show how continually this principle is in operation around us. If a boat be rowed across a river where there is no current, it will reach the point opposite to that from which it started, if rowed in a direct line towards it. But if there be a current, it will be necessary, in order to prevent being carried to another point, to direct the head of the boat in some degree towards the current. For, suppose that the boat requires a quarter of an hour to cross a river, and that a current is setting down the stream at the rate of four miles an hour; the boat, if rowed in the direction of the opposite point, would be carried a mile down the stream, during the time required to cross it. Hence it will be necessary to direct it towards a point higher by a mile than the opposite one; and the force expended by the rower in urging it forwards in this direction, combined with the force of the current, will cause the boat to move across the river in the direction required. A similar effect will be produced by the combined action of wind and tide acting upon a vessel in different directions. Thus, if the wind be driving it forwards in the line of the keel, and the tide be drifting it sideways, the real movement will be in a diagonal to a parallelogram, of which the sides represent the direction and force of these two propelling powers.

165. The fall of a body from the top of the mast of a vessel that is moving rapidly through the water, is another instance of the composition of motion. It might have been expected that, as the vessel sails forwards a certain space during the descent of the stone, this would fall so much behind the foot of the mast. But such is not the case; for, if the vessel be in regular motion, the stone will fall in exactly the same place as if the vessel were not moving at all. The reason is this. The stone, at the moment at which it is let go from the hand, partakes of the onward motion of the vessel; and it does not really fall, therefore, in a perpendicular line, but exactly in that curve which it would describe, if thrown from the top of a house with a force which would carry it to the distance from its bottom, that the vessel would traverse in the same time. Thus, suppose the height of the mast to be 144 feet,—the stone would fall through this space in 3 seconds (§. 242). The vessel moves onwards during this time at

the rate of 8 miles an hour, which is at about 35 feet in three seconds. The stone having, when it is let fall, an onward motion of 8 miles an hour, will fall at a point 30 feet distant from the perpendicular through which it would otherwise have descended; but if the vessel have been moving on regularly during that time, this will be exactly at the foot of the mast.

166. In the same manner are produced some of those feats of horsemanship, which, when thus explained, are found to be less wonderful than they seem to be at first sight. Thus a man, standing on the saddle of a horse at full gallop, jumps from it, and alights again at some distance, the horse having passed over a space of many yards in the time between his leaving the saddle and returning to it; but he does not really jump forwards; for, as he partakes of the motion of the horse at the time of leaving it, this carries him on at the rate necessary to bring him down upon the saddle again, after having jumped upwards as if he were leaping from the floor. In this manner he may be carried through a hoop, without any other than an upward jump. But if he were to jump from the saddle of a horse at rest, and the horse were to be put in motion at that instant, he would be left behind by just the amount that the horse had moved in the interval. Or let it be supposed that the horse had been in rapid movement at the time of his upward leap, and had then been suddenly checked or retarded, he would be thrown forwards, as in the first instance, and would descend again just where the saddle would have been, if the horse's rate of motion had continued.

167. There is a very interesting example of the composition of motion, from which direct proof may be drawn that the Earth really turns round upon its own axis. A stone let fall from the top of a high tower, does not fall exactly at its foot, but at a certain distance to the eastward. The reason is simply this. The top of the tower and its base are not moving round the earth's centre at the same rate; for the top moves through a larger circle than the bottom, and will, therefore, pass over a longer space in the same time. This will be seen by reference to Fig. 12 (§. 93); for, if A *a* be the tower, and it be carried by the earth's revolution into the position B *b*, the top will pass over

a larger space than the bottom. The stone, when let fall from the top, partakes of *its* movement; and as, during the time of its descent, the bottom of the tower has not passed on quite so far as it has itself done, it will fall a little in front of this,—that is, in the direction of the earth's movement, or eastwards.

168. The most constantly-acting example of the composition of force, is that in which the motion of a body in a curve is produced by the combined action of two forces. It has been seen that, by the first law of motion, the action of a single force can never cause a body to move in any thing else than a straight line. And when two or more forces, each of such a nature as to produce a uniform motion, act together upon a body, an intermediate straight path is the result. But if one of these forces be of such a kind as to produce a uniform motion, and the other a uniformly accelerated motion (like that of a falling body, Chap. VIII.), a curved line will be the result. Thus, suppose a ball fired by a cannon from the top of a high tower, exactly in the horizontal direction;—from the moment it has left the mouth of the gun, the force of gravity begins to act upon it, and draws it towards the ground. This force produces exactly the same effect upon it, as if it were falling perpendicularly from the tower, so that it will reach the ground in the same time; but it will have moved to a greater or less distance from the tower, according to the force with which it was projected from the gun. In its descent it will describe a peculiar curve, termed the *parabola*.

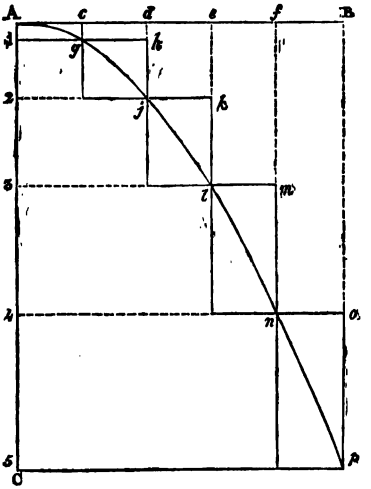


FIG. 37.

169. This will be made evident by the preceding diagram. Let the line A B express the distance to which the body will have moved in 5 seconds, if influenced only by the force of the gun, at a uniform velocity indicated by the equal divisions *c*, *d*, *e*, *f*. And let A C represent the line through which the body would have fallen in 5 seconds, if influenced only by the attraction of the earth; this movement is not uniform, being at the rate of 16 feet the first second, 48 the second, 80 the third, 112 the fourth, and 144 the fifth (§. 241). Now in order to determine the course of the body, when acted on by both forces, we draw a line from *c*, parallel and equal to A 1; then, according to the law already stated, the point *g* will be that reached by the body in the first second, under the combined influence of the forces A *c*, and A 1. In order to determine its course in the next second, we draw a line, *g h*, in the same direction with *c d*, and equal to it; this line, therefore, represents the action of the force *c d* upon the body when its position has been changed from *c* to *g*. In like manner, a line, *h j*, parallel and equal to 1 2, will represent *its* action on the body; and the point *i* will be that which it will have reached at the end of the 2nd second. In the same manner, by making *j k* and *k l* parallel and equal to *d e* and 2 3 respectively, we shall find *l* the position of the body at the end of the 3rd second; and continuing the same process, we shall get *n* the position of the body at the end of the 4th second, and *p* its place at the end of the 5th second.

170. Thus we see that by the combined action of these two forces, the body will pass from A to *p* in 5 seconds; and the same point would have been attained, if both the forces had been uniform. In that case, however, the body would have moved in a straight line; in the present case it does not, but passes through the points *g*, *i*, *l*, and *n*. For exactly the same reason, it does not move in a straight line from A to *g*, or from *g* to *i*; because the force of gravity is producing a continually-accelerated motion downwards, during any single second, as during the whole five. The line which is actually traversed is the curve which has been drawn through the points A, *g*, *j*, *l*, *n*, *p*;

and this curve, known in geometry by the name of the parabola, is that through which a falling body will always move, when it has received a horizontal impulse at the same time. The same curve is described by a ball fired from a gun directed partly upwards; for in this case, according to the law of retarded motion (§. 250), the upward direction will be gradually, and at last completely, neutralized by the action of the earth; and it will then begin to descend, its onward motion being continued during the whole time.

171. Another most important case of movement in a curved line, results from the combined action of two forces, one of which constantly draws the body towards a given centre, round which it is thus made to revolve. Thus let a body have received an impulse which would carry it in the direction A B; and at the same time be subject to an attraction which would constantly draw it towards the centre C. If this attraction be such as would cause the body to fall through the space A *f* in the same time that it would have moved through A *e*, it will in reality reach the point *g*. But at that time the direction of its movement is so changed, that, if left to itself, it would pass on in the line *g h*; the direction of the attraction is also changed, so that if the body were influenced by it alone, it would fall in the line *g i*. But the two forces being balanced against each other as before, its real movement will be to *k*; and in this manner it will describe a circle round the point C.

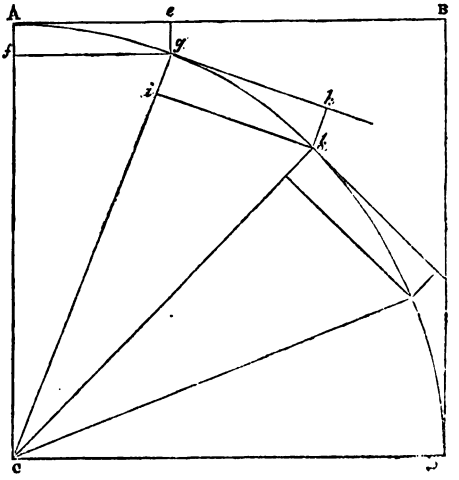


FIG. 38.

172. But in order that a *circle* may be the curve in which the body will move, there must be a fixed proportion between the force by which it was first projected, and the force of attraction; and if this proportion do not exist, or any cause alter it after it has once existed, a different curve will be the result. For suppose the attraction towards the centre to be greater in proportion to the projectile force than it was in the former case, then the body, instead of moving through a space equal to A *f*, whilst it is traversing A *e*, would fall through A *l*; and, consequently, instead of moving through the curve A *g*, which is part of a circle, it will traverse A *m*, which is part of an ellipse or oval. By the time it has got to *m*, it will have approached nearer to the centre of attraction; and the force with which it is drawn towards that centre, instead of continuing the same, as in the former case, is increased, according to the law already stated (§. 90). This increased attraction will cause it, in its next period, to perform such a curved movement that it will approach the centre still more; and the same result will continue, until it reaches the point D, on the opposite side of the centre of attraction to that from which it started, by passing through a curve which will form half of an ellipse or oval. But when it reaches that point, its motion towards the centre will cease, and it will begin to recede from it again; and this results from the following cause. Whilst the body has been moving from A to D, its velocity has been continually increasing, in consequence of its having been influenced by a constantly increasing attractive force, just as if it were falling directly towards the centre; hence

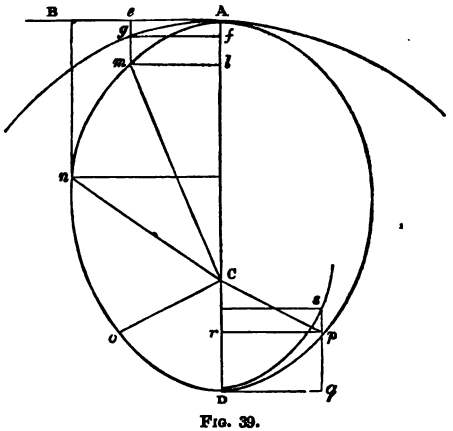


FIG. 39.

it will pass through, in equal times, the portions of the curve A *m*, *m n*, and *n o*. If its distance from the centre have diminished to one-half A C, the velocity will be doubled; so that the body would only occupy half the time to go through the space D *p* that it formerly would have required to traverse A *m*. This increased velocity will tend to balance the increased attraction; so that instead of still further approaching C, or moving in a circle to *s*, the body will continue to move in an elliptical curve, and will recede to *p*, by the combined operation of the projectile force that would have carried it to *q*, and the force of attraction which would have drawn it to *r*. By the continuance of the same mutual action, the body will be carried through the remainder of the ellipse, and will arrive again at A, ready to go through the same rotation.

173. The *ellipse* is the curve which the planets traverse in their rotation round the sun. The orbits (or paths through which they move) of none of them are quite circular; and they depart more or less from that form, in proportion as there was a want of the original balance between the two forces. In some instances, the length of the oval is very much greater than its breadth; and the orbit is then said to be very eccentric. This is the case especially with the comets. In these instances, the movement of the body in the portion of the ellipse nearest the sun is accomplished with enormous rapidity; whilst in the most distant part of the orbit, it is proportionally slow. When a body moves round another in an ellipse, the place of the latter is not in the centre, but in one of the two points termed by mathematicians the *foci* of the ellipse; the situation of which is such, that if a line be drawn from each of them to any point of the curve, the sum of these two lines shall be constantly equal. (The point C, in Fig. 39, is the situation of one of these foci.) In proportion as the eccentricity of an ellipse diminishes, the foci approach each other; and at last they meet, so that the figure becomes a circle. The application of these statements to the movements of the heavenly bodies will be explained in a subsequent part of the volume.

174. *Resolution of Forces.*—It has been shown that we may

represent the combined action of any two uniformly-acting forces, by a straight line drawn diagonally in an intermediate direction (§. 162). But we may also represent a known force equivalent to A D (Fig. 35) as resulting from the combined action of two supposed forces, acting in the directions A C and A B, and equal to them in amount. This it is often necessary to do, in order to ascertain the really acting portion of a force, of which a part is resisted; and the process, being the reverse of that which has been explained as the *composition* of forces, is termed their *resolution*. A good illustration of it is derived from the action of a side wind upon a vessel in propelling it forwards. In order to take advantage of such a wind, the sails are set in such a way that their hinder surface shall be obliquely directed towards it.

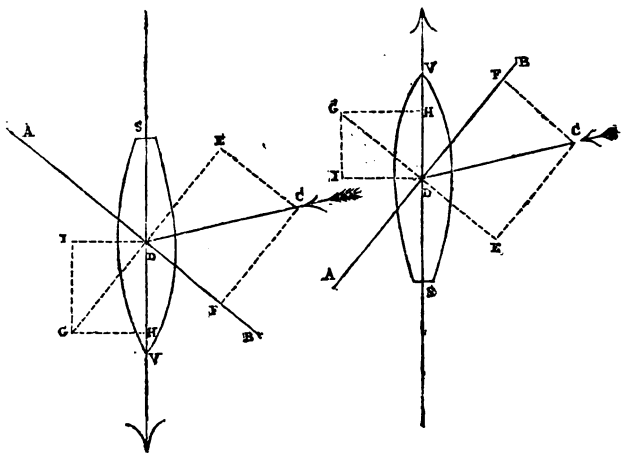


FIG. 40.

Let V S be the line of the vessel's length, C D the direction of the wind, and A B the direction in which the sail is set. Now the line C D, which represents in amount and direction the force of the wind, may be considered as the diagonal of a parallelogram, of which one side is formed by the surface of the sail, and another by the line E D perpendicular to the sail; and the force of the wind may thus be resolved into the two forces E D and D F.

The latter has obviously no influence on the sail, so that the former represents the real action of the wind upon it. If the vessel were so formed as to move through the water with equal readiness in any direction, it would thus be propelled by the wind in the direction D G; but this is not the case, since the vessel will move much more readily in the line D H than in the line D I. In order to ascertain the real action of the force D G, therefore, we must construct a parallelogram of which this shall be the diagonal, and D H and D I the two sides; D H will then represent the force with which the wind propels the vessel forwards; and D I the amount to which it would be drifted sideways, if not resisted by the water. This drift is termed *leeway*; and the amount of it will depend greatly upon the build and rigging of the vessel. By a comparison of the adjoining figures, the mode in which the same wind will propel two vessels in opposite directions, with the same or different amounts of force, will be readily understood.

Third Law of Motion.

175. The laws of motion which have been illustrated in the preceding cases, apply to every single mass of matter; and if one alone existed in the universe, they would still hold good. But the third law relates to the action of different bodies upon each other; and for its operation, therefore, two bodies at least are necessary. It is expressed in the following terms:—*Action must always be equal and contrary to reaction; or the actions of two bodies upon each other must be equal, and directed against contrary sides.*

176. The simplest case of the action of one body upon another, is where a body in motion, which we may call A, strikes upon another equal body at rest, which may be termed B; if there is nothing to resist B's movement, it will be put in motion by the force impressed upon it by A, and the two bodies will move onwards together. But their motion will be only half as rapid as that of A was previously; for its force has now to be distributed over twice the quantity of matter, and will therefore

only move it with half the velocity. The force with which A and B respectively move, is the same, since they have the same weight and the same velocity; and thus it is evident that B has received from A an amount of force equal to that which A has lost. Hence B's *reaction* on A is equal to A's *action* on B. In all such cases, the rate of motion after the *impact* (or *blow* of one body upon the other) is the same for both bodies, whatever be their relative size; and hence the moving body must be retarded by the loss of the amount of force, which has been expended in putting in motion the body that was previously at rest. Thus supposing that A were only half as heavy as B, and its rate of motion, previously to the impact, were 12 feet per second, it is obvious that the force with which A was previously moving, will have to be distributed over the combined masses of A and B, an amount equal to 3 times its own; and the velocity of the two, after impact, will, therefore, be only one-third of A's previous velocity, or 4 feet per second. Or supposing that A be equal to twice B, the conjoined weight of the two will be to A's original weight as 3 to 2; and their united velocity after impact will be to A's previous velocity as 2 to 3, or 8 feet per second. In any case, then, the re-action of B in retarding A's motion is precisely equal to the action of A in communicating motion to B.

177. The amount of force with which a body in motion will strike against a *fixed* obstacle, or against another body in motion, depends upon two conditions,—its weight, and its velocity or rate of movement. This must be evident to every one. If two bodies of the same weight are moving with different velocities, *that* one will strike the hardest which is moving the fastest; or if two bodies of different sizes be moving with the same velocity, the larger will strike the hardest. Thus we see that the force depends upon both the weight and the velocity; so that a small body, moving very fast, may strike with more force than a very large body, moving at a comparatively slow rate; or a very large body, though moving slowly, may have more force than a much smaller one having many times the speed. The force of a body in motion, thus compounded of its weight and velocity, is termed its *momentum;* and it is estimated by multi-

plying these two quantities together. In this manner, we may always compare, without difficulty, the momenta of bodies having very different weights and rates of motion; and may ascertain which will be most effectual for a given purpose.

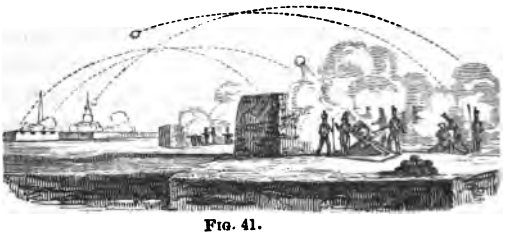


FIG. 41.

Thus if the velocity of a cannon-ball, weighing 20 lbs., be 1200 feet per second, its momentum will be represented by the number 24,000. Now suppose a battering-ram, such as that employed by the ancients, to weigh 2000 lbs., and to have a motion of 8 feet per second; its momentum will be only 16,000; and it is therefore a much less effectual instrument than the cannon-ball for demolishing a wall. But, on the other hand, let a large ship, weighing 2000 tons, be moving though the water at a rate of $1\frac{1}{2}$ foot per second (about 1 mile per hour); and it runs foul of a small vessel moving in the opposite direction at the rate of 10 feet per second, but only weighing 200 tons; the momentum of the large vessel will be very much the greater of the two, since $2000 \times 1\frac{1}{2} = 3000$, and $200 \times 10 = 2000$. It is from the amount of motion distributed through its vast bulk, that a large ship, whose advance is so slow as to be scarcely perceptible, crushes, in an instant, a strong boat

FIG. 42.

which may happen to intervene between itself and a pier which it is nearing; and it is from the action of a similar amount of force, which shows itself in giving a much more rapid movement to a smaller body, that the cannon-ball derives its destructive power.

178. The same amount of force, expended in putting in motion a heavy and a light body, will always produce the same amount of momentum; and thus the velocity communicated will be in the contrary proportion to its size. Thus a force which will move a weight of 20 lbs. through 100 feet per second, will move 100 lbs. through only 20 feet per second; for it will have the same effect, in the latter case, as if it was exerted on five bodies weighing 20 lbs. each; and it is easy to understand that a given amount of force will have only one-fifth of the effect upon each of five balls, that it would have on one. It is through this cause, that a man having a heavy anvil laid upon his chest, can allow it to be struck with a sledge-hammer; the same blow from which, if struck directly upon his chest, or upon a thin plate of metal lying upon it, would be fatal. For the sledge-hammer is a small body which derives its force from its velocity; and when it strikes upon the anvil, it cannot communicate to it the same amount of velocity, but an amount proportionally less as the weight of the anvil is greater than its own.

179. It is on account of the momentum which the particles of bodies acquire by rapid motion, that even the hardest substances may be cut or perforated by substances much softer than themselves; or large solid masses by minute separate particles, if they are made to move fast enough. Thus a piece of hard steel, such as a file, may be cut with great ease by the edge of a circular plate of soft iron, made to revolve in a lathe with great rapidity; and the lapidary cuts the hardest stones (except the diamond) by means of emery powder, the particles of which are carried round by a plate of soft iron, or lead, made to turn in a similar manner. It is on the same principle, that a deal-board may be perforated by a tallow-candle fired from a musket. This curious effect partly depends upon the principle, that, in all cases in which one body acts upon another, a certain *time* is required

for the communication of the action through the several particles of the latter; that is, in the case of the impact of one body upon another, a certain time elapses before the parts in the immediate neighbourhood of the part struck are affected by it.

180. Hence, even soft bodies may offer a considerable resistance, though only for a moment; since the surface struck cannot yield, until the parts beneath or around are displaced; and for this a certain time is required. If we strike the palm of the hand flat upon the surface of water, the blow is resisted at the first instant almost as though by a solid body; and many persons, who have jumped into water, head foremost, from a great height, have been killed by the impact of the head upon the water, the reaction of the liquid being, for the moment, almost as great as that of a solid would have been. The method of avoiding such an injury, is to bring the two hands together above the head; so that the hands and arms form a kind of wedge, which cleaves the water, and prepares the way for the head. The resisting power momentarily exerted by water, is well seen in the common boy's sport of "ducks and drakes;" which consists in throwing a flat stone, or pebble, along the surface of water, in such a manner that, whilst it has a rapid onward movement, it shall be caused by the force of gravity to descend, so as to impinge or strike upon the liquid; the *momentary* reaction of the water upon it is almost the same as if the surface had been solid, and the stone glances upwards again, continuing its onward movement; it is presently again brought downwards by the force of gravity, and again glances upwards; and the same kind of movement continues until the onward force is nearly spent. The same thing happens even with a round cannon-ball (to which the water offers less resistance than to a flat surface), in consequence of its great velocity; and it is said that a leaden bullet has even been flattened by water, from the like cause.

181. The same explanation applies to many other curious effects produced by bodies in rapid motion. A musket-ball will pass through a pane of glass without cracking it; for the impression produced on the surface by the first impact, has not time to propagate itself so as to crack the pane, before the ball has passed

through. The same ball moving more slowly, as when thrown by the hand, would shatter the whole pane; because its force is not sufficiently great to enable it to pass through the glass, before the effect of its impact has been distributed. In like manner, a sheet of paper placed edgeways, may be perforated by a pistol-ball, without being knocked down; and a door half open, may be pierced by a cannon-ball without being shut. A cannon-ball has been known to carry off the extremity of a musket, without the stroke being felt by the soldier who carried it; as the head of a thistle may be struck off by a rapid blow, without the stalk being bent. It is for this reason that the effects of cannon-balls are so different, according as they strike the object soon after they have quitted the gun, or are *spent*—that is, have lost the greater part of their velocity, by the resistance which has been opposed to their movement. A ball moving with great velocity, passes through a thick mass of wood, as the side of a ship, leaving a clear aperture; whilst a spent ball splinters it. The military or naval surgeon is well aware of the difference between the effects of the two; for whilst a ball moving with great velocity may take off a limb with almost as regular and clean a wound as if it had been cut with a sharp instrument, the blow of a spent ball shatters the limb without separating it, splintering the bone, and bruising the soft parts in a way that makes the injury really more severe than the former. The fact should be borne in mind, that even sharp clean wounds may be produced by a blunt instrument, if moved sufficiently fast; for cases of suspected murder have arisen, in which much depended upon the determination of the instrument with which the wounds were inflicted; and a difficulty has been sometimes felt, in consequence of a want of apparent correspondence between the wound and the suspected instrument, which a knowledge of this principle would have removed.

182. Impact occurs, however, not only when a body in motion strikes another which was previously at rest; but when a body in motion overtakes another in less rapid motion in the same or a similar direction, or meets another which is moving towards it. When a body in motion overtakes and impinges on

another moving less rapidly in the same direction, the two will move onwards after impact at an equal rate; and this rate will be such, that the combined momentum (§. 177) of the two bodies will be equal to the sum of the momenta which they previously had. Thus, if a body, A, of 10 lbs. weight be moving at the rate of 12 feet per second, and it *overtake* another body, B, of 15 lbs. weight, moving at the rate of 7 feet in a second, the two will move onwards together at a rate of 9 feet per second. For the momentum of A is (10 × 12) 120, whilst that of B is (15 × 7) 105; the sum of the momenta is, therefore, 225; and this is equally distributed through the combined weights A and B (25 lbs.), giving to each of them, therefore, a velocity of 9 feet per second.

183. On the other hand, if a body moving in one direction *meet* another moving in the opposite direction, their momenta, if previously equal, will neutralize or destroy one another, so that both bodies (if not elastic) will remain at rest. But if one have a greater momentum than the other, this will neutralize or destroy the less momentum of the other, and will itself be neutralized or destroyed to the same amount; whilst the force that remains will be expended in giving to both bodies a movement in the direction in which it acts. Thus if A have a weight of 10 lbs., and it be moving with a velocity of 12, and it meet B having a weight of 15lbs. and a velocity of 8, they will both come to rest; since their momenta, exerted in opposite directions, are equal, being both expressed by the number 120. But if A have a weight of 10lbs., and be moving at the velocity of 12, whilst B, having a weight of 15, has a velocity of only 3, the momentum of A will be to that of B as 120 is to 45. Of A's momentum, therefore, a quantity equal to B's will be expended in checking its motion; whilst the remainder, 75, will be distributed equally through A and B, as if B had been at rest; and, since their combined weight is 25, it will give them a velocity of 3. In these, as in the previous instances, therefore, the reaction of B upon A is precisely the same as the action of A upon B.

184. In the case of two non-elastic bodies, therefore, there

is an absolute loss of motion, when the impinging bodies move in opposite directions. The results which will occur, if the bodies are not moving in directions exactly opposite or exactly the same, may be easily calculated according to the law of composition and resolution of forces. Thus, let the body A, moving in the direction A C, meet the equal mass B, moving in the direction B D. Now if these lines, A C and B D, represent their respective velocities before impact, their velocities and directions after impact must be thus determined. A line, M G, must be drawn through their centres and the point of contact; and the force of each body must be resolved into two forces, one moving in the direction of that line, and the other at right angles to it. Hence A C will be represented by the two sides, A M and A L, or M C and L C, of the parallelogram of which it is the diagonal; whilst B D will be represented in like manner by B K and B N, or K D and N D. Now it is obvious that the forces L C and K D can have no action on each other; whilst N D and M C are directly opposed to one another, and their effect must be determined by the rules already given. Let it be supposed that the masses A and B, and their previous velocities M C and N D, are such, that, after the stroke, the body A, if previously moving in the direction M C, would have been retarded so much as only to reach O, whilst the body B is driven back to G. Then the motion of A after the impact will be compounded of C I, the line in which it would have moved if operated on by L C alone, and C O, the amount it would have traversed if carried on by what remained of M C; and this combined force is represented by the diagonal C E. In like manner, the motion of B after the impact is expressed by D G and D H combined into the diagonal,

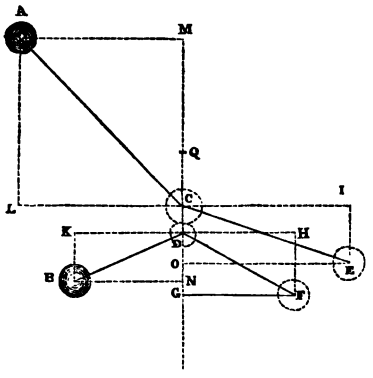


FIG. 43.

D F. The results will be very different if the bodies are elastic. (§. 193.)

185. Many examples of the operation of this law of motion are to be found in occurrences daily taking place around us. One of the most familiar is the *recoil* of fire-arms when discharged. The motion of the ball is given to it, by the sudden and enormous expansion of a small quantity of solid gunpowder, into gases whose bulk is very great in proportion. This expansion exerts an equal force in all directions; and the tendency of the gun to move backwards is exactly the same as that of the ball to move forwards; but as it has to act upon a much larger mass of matter in the one case than in the other, the velocity given to the cannon is not so great as that imparted to the ball, though its momentum is precisely the same. A light gun, therefore, will recoil more than a heavy one. If the recoil be prevented by the fixture of the gun, its force is exerted upon the fastenings, and the mass to which they connect it. When the whole broadside of a ship of war is fired at once, the combined force of the recoil of the several guns is enough to make her *heel* towards the opposite side; and if guns be discharged from her stern alone, they will accelerate her forward movement through the water, whilst if fired from her bows they will retard it. So decided is the effect thus produced, that it has been proposed to propel a ship in a calm, by firing guns from the stern at short intervals; but it has been found that the advantage gained does not compensate for the quantity of powder employed.

186. Another kind of illustration may be drawn from the effects of impact on our own bodies. If a man, walking or running, encounter another man standing still, each suffers equally from the collision; and this will be more severe, in proportion as the movement is quicker. On the other hand, if two persons, walking or running in opposite directions, encounter one another, they will suffer as much as if one had been at rest and the other had been moving with the combined velocity of both. When the fist of the pugilist strikes the body of his antagonist, it sustains as great a shock as it gives; but the proportion of pain and injury experienced by each will depend on

the nature of the part struck. When fist meets fist, however, each part is alike sensitive; and the effect is doubled by the motion of the two, being the same as that which the fist of each would experience, if it were struck by the other moving with the combined velocity of both. In the same manner, when two ships run foul of each other at sea, the injury is as much greater than that which either would sustain by running against a fixed obstacle, as the conjoined momentum of both ships is greater than that of either singly. Thus, if a ship of 500 tons burthen be sailing in one direction at the rate of eight knots an hour, and a ship of 400 tons be sailing in the other direction at the rate of ten knots an hour, the momentum of each will be 4000; and the injury sustained will be the same, as if either had run with twice its velocity on the other whilst remaining at rest. It is quite a mistake to suppose that, when a small and a large body encounter, the small body suffers a greater shock than the large one: the shock which they sustain must be the same, but the large body may not show its effects so obviously. Thus, when a large vessel *runs down* a smaller one at sea, it does so in virtue of its greater momentum, which prevails over the less momentum of the small vessel, forcing it to a sudden change of direction, in which change it is upset and sinks. The velocity of the large vessel is retarded just in proportion to the momentum of the vessel which it has struck; but it does not manifest any other injury from the collision, because its stronger construction enables it better to sustain its effects. In the case of the pugilist, however, the effect of the blow may be felt much more by the large body struck, than by the fist that strikes; on account of the greater sensitiveness of the part.

187. The effects of impact are modified in a considerable degree by the nature of the substances which come into collision. In the rules which have been given for determining these effects, they have been hitherto considered as unpossessed of elasticity (§. 57),—that is, as being incapable of recovering their original state, after this has been altered by some external force. But most bodies possess a certain degree of elasticity; and the effects of this will modify considerably the results of

impact. We shall first consider these results, in the case of bodies that are perfectly elastic; but it will be desirable to give a previous example of the operation of elasticity. Suppose a string to be stretched between two fixed points, and extended with a great force applied to its ends. Now a moderate force applied to its side, especially at the middle of its length, will draw it out of its straight position, the elasticity of the string permitting a certain amount of extension of its length. The greater the force used, the more will the string be drawn out of the straight line. Now if let go, the string will not only return to its original situation, but will go far beyond it on the other side; nearly as far, in fact, as it had been at first. It will then again return to the centre, and pass it; and will reach a point near that from which it had been at first let go. The same action will then take place again and again; and a series of vibrations will thus take place, resembling those of the pendulum, each less than the preceding, until the string comes at last to rest. It is by these vibrations, communicated to the air, that sound is produced.*

188. If the string were perfectly elastic, and its vibrations took place in a vacuum, they would have no tendency to come to an end; but would continue for ever, like the vibrations of a pendulum in a vacuum and without friction (§. 144). The reason is this. When the string has been drawn out of the straight line, it has a certain tendency, in virtue of its elasticity, to return to it; and this tendency is the greater, in proportion to the alteration in form which the string has sustained. Its particles are thus all put in motion; and by the time that the string has reached the straight line (at which point the original moving force ceases) they have acquired momentum sufficient to overcome the resistance of the string, to an amount equal to the force which caused their return to the centre; and thus the string will vibrate on the other side (if its elasticity be perfect, and there be no resisting force) to a point as distant as that from which it set off, and will continue the same succession of movements.

189. Now let it be supposed that the string, instead of being

* See Treatise on SOUND.

drawn to one side by the finger, is struck by a hard body in motion, with a certain amount of force. The further the string is carried out of the straight line, the greater will be its resistance; and at a certain point this resistance will be sufficiently great, to neutralize and destroy the force with which the body was moving; so that the string, when carried to that point, does not move beyond it, and the onward motion of the body is itself completely checked. The elasticity of the string will then tend to carry it towards the straight line, with a force equal to the resistance it was previously exercising; and it will communicate this motion to the body which struck it, so as to send it back again towards the point from which it came, with a force equal to that which it originally possessed.

190. This illustration is applicable, not to strings only, but to all kinds of elastic bodies; for every one of them suffers some change of form by a stroke or pressure, and afterwards recovers its original form by reacting in the contrary way, with a force which, in perfectly elastic bodies, is equal to the pressure or stroke received. Hence, if two such bodies of equal size, and moving at the same rate in contrary directions, impinge on one another, they will not remain at rest as they would do if perfectly inelastic (§. 183), but will recede from each other with a force equal to that by which they previously approached.

191. Hence we see that, in the impact of perfectly elastic bodies in opposite directions, no force is ever lost; and that, at any given time after the impact, their distance from each other will be precisely that which they had at the same time before the impact. But this results from a remarkable interchange which will occur between them, if their weights be equal, but their previous velocities unequal; for each will possess after impact the velocity of the other. If A in motion strike B at rest, it will communicate its motion to B, which will go forwards with the velocity that A previously had; whilst B will remain at rest. But if A have a velocity of 5, and it strike B moving in the opposite direction with a velocity of 3, their velocities will be interchanged after impact, so that A will move back with a velocity of 3, and B with a velocity of 5.

192. When the masses as well as the velocities are unequal, the principle becomes more complex. It may be that both bodies move after impact in the same direction; but the rule still holds good, that they will recede from one another to the same amount in any given time. Thus, suppose a very large mass A to be moving slowly onwards, and to meet a small mass B moving more rapidly in the opposite direction; the momentum of B will not be sufficient to check the motion of A, though it will retard it; but the direction of its own motion will be reversed; and its velocity will be so much increased, that, at any period after the impact (say a minute), B shall have receded from A, though both now move in the same direction, to the same distance as they had at a minute before impact, when they were moving in opposite directions. If the two bodies be moving in the same direction before impact, they separate afterwards, so as to regain, in the same period, their previous distance from each other. The velocity of the foremost is augmented, whilst that of the hindmost is either retarded, altogether destroyed, or made to change its direction, according to the relative momenta of the two bodies before impact.

193. It would be impossible to explain in more detail, without the aid of mathematics, the principles regulating the impact of elastic bodies; and we shall therefore go on to show, by the aid of the subjoined figure, the mode in which we find, by the laws of the composition and resolution of forces, the results of the impact of two elastic bodies moving obliquely towards each other. Let A be one body, moving in the direction and with the velocity A C; and B the other body, moving in the direction and with the velocity B D. In order to ascertain their action on one another, a line M R must be drawn through their centres and the point of contact; and the force of each must be resolved into two forces, the one acting in the direction of that line, and the other at right angles to it. Thus A C is the diagonal of a parallelogram of which A L and A M are two of the sides; and the force with which A moves towards C may be considered, therefore, as made up of A L or M C, and A M or L C. In the same manner, the force B D is represented

by B K or N D, and B N or K D. Now it is obvious that two bodies moving along the parallel lines K D and L C would have no action on one another; and that the effects of the impact must be determined by the forces M C and N D. Supposing it to be determined, by the laws already explained, that the body A moving in the direction M C, and meeting B moving in the direction N D, will recoil to Q, whilst B recoils to R; then the subsequent motion of A will be compounded of the force C Q, and of the force C I, the continuation of L C, which will give the diagonal C Z; and in the same manner the subsequent motion of B will be compounded of D R, and of D Y, the continuation of K D, which will give the diagonal D X. If this figure be compared with that illustrating §. 184, the difference in the results of the collision of elastic and non-elastic bodies will be obvious.

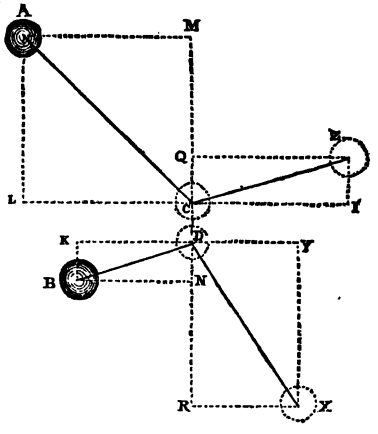


FIG. 44.

194. Our attention has been hitherto chiefly directed to those cases, in which two bodies in motion come into collision; or in which one body in motion strikes a movable body at rest, and communicates motion to it. But the case is different when one of the bodies is not only at rest, but is also

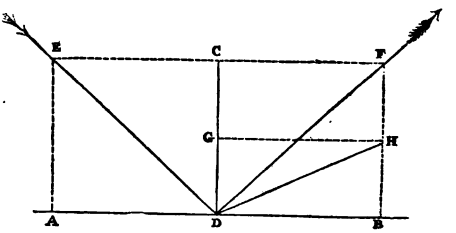


FIG. 45.

immovable. Thus, let A B be a flat surface, and C a body moving towards it in the direction C D. Then if C D be perpendicular to A B, the whole force of C D will be resisted by A B; and, if the surface A B and the body C be perfectly inelastic, the whole of C's motion will be destroyed at D, and it will remain at rest. But if either or both be perfectly elastic, the body C will be thrown back in the direction, and with the force, which it had when approaching A B. But suppose that the inelastic body E, moving in the direction E D, strike upon the inelastic surface A B at D, the result of this impact must be determined by the law of resolution of forces. For the force E D is equivalent to the two forces E A and E C, the one perpendicular to the plane, the other acting along its surface: the former is completely opposed and neutralized by the plane, as in the last case; and the latter would carry on the body in a direction parallel to E C, namely D B. A perfectly inelastic body impinging at any oblique angle upon an inelastic plane, therefore, will move after impact along the surface of the plane.

195. But the case is entirely different if the body or the plane be elastic. For the force E D being resolved as before into the two forces E A and E C, the perpendicular force E A, or C D is returned to the body in the contrary direction; so that, if acted on by it alone, the body would be carried from D to C. But as it is acted on by the continuation of the force E C or A D at the same time, it will move in the resultant of these two, that is, in the diagonal D F. Since, when the body is perfectly elastic, the force C D which it would have after impact, is the same as that which it had before, and since D B is equal to D A, the whole parallelogram C D B F is equal to the whole parallelogram E A D C; and the diagonal D F is equal to E D. The angle C D F, which is termed the angle of reflexion, will thus be invariably equal to the angle E D C, which is termed the angle of incidence; and this law holds good with respect to the reflexion of light, sound, &c., as well as with regard to solid elastic substances. But if neither the body nor the plane be perfectly elastic, the force with which the body returns in the perpendicular direction will be less than that with which it im-

pinged; and the angle of reflexion will always be greater than the angle of incidence, so that the direction of the body is nearer that of the plane surface after impact than it was before. Thus, let the elasticity be such, that the body C falling upon A B at D is only thrown back as far as G; then, if a body move from E to D with the combined forces E C and E A, it will be reflected in the direction D H. For the force A D acts as before, in producing a motion parallel to A B, and equal in amount to E C; and this alone would carry it to B, whilst the force produced by the elasticity would of itself carry it to G; the diagonal D H, therefore, expresses their combined action.

196. Illustrations of these laws of impact may be found in the game of billiards; which, in fact, can only be played well, when a thorough practical knowledge of them has been acquired. This game not only requires that the ball should be struck in a particular direction with the stick moved by the hand, but that it should strike another ball in such a manner, that one or both of them may roll into pockets at the corners and sides of the table. Very great skill in the calculation, and dexterity in the execution, are required to effect this in the various positions of the balls. Very much depends upon the precise point at which the second ball is struck; since, if the moving ball impinge on that part of its face which is opposite the line of its own motion, the stationary ball will be moved onwards in the same line; but, if the point of impact be ever so little on one side or the other of this, the motion of the stationary ball will be oblique. The skilful billiard-player, therefore, adapts the direction of his stroke, so as not merely to hit the second ball, but to drive it towards either one side or the other, as he desires. The material used for these balls is ivory, which, though hard, is also very elastic. The elasticity of this material is shown by the simple experiment of letting a ball strike upon a hard surface which is covered with a thin layer of oil; if neither the ball nor the surface were altered in form by the impact, the oil would not come in contact with more than a very minute portion of the ball; but the fact is, that the ball exhibits a considerable spot of oil, larger according to the force of the blow, indicating that it

has undergone a momentary flattening, though its elasticity has caused it to recover its shape.

197. The elasticity is imperfect, however; and the results of the impact are therefore not exactly what they have been shown by theory to be. Thus we have seen that, when an elastic ball in motion strikes another at rest, the latter is set in motion in the same direction, whilst the former remains stationary. This is not generally the case with ivory balls, however; for we may usually notice that both balls move after the impact; but it sometimes occurs that, when the stroke is a very sharp one, the result just mentioned does occur. This is probably due to the fact, that the friction of the balls on the cloth-covered table, the imperfection of the elasticity, and other causes of disturbance, have less influence when the body is acted on with a powerful stroke, than when it is moving with an inferior force. The equality between the angles of incidence and reflexion, is another general principle which the billiard-player keeps in mind; for it often happens that he cannot produce by a direct stroke the effect he desires, but that he can accomplish it by a slight manœuvre. For if he strike the first ball obliquely towards the elastic cushion that runs along the side of the table, it will rebound in such a direction as to strike the second ball in the manner he wishes, if he have properly calculated the angle at which his first stroke was made. These are the general principles of the game; but several of the most curious effects are produced by strokes, that give a tendency to the ball to rotate on its own axis, as well as to roll forwards in the direction of the stroke. One of these will be noticed hereafter (§. 231).

198. There is an effect of the impact of elastic bodies, which is, at first sight, not a little surprising. If we place 2, 3, 4 or more ivory balls in a straight row on a smooth flat surface, nearly or quite in contact with each other, and then cause another ball moving in the same line to strike one end of the row, it will not cause the whole row to move forwards, but will appear to act only on the ball at the farthest extremity from it, causing this to separate from the rest, and to roll on in the direction in which the first ball was moving. Thus, if we designate

the ball at first in motion by the letter A, and it strike against another ball B, which is the first of a row B C D E placed in the same direction, the balls B, C, and D will remain stationary, and E only will be moved. However strange this result may appear, it is at once understood, when we try it by the law regulating the impact of elastic bodies of equal size, of which one is at rest and the other in motion. For, according to this law, the moving ball A communicates the whole of its motion to B, itself remaining at rest. But B cannot move onwards without impinging on C; and, according to the same law, C communicates to D the whole of its movement, itself remaining at rest; and, in like manner, D transfers the same force to E, which moves onwards, there being no other ball to receive its impulse. The same thing would happen with any number of balls, provided they were perfectly elastic; but as the compression of each mass and its return to its former shape ocupies a certain time, the motion of the last ball will not follow instantaneously the impact upon the first; and the interval will be longer in proportion to the number of balls.

199. The experiment may be varied by suspending the balls by threads, in such a manner that they shall hang side by side, and then causing one of the balls, A, to swing like a pendulum against the rest; the ball at the other end of the row, E, will be driven from it in the same degree, if the material is perfectly elastic, and on its return-swing will produce an impact that will send A off again to A′. In this manner the balls A and E will be kept in continual oscillation, without B, C, or D being perceptibly displaced; but the oscillation will soon cease, being checked not only by the resistance of the air and by the friction

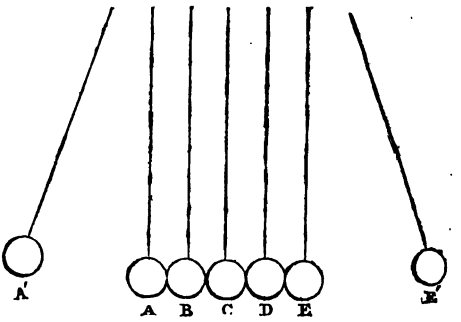


FIG. 46.

of the strings on their points of suspension, but also by the imperfect elasticity of any material we possess, which will prevent the whole force of A's stroke from being communicated to C. We shall presently find that the interposition of elastic substances amongst hard materials, is a means often very usefully employed to diminish the jar or shock which would otherwise occur, when one part of a dense structure receives a severe blow.

200. Although we have spoken of the motion of bodies as altogether neutralized or destroyed, under certain circumstances, this never happens instantaneously; because there are no bodies which are so perfectly hard, that their particles cannot be in some degree displaced upon one another. The less elastic a body is, the more likely it will be to break, when it impinges violently on another which opposes its progress; and for this reason. The particles in that portion of the impinging body, which comes into immediate contact with the immoveable obstacle, are those first brought to rest; those immediately behind them retain for a time the force of *their* movement, and press directly on the first,—those behind these on *them*, and so on; until the momentum of each in succession is destroyed by the resistance of those before it. This action is illustrated, on a large scale, by the terrible accidents which have several times occurred to railway-trains, in consequence of the sudden and complete stoppage of the engine in front, by a fall of earth, by running against a bank, or by other causes. The whole train may be regarded as a single mass, of which the cohesion among the separate parts is not sufficiently strong to resist the force created by the momentum of the hinder parts, when in rapid motion; for this momentum of the hinder part of the train causes the carriages immediately behind the engine to be so pressed on by them, as to be crushed to pieces. In the late terrible accident on the Versailles Railway (May, 1842), it is stated that the crushed carriages formed a pile of 30 feet high; so great was the force created by the continued tendency to onward motion in the long train behind them.

201. There is another cause, however, of the fracture of bodies by impact, which is not illustrated by such occurrences.

We have hitherto spoken only of the parts which lie immediately behind the point or points of impact; but as these points frequently bear but a small proportion to the whole surface of the body, it is obvious that the motion of a large part of its mass will not be *directly* resisted by the collision; and that it is, in fact, only by the cohesion of its particles among themselves, that their motion is checked at all. We see what takes place when this cohesion does not operate, in the spreading of the particles of liquids, as they drop upon a resisting surface. Let us suppose that three railway-carriages fastened together *side by side* were moving rapidly onwards, and that the central one met with an obstacle which suddenly and completely checked its motion: the carriages on either side would exert a violent strain upon the fastenings by which they were united with it; and, if this strain were strong enough to break them, would move onwards by their own momentum; whilst if the fastenings were strong enough to resist the strain, the side carriages would be brought to rest with the central one. Just so it is with a solid body, the cohesion amongst the particles of which is capable of exercising only a certain amount of resistance; if the momentum of the particles, which are not directly resisted by other particles before them, is superior to this cohesive force, the body will break; but if the cohesion is strong enough, the body will all be brought to rest together.

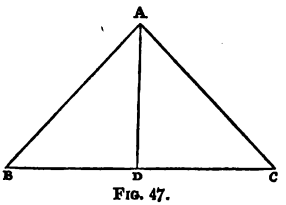


FIG. 47.

202. It is obvious, therefore, that much depends on the form of the impinging surface. For instance, let a mass of a triangular form be in rapid motion with the point A foremost; when A impinges on a fixed obstacle, only those particles which are in the line AD are directly resisted by it; and the forward impulse of all the particles situated on either side of that line will cause the body to split, if its cohesion be not sufficiently great to resist it. But if the same body were moving with the same rapidity, but with the flat side B C foremost, when this impinges on an obstacle also having a flat surface, all the

particles along the line B C are brought to rest by it; and as all those behind it are directly resisted by these, the body will have no other tendency to break than that which is produced by the pressure of one part of it on another. But suppose that the obstacle, instead of having a flat surface which meets B C, has a projecting edge, against which B C strikes at D; the effect will then be much the same as when the point A struck the obstacle; for of all the particles in the body, none will be directly resisted but those in the line D A; and those on each side will have such a tendency to continued onward movement, as very probably to break away from these, unless their cohesion is extremely strong.

203. The more elastic the body,—that is, the more its particles are capable of motion amongst each other, without losing their cohesion or their tendency to return to their original situations,—the less liable it will be to be broken by impact. Thus we can scarcely conceive of a force that should break a piece of India-rubber. And yet such a rapidity of motion *might* be given to it, that it should not have time to exercise its elasticity, its particles being detached by their separate momenta, without yielding. Or let us suppose a piece of whalebone to be moving onwards, not endways, but in the direction of one of its sides: if it came against a projecting obstacle which checks the motion of its centre, the ends would be carried forward by their separate momentum, and the whalebone would thus be bent, until the resistance afforded by the elasticity had destroyed that momentum. But we might here also imagine, that such a rapidity of motion might be given to the whalebone, as to cause it to divide at the point where it is met by the obstacle. The case may be reversed by supposing the whalebone to be supported at its two ends on fixed points, the backs of two chairs for instance; and that a stroke be given by a sharp instrument across its centre: the question whether or not the instrument would sever it, or whether the whalebone would sink before it, would be decided by the degree of resistance offered by the whalebone (which would vary according to its thickness, the distance between its ends, &c.), and by the rapidity of movement of the cutting

instrument, at the moment of impact, which might be such as not to give time for the part immediately in front of it to yield. The contrast between mere brute force and dexterity, as to the effects of blows with sharp instruments, has been well set forth by Sir Walter Scott in his novel of the Talisman. He represents Richard Cœur-de-Lion and Saladin as competing with each other in the use of their swords; and whilst Richard, with his giant strength and ponderous weapon, severs the handle of a massive iron mace, Saladin, with a light but keen-edged scymitar, and a rapid stroke, cleaves an elastic cushion, which had been placed upright before him, and even divides into numerous pieces a veil floating in the air.

204. The same tendency to permanence of motion, which breaks asunder the parts of bodies whose cohesion is not sufficiently strong to resist it, may produce that violent agitation among them, which is known as a *jar*, even when they hold together firmly. This we ourselves experience, when the body receives a shock in a direction upwards from the heels or from the bottom of the spine. The arch of the foot is admirably contrived to prevent such a shock, by the elasticity of the ligaments which hold together the bones; and in ordinary walking or running, when we place the toes on the ground first, or the whole sole at once, the force of the collision is taken off by the *spring* of the foot. But if the heels come first in contact with the ground, especially when it is descending from any height, we experience a painful sensation through our nervous system, which makes us conscious that this jar has been communicated through the body. From the bones of the legs it is propagated to the lower part of the spine; and, if this were composed of one bone, or of a set of bones piled on one another in a straight line with little or no elastic substance between them, the jar would be communicated to the head, and would produce a very injurious effect on the brain. The spine, however, is composed of a number of bones, with a thick piece of elastic substance interposed between each pair; and they are not arranged in a straight line but in a curve, so that the shock is not transmitted directly upwards, but has a tendency rather to bend the spine. Still it

may happen that the momentum of the upper part of the body is so great, that after all the elasticity of the spinal column has operated, the resistance is transmitted with sufficient suddenness from the feet to the head, to cause the brain to be severely affected; and thus a man, who falls upon his legs from a great height, may be stunned or even crushed to pieces. The same kind of shock is experienced in a less degree, when a person falls backwards in consequence of his seat being suddenly taken from under him.

205. A familiar example of the effect of elasticity in modifying the results of impact, is seen in the operation of carriage-springs. In an ordinary cart, the body is fixed upon the framework with which the wheels are connected; and every time that the wheels pass over the slightest obstacle, the body must be raised accordingly. From the unevenness even of the best ordinary roads, a continual *jarring* is thus produced, which is very disagreeable to those unaccustomed to it. But when the body of a vehicle is hung on good springs, it is not nearly so much affected by the unevenness of the road; for, if the wheels are raised by a small obstacle, their first action is to compress the springs, since these yield to a considerable degree, before the body (in consequence of its inertia) is much raised. If the carriage were to be checked, just at the time that the wheels are passing over the obstacle, the body would be lifted by the reaction of the spring; but, in general, the spring does not begin to react until the wheel has come down to its former level; and then the reaction only serves to keep the body at its proper height. For the same purpose, springs are applied between the carriages of a railway-train, to deaden the influence of slight shocks suffered by one or more of them at a time. If it were not for these, any obstacle that interrupted for an instant the movement of the engine or of any carriage, would be immediately felt as a *jar* through the whole train; but its first effect is to cause a compression of the spring of the carriage behind; and the reaction of this will be chiefly expended in pushing forwards the mass in front, when the resistance has been overcome.

206. It is the tendency to permanence of motion in all the

particles of a moving body, that makes the force of *impact* greater than any force of simple pressure, and that thus creates in the *hammer* a power to which no other is comparable. It must be remembered that no substance in nature is *perfectly hard*,—that is, capable of resisting any force applied to it. There are many which bear enormous pressures, and which yet may be acted on by very minute particles, in sufficiently rapid motion (§. 179). In the same manner, a slight blow from the lightest hammer will make an impression on a surface, which would not yield to the direct pressure of a ton weight. We see, therefore, that there is in reality no such thing as the complete destruction of motion; for when a moving body impinges on a fixed obstacle, the particles of the latter are more or less displaced, in offering resistance to its momentum; and this displacement is greater or less, in proportion as the cohesive force is less or more able to offer resistance to it. It appears that the state of vibration into which the particles of a body are thrown by blows, weakens their cohesion. This vibration is made evident to us, in the case of many bodies, by their sound; and it is well known that a glass vessel may be set in such rapid vibration, by the sounding of its own note close to it, without being itself struck, as to scatter water contained in it, and even to fly into fragments. Again it will be found that a stone, which has been several times struck against another, may afterwards be broken by a blow which would not previously have made any visible impression on it. In this case, as in that of the iron formerly mentioned (§. 22), there seems little doubt that the internal structure of the body must be in some degree changed by the action of the successive blows, so as to weaken its cohesive force, although its external form and appearance are not affected.

207. It is beautiful to observe how the forces of impact, cohesion, and elasticity, are balanced in nature, so as to become subservient to the wants of the living beings inhabiting this earth, and especially to those of man. It has been well remarked that "there is no machine comparable to the hammer. The force of heat, indeed, insinuates itself between the pores and interstices of bodies, and operating there, separately, upon their particles,

breaks them up in detail; but the hammer encounters the *accumulated* force of their cohesion, and overcomes it. The hardest rocks and the most unyielding metals submit to it. If man reigns over inanimate matter, shapes out the face of the earth to his use or to his humour, and puts the impress of his skill and his labour upon the whole face of nature, it is chiefly with the aid which this mighty force of impact gives him. It is this that clears away for him the trees of the forest, that shapes for him the materials of his dwelling, that beats out for him the instruments of tillage, that digs and hoes up the earth, that after having cut for him his corn, threshes it and crushes it into flour, that tames for him his cattle, shapes and bends together his wagons and carts, and makes his roads; in short there is no use of society for which this force of impact does not labour, and there is no operation of it which does not manifest this tendency of communicated motion to permanence."* We have seen the importance of elasticity, in obviating the injurious effects of impact; but if all bodies had been elastic, we should have had none firm enough to afford that resistance, which we are continually requiring in the construction of our dwellings, our ships, our machines, our furniture. And if all bodies had been hard and destitute of elasticity, we could not have taken a step, or moved in a carriage, or even touched anything with our hands, without a *jar* that would have made each action painful. The shocks continually produced by movement of any kind, would then be propagated without interruption in every direction; and the slightest impact would leave permanent effects. But by the arrangements of an all-wise Providence, the qualities of hardness and elasticity are so distributed among the bodies which surround us, that there are few, however dense, which have not some power of yielding to an impact, and afterwards recovering themselves; whilst there are none possessing the solid form, which are so completely elastic as not to be able to offer some resistance. By this admixture of properties in the same substances, the wants of man are supplied far more perfectly, than if they were possessed in a higher degree by distinct bodies.

* Moseley's "Illustrations of Mechanics," p. 239.

CHAPTER VII.

OF CIRCULAR MOTION.—CENTRIFUGAL FORCE.—RADIUS OF GYRATION.—CENTRE OF PERCUSSION.

208. It has been shown in the preceding chapter, that bodies, when set in motion by a single force, continue to move in a straight line; but that, when acted upon by two forces, of one of which either the direction or the intensity is continually changing, the motion takes place in a curved line. The nature of this curve depends on the mode in which the forces act. Thus it was shown that, when a body receives an impulse which of itself would carry it in a horizontal direction, and is at the same time subjected to the force of gravity, which draws it perpendicularly downwards with a continually-increasing velocity, the curve is a parabola (§. 169). But when the body receives an impulse which would of itself carry it in a straight line, and is at the same time acted on by another force, which tends to draw it towards a given centre, it will move round that centre (if the two forces are properly balanced against each other) either in an ellipse or in a circle (§. 171). The motion of bodies in a circle is one of such frequent occurrence, and is connected with so many points of interest, that it is desirable to give it a separate consideration.

209. It is necessary, in the first place, to recal the principle, that motion has not, in itself, any tendency to cease (§. 140); and that, if two forces be acting upon a body, and the influence of one of them be suspended, the movement will go on under the influence of the other alone. It has been shown, that the movement of a body round a centre, is due to the combined action of two forces, both acting in straight lines; one of these

being an impulse which urges it directly onwards, and the other, an attraction drawing it out of that straight line towards the centre. Now, if the attraction towards the centre were to be suspended at any part of the revolution, the body would continue to move in the direction it had at that moment; and it may be shown, from the properties of the circle, that this direction will be that of the line known as a *tangent*, which touches the circle at that point, and which is at right angles with the radius drawn to it. Thus in Fig. 47, A B, *g h*, and *k l*, are tangents to the circle at the points A, *g*, and *k*. The body originally received an impulse in the direction A B; but it was drawn out of that straight line into the circular curve A *g k* D, by the central attraction, which, if it acted alone, would carry it to C. By the time it reaches *g*, the direction of its movement has been so far changed, that, if the central attraction ceased to act at that moment, the original impulse would carry it on in the line *g h*, which may be regarded as the continuation of that in which it was moving at the particular instant when the central attraction ceased to act. Or, if it proceed to *k* before it is liberated, the direction of the first impulse will have been further changed, so that it will fly off in the tangent *k l*; and the same law governs its course, if set free at any part of its revolution.

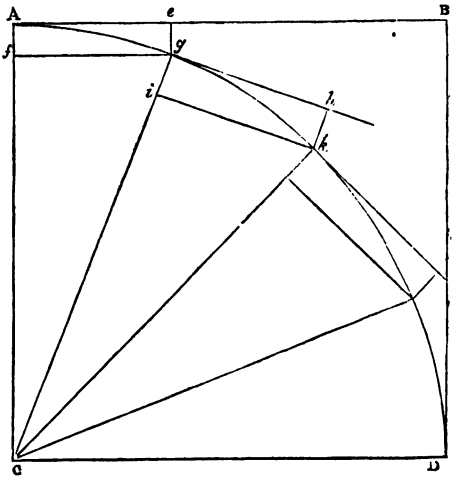


FIG. 47.

210. Many familiar illustrations of this fact may be adduced. If we tie a weight to the end of a string, and whirl it round,

holding the other end in the hand, we shall feel a considerable strain upon the string, which increases with the rate of the movement. If we let go the string at any moment, or it breaks in consequence of the increased strain upon it, the weight will fly off in a straight line, or rather in a line which would be straight, if it were not for the force of gravity, which converts it into a parabola (§. 168), drawn as a tangent to the circle at the point at which the body was at that instant. Upon the same principle depends the action of the sling and stone, which, in practised hands, is so efficient a weapon. The sling, holding the stone at its end, is made to move through a part of a circle with great rapidity; and the stone is let go at such a point, that it shall have an onward movement in the required direction. The islands which we know under the names of Majorca, Minorca, and Iviza, were termed Balearic* Islands by the Greeks and Romans, from the expertness of their inhabitants in the art of slinging, to which they were trained from their infancy. Their dexterity as slingers, while serving in the Carthaginian and Roman armies is often noticed by ancient authors; and it is recorded, as an instance of their early training to this practice, that it was customary to place the breakfast of a young person on the top of a high pole, that he might bring it down by the use of the sling. We see, then, in these instances, that the body does not fly off by means of any new force impressed on it at the moment of its departure, but merely by the continued action of that which was previously causing it to revolve round the centre, under the influence of the restraining power of the string.

211. It is commonly said, that a body revolving round a centre is kept in its path under the influence of two forces balanced against each other,—a *centripetal*, or centre-seeking,—and a *centrifugal*, or centre-flying force. The centripetal force may be the attraction of a large mass, such as that which the sun has for the earth, or the earth for the moon; but we may substitute for it the restraining power of a cord, as in the examples just adverted to. There is this important distinction in the two cases.

* From the Greek word *ballo*, to throw.

Supposing that, during the earth's revolution round the sun, or the moon's round the earth, its onward movement were suddenly checked at any period,—the centripetal force would then act alone, and would draw the revolving body towards the central mass in a straight line. But when a weight is whirled round at the end of a string, there is no real centripetal force; for if its revolution be checked, it has no tendency to move towards the centre, but rather falls to the earth. Again, the term centrifugal force is very liable to be misunderstood; since it would seem to imply a force which, acting alone, would cause the body to fly directly away from the centre, which we have seen to be very far from the truth. We must constantly bear in mind that its only proper use is to express the tendency, which the continued action of the force at first impressed upon the body has, to carry it on in the straight line that touches the circle, at the point at which the centripetal force ceases to draw it from that line. The body will, it is true, then recede from the centre; but it will only do so by passing along the tangent, the distance of which from the centre is continually increasing, and not by flying in a direction opposite to that of the central attraction. Its action will be, however, to cause the particles of a body in rapid revolution, to take their places at the greatest possible distance from the centre; of this, several examples will be presently given.

212. The simplest mode of showing by experiment the real action of these two forces is the following. Let a hole be made through a very flat board; and let the edge of this hole be made very smooth. A smooth ball of hard wood or ivory is to be fastened to one end of a string; and this string being passed through the hole in the board, a small weight is to be hung to the other end of it. The action of this weight will of course be to draw the ball towards the hole, if no other force be operating. Now let the ball receive an impulse which would of itself carry it in a direction at right angles to that of the string; its course will be so far modified by the centripetal force exercised by the weight, that it will move in a curve; and the nature of that curve will depend upon the proportion between the weight and

the force with which the ball is set in motion. These may be so balanced against one another, that the ball will move (for a time at least) in a circle; but if the impulse be too great, the ball will be less turned aside from its straight course by the action of the weight, and its superior centrifugal force will cause it to recede from the centre, and thus to draw up the weight; whilst, on the other hand, if the impulse have not been strong enough, the centripetal force will prevail, so that the ball will not describe a circle, but a spiral, being drawn nearer and nearer the centre at each revolution, by the descent of the weight, until at last it reaches the centre itself. In any case this will be the ultimate result; for the friction of the ball will gradually retard its motion, and destroy the force of the impulse it received; whilst the centripetal force remains the same as at first. Now if, whilst the ball is revolving in a circle, the string to which the weight is attached be cut, the centripetal force will cease to act, and the ball will change the direction of its motion from a circular curve to a straight line. On the other hand, if it meet with an obstacle which suddenly checks its revolution, it will be drawn towards the centre by the action of the weight.

213. With an instrument constructed upon a plan of this kind, and known under the name of the whirling-table, the laws which govern the action of the centrifugal force may be shown by experiment. Thus, if two equal weights, one at double the distance of the other, are whirled round with the same angular velocity (so as to pass through their whole circles, or any similar portions of them, in equal times), the one which is farthest from the centre will raise twice as great a weight as the nearer one. But if two equal weights be whirled at equal distances, and the one have double the angular velocity of the other (that is, makes its whole revolution, or any similar portion of it, in half the time), the former will raise four times the weight which the latter will raise,—or, which is the same thing, will exercise four times as great a strain upon the string. Hence we perceive that, if other things are equal, the centrifugal force, (measured by the weight that will be drawn up at the centre,) which causes a body to exercise a strain upon the cord that keeps

it in its circular revolution, increases with the simple distance of the body, when the time of its revolution remains the same; but increases with the square of the velocity when the distance remains the same. If the weights of the moving bodies be different, whilst their distances and velocities be the same, the centrifugal force will be proportional to the respective weights.

214. The tendency of the centrifugal force to cause the bodies on which it acts to recede from the centre, as far as they are permitted to do by other restraining forces, may be shown in a great variety of different modes. The simple apparatus represented in the accompanying figure, gives an illustration of it, which is the more interesting, as it makes perfectly obvious the cause that has changed the form of the earth from a perfect sphere to that of a spheroid (§. 95). The two hoops are made of an elastic material, such as thin brass or steel; they are fixed at the bottom to the spindle, but are left loose at the top, so that they can slide up or down, as far as their elasticity permits. If they be rapidly whirled round, the tendency of their particles to recede from the centre causes them to bulge out midway between the upper and lower ends of the spindle; the portions through which the spindle passes are flattened in a corresponding degree; and the upper part of the hoops is drawn down towards the lower. The greater the velocity given to them, the greater will be the change of form; and it will go on, until the resistance offered by the material of the hoops to any further alteration, counterbalances the influence of the centrifugal force.

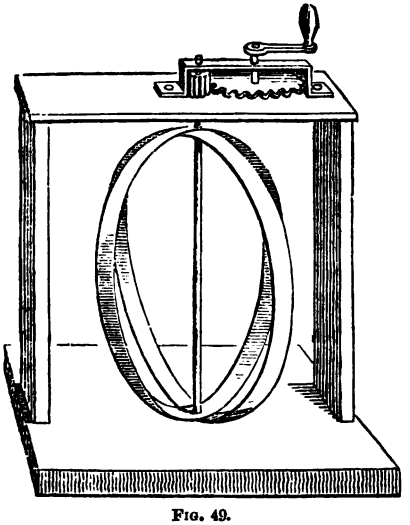


Fig. 49.

215. In the case of the earth and other planets, there is reason to believe that the matter of which they are composed once had a condition sufficiently fluid to permit a similar change of form; and in this case, the limit to the change would be the attraction of all the particles towards the centre of the mass, which would prevent any of them from greatly increasing their distance from it, unless the centrifugal force were very much increased. Now the planet Jupiter turns upon its axis more than twice as fast as the earth; and the centrifugal force that acted upon it, whilst its form could be changed, has therefore produced a much greater effect. This planet is so much flattened at the poles, that a difference between the two diameters can be seen by the eye alone (when aided by the telescope), though it it is more precisely determined by measurement. It has been ascertained by calculation, that the form which both Jupiter and our Earth actually possess, is precisely that which would be the result of the balance between the centrifugal force and the mutual attraction of their particles, supposing them to have been allowed, by the liquid condition of their masses, to take any form which these forces would produce.

216. The effect of centrifugal force is further shown by the very simple and familiar fact of the sprinkling of water from a mop that is being trundled; or the flying-off of dirt from a carriage-wheel in motion. Supposing that the axle of the wheel were at rest, and the wheel were made to turn rapidly upon it, the dirt would fly off, as soon as the centrifugal force produced by its rotation should be strong enough to overcome its adhesion to the wheel. It would at first be thrown off in the tangent *a b*; but the attraction of the earth causes it to descend towards it, and

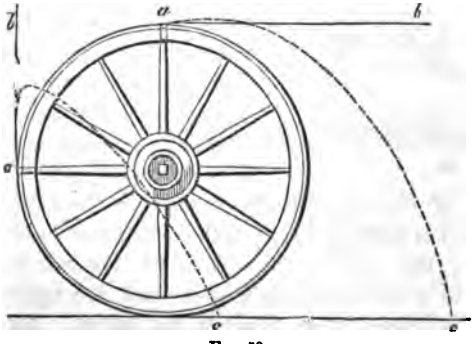


FIG. 50.

to describe the curve *a c*, like any other body similarly circumstanced (§. 168). When the wheel has an onward motion, however, as well has a rotation on its axle, the course of the substances thrown off from it will be somewhat different; since they partake of the motion of the wheel at the moment of quitting it.

217. Again, if we suspend a cup of water by a string, it is possible to whirl it round so dexterously, that none of the water shall fall out, though the mouth of the glass is downwards in part of every revolution. This is due to the tendency, which the particles of the water derive from the centrifugal force, to recede as far as possible from the centre. The chief difficulty lies in commencing and terminating the movement; since the velocity is not then sufficient to give the required centrifugal force. The action of the centrifugal force upon liquids is further shown in a very interesting manner, by hanging a bucket by a long cord, and then twisting the cord; so that the bucket, when set free, shall have a rapid revolution. The tendency of the water to recede from the centre will cause it to rise up within the sides of the bucket, and to sink down in the middle; and if the revolution be sufficiently rapid, the water will flow over and be sprinkled all around. If the sides of the bucket be inclined, so that the water is further from the centre at the top than at the bottom, it would be possible to get rid in this manner of the whole of the water which the bucket originally contained; but, if the sides are upright, the last portions of the water will not rise to the top or flow over. This principle has been employed in a very ingenious and simple contrivance for raising water through small heights. It consists of a vessel shaped like a bucket with slanting sides, open at the top and bottom, and having three or four pieces projecting inwards. Now when the vessel is made to revolve rapidly, and its lower end be immersed in water, the portion of the water within its sides will be carried round with it by these projections; and it will thus acquire a centrifugal force, which will cause it to rise along the inclined sides of the vessel, and at last to flow over its edge into a circular trough adapted to receive it. By continuing the motion of the vessel, a continual supply

may thus be obtained; since a fresh quantity of water will be constantly taken in by it below, and will receive the same motion.

218. There is another contrivance for raising water, acting on the same principle, which is known under the name of the centrifugal pump. This is represented in the accompanying figure. It consists of an upright pipe *a a*, which is inserted into the vessel of water, *e*; into the top of this pipe are fitted two cross pipes, *a b*, and *a c*; and the whole is put into rapid motion by the handle *g*. Now, supposing that the pipes are full of water at the moment when they begin to turn, the centrifugal force communicated to the water in the cross tubes will cause it to be discharged, by holes at their extremities, into the cistern *d*; and a void or vacuum will be created in the pipe, which will be supplied by water pressed up by the air from below, through the pipe *a a*. So long as the motion is continued, so long will the cross pipes deliver water, and the upright pipe draw it up. When the motion ceases, the water would flow down again into the lower cistern, were it not for valves in the pipes, opening upwards only, which prevent its return. Thus, when the pipes have been once filled with water, the apparatus is always ready for action. This first filling is necessary in order that the water may rise through the pipe *a a*; for it has no tendency to do so by the mere revolution of the pipe; and it only takes place when a vacuum has been created by the emptying of the cross arms. The aperture at *f* serves to discharge the water from the reservoir *d*, into which it is raised by the pump.

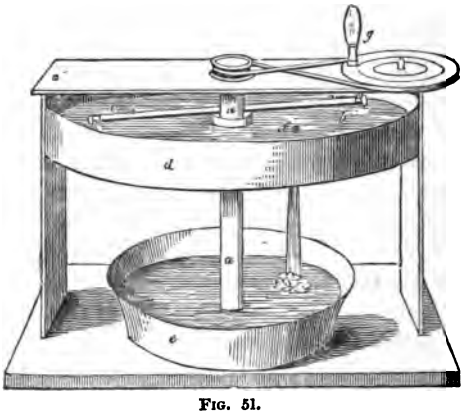


FIG. 51.

219. Another illustration of the action of centrifugal force upon liquids, is to be found in the mode in which an eddy or whirlpool is produced. When a rapid stream of water meets with an obstruction which causes it to take a considerable bend, it is thrown from its straight line into a curved direction; and, as a consequence of this, the surface of the water takes a form resembling that which it presents when whirled round in a bucket; being deeply hollowed in the centre and raised towards the outside. Any floating substance, which has been brought within this vortex, will be drawn down towards the centre of it, that being the point to which the stream is directed; and hence arises the danger to swimmers from eddies in rivers; or to ships and boats from the larger whirlpools which are sometimes to be met with on the sea-coast. The whirlpool of Charybdis in the straits of Messina has been celebrated from ancient times; there is also one on the coast of Norway, termed the Maelstrom; this is caused by the rapid flow of water, during the strongest part of the tide-stream, between two rocky islands; and it is impossible to navigate this channel when the tide is strong, without great risk, many vessels having been drawn in by the Maelstrom. The same occurrence may be seen, on a small scale, whenever a circular vessel, especially one of a funnel-shape, is being emptied by a hole in its bottom; a circular motion of the water takes place, and there is consequently a depression or even a hollow in the centre, and a rise all around; this is best seen when the water has been nearly all discharged, or when we pour a stream of water obliquely into a funnel.

220. Of the influence of centrifugal force upon solid bodies, it would be easy to multiply examples, in addition to those which have been already given. Its agency is very important in the common operation of grinding corn. The corn is placed in a kind of funnel, termed the hopper, which lets it pass gradually through a hole in the centre of the upper mill-stone; the revolving motion causes the grain to fly off towards the outer part of the space between the mill-stones, where they are closer together; and thus it is ground finer and finer, until at last it is thrown out beyond their edges, in the form of flour. Just in

the same manner would the centrifugal force act upon bodies placed on the earth's surface, if it were not for the attraction of its mass, which keeps them in their places. A body situated *exactly* on either of the poles, being at the centre of the circle, would have no tendency to fly off; but if placed anywhere else, it would, of course, move in a circle of greater or less size, in proportion to its distance from the pole, and would acquire a corresponding amount of centrifugal force, which would cause it to increase its distance. But though this force is much more than counterbalanced by the attraction of gravitation, it still operates; and at the equator, where it is greatest, it tends to oppose the pressure of bodies towards the centre of the earth, and is one cause of the diminished weight which they have there, as compared with their weight at or near the poles.*

221. It has been calculated, that if the rotation of our earth were seventeen times faster than it is, the bodies at the equator would have a centrifugal force equal to their gravity, so that they would exert no downward pressure; and a little more velocity would cause them to fly off altogether, or to rise and form a ring round the earth like that of Saturn. An occurrence of this kind sometimes takes place, when the velocity of the large grinding-stones used in the manufacture of various articles of iron and steel, is so much increased as to create a centrifugal force, greater in amount than the cohesive attraction of the particles of the stone; large fragments of the stone have been known to be carried through the roof of a building, and hurled to a considerable distance from the spot where it was worked, carrying death and destruction in their course. It has been remarked, that the wreck produced by the *disruption* (or breaking asunder) of one of these stones, resembles nothing so much as that occasioned by the bursting of a steam-engine boiler.

* A body that presses towards the earth's centre, at the equator, with a force of 289lbs., will press downwards, at the poles, with a force of 290lbs. When we add this difference to that which results from the flattening of the earth in the polar regions (§. 95, 96), we find the whole difference between the downward pressure of a body at the equator and the poles, to be 1-194th of its entire weight; that is, a body weighing 194lbs. at the equator, will weigh 195lbs. at the poles.

222. The centrifugal railway, which has been recently constructed upon a large scale, is a very ingenious device for exhibiting the effect of centrifugal force. The roadway, on which the carriage moves, first passes down a steep incline, A B; t then rises rapidly in a curve, B C, which completes by its descent a circle in the air; and lastly, it ascends in an incline, B D, which brings it to a point nearly as high as that from which it started. The carriage descending the first incline, acquires, by the time it reaches the bottom, a velocity which would carry it up another incline of the same height, if friction, and the resistance of the air, did not in some degree diminish it. But its motion is altered, by the direction of the roadway, from a straight line to a curve; and in rising through the circle which it is then made to describe in the air, a sufficient amount of centrifugal force is produced to counteract the earth's attraction; so that the carriage and the persons it contains are enabled to pass along the highest part of the circle, in which they are completely unsupported, just as water remains in the cup that is being whirled round (§. 226). In the passage of the carriage down the descending part of the curve, it acquires enough velocity to carry it up the second incline, when the direction of the roadway again changes to a straight line. It is essential, to the successful use of this contrivance, that the highest part of the circle should be considerably lower than the point from which the carriage first starts; since the velocity it acquires in its descent will not otherwise create a sufficient amount of centrifugal force to keep the carriage against the roadway at the part C, where it is completely unsupported. Any accidental cause that retards the descent of the carriage will prevent its traversing the circle with safety; and, as several accidents have occurred from this cause, it cannot be right to put human lives in peril

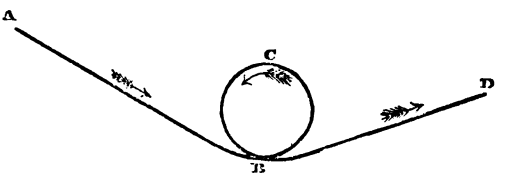


FIG. 52

for exhibiting what is at best but a philosophical toy, since it can never be applied to any beneficial purpose. A centrifugal railway, on a small scale, may be constructed by simply bending a bar of iron into the form represented in Fig. 52; and making a small heavy carriage run upon it, by means of flanches or projections upon its wheels, which shall keep it from rolling off.

223. The apparatus termed the *governor*, which has been long used for the regulation of mill-work, and which was very ingeniously applied by Mr. Watt to the steam-engine, acts upon the principle of centrifugal force. The mode in which this is brought into operation may be understood from a very simple experiment. Let the fire-tongs be suspended by a string fastened at the top, and be made to turn by the twisting or untwisting of the cord; the legs will separate from each other with a force proportioned to the speed of the rotation, and will fall together again as the motion slackens. If the legs had heavy balls at their extremities, the force with which they would separate would be much greater. Such an apparatus forms the essential part of the *governor*, which is made to revolve by the action of the steam-engine or other moving power. The purpose of it is this. In almost any kind of mill, the quantity of work to be done is continually varying; so that, if the same power were always applied, it would be sometimes too great, driving the machinery too fast, and sometimes too small, scarcely propelling it at all. Again, there may be irregularity in the moving power; as by variation in the heating of the boiler, in the case of the steam-engine; or by inequality in the force of the wind, or in the amount of water, in the case of a wind-mill or water-wheel. The governor affords the means of constantly keeping the power accurately proportioned to the work to be done; and this is accomplished as follows. By a system of levers, its legs are so connected with the apparatus by which the power is regulated, that, when the balls fly asunder, they diminish the power, and when they close together they increase it. In the steam-engine, the governor opens and closes a valve, which regulates the quantity of steam admitted from the boiler. In the

wind-mill, it acts upon the sails in such a manner as to make them present a greater or less amount of surface to the wind. And in the water-mill, it opens or closes the sluice that regulates the quantity of water which acts upon the wheel. Now if, by a diminution of the labour to be performed, or an increase of power, the machinery begins to turn too rapidly, the balls fly out and partly close the valve of the steam-engine, or the sluice of the water-wheel, so as to diminish the power. But if more work is to be done, or the power is unequal to it, so that the machinery moves too slowly, the balls fall together, and the power is increased by the contrary action.

224. One more illustration of the action of centrifugal force may be given, and that of a very familiar kind. When a carriage, a horseman, or a pedestrian, moves rapidly in a curve of small diameter (as when suddenly turning a corner), a considerable amount of centrifugal force is experienced, which gives the body a tendency to fall outwards. This the rider or pedestrian counteracts by inclining his body inwards, or towards the centre of the curve; but the carriage cannot make this adjustment; and, if the centrifugal force be sufficiently great (either in consequence of the rapidity of its movement, or the smallness of the circle in which it is turning), the carriage will be upset *from* the centre, or, in other words, away from the corner round which it is turning. The centrifugal force produced in this manner, is of great assistance to the riders who perform feats of horsemanship; since they can do many things, when moving in a small circle, which they could not accomplish if they moved in a large circle or in a straight line. A man who would ride in a straight line, standing upon his saddle, would find it almost impossible to keep himself balanced upon it. But, when he rides with great rapidity in a small circle, a new and very considerable force comes into operation, which gives him great assistance. This is the centrifugal force. It gives both to his body and to that of the horse an outward tendency, which they counteract by bending inwards, or towards the centre of the circle. Hence a less portion of his weight is supported by the saddle; and it is much easier for him to keep his balance, which

is done by simply regulating his speed. For if he should find himself likely to fall inwards, he has merely to quicken his speed, and thus to increase the centrifugal force, which will carry his body outwards, and thus restore his equilibrium. If, on the other hand, he be in danger of falling outwards, he slackens his speed, and thus diminishes his outward tendency.

225. When a body revolves about a fixed axis, each of its particles receives an amount of centrifugal force, which is proportional to its respective distance from the centre (§. 213). Hence there will be a number of different forces acting on different sides of the axis; and these will or will not balance one another, according to the form of the body, and the position of its axis. If the particles be so arranged, that equal weights exist at equal distances on every side of the axis, the centrifugal forces will be exactly balanced against each other, so that they will exert no pressure on the axis. This is the case when a globe, composed of the same material throughout, is made to rotate on any axis that passes through its centre; or when a cylinder or drum is made to rotate on an axis that passes through the centres of its two circular ends; or when a square or circular plate is made to revolve about any of its diameters, as in the common act of spinning a coin or counter. But if the particles are not arranged about the axis of rotation with this uniformity, the centrifugal forces created by the movement of the different portions will not balance each other; and there will consequently be a strain upon the axis, which will tend to pull it towards the side on which the centrifugal force is the greatest. This may be illustrated by a very simple experiment. If we whirl round a ball at the end of a string, the centrifugal force is all on one side; and we feel its tendency to draw the hand out of its place, which it requires some force to resist. But if we fix two balls of the same weight to the two ends of a stick, and whirl these round upon an axis held in the hand, and passing through the middle of the stick, we feel that there is no more strain upon the axis than that which is created by the weight of the stick and balls. Let us further suppose that the axis does not pass through the middle of the stick, but that one end

of it has twice the length of the other; then a weight, A, of 1 lb., at a distance of 2 feet from the centre, will counterpoise a weight, B, of 2 lbs., at a distance of 1 foot from the centre, as will be explained in Chapter X. If the stick be then whirled round, the centrifugal forces of the two weights will be still equal; for, according to the laws of centrifugal action (§. 213), the weight A will have twice the centrifugal force which would be possessed by an equal weight at the distance of B; whilst the weight B, being double of A, has twice the centrifugal force which A would have at B's distance. Hence the axis will still be free from any strain produced by the centrifugal forces. But if the axis pass through any other point of the stick, the centrifugal forces of A and B will no longer be equal; and there will be a strain upon the axis towards one body or the other, according as the situation of that axis is more distant from it than in the case just mentioned.

226. In any mass of matter, however irregular may be its form, there are at least three lines round which its parts are so arranged, that, if it be made to revolve upon either of them as an axis, the centrifugal forces are so balanced, as to exert no strain upon the axis, and, consequently, to have no tendency to alter its position. These three axes, which are called the *principal axes*, are at right angles to each other. Their length will be different in almost every instance; and this difference produces an important influence on their respective properties. Where they are all of the same length, the body will rotate equally well on either of them; this is the case, for instance, in a globe, the number of whose principal axes has no limit. But in a body whose principal axes are unequal in length, it will rotate *securely* on the *shortest* of these axes only; that is, if, whilst rotating upon either of its longer axes, it be slightly thrown out of its position, it will have no tendency to recover itself, but will alter its condition altogether, and will revolve round the shortest axis; and for the same reason, if, whilst rotating around the shortest axis, it receive any disturbance, it will recover itself, and thus constantly tend to maintain its position. This principle may be familiarly illustrated by a simple experi-

ment, made upon an ordinary sea-shore pebble. We will suppose this to be of a regular oval form, having two surfaces somewhat rounded. The three principal axes of such a pebble will be,—1. The long diameter of the oval side, or the line joining its two furthest points;—2. The short diameter of the oval side, or the line joining its two nearest points;—3. The line joining the centres of the two sides, thus crossing the thickness of the pebble. Now we can spin such a pebble upon one of its ends; that is, around its long diameter as an axis; but it will be very unsteady, and the slightest variation of its position will overturn it altogether. The same will occur when it is spun upon the side-edge; that is, around its short diameter. But when it is spun upon one of its rounded surfaces, it has no tendency to fall over, since it is then revolving around its shortest diameter; and if disturbed, it will return to the same position. Hence the axis around which a body of any form is in rotation, will always tend to change into the direction of its shortest principal axis, and will then remain settled.

227. This experiment must not be regarded as fully illustrating the principle; since, when the axis is supported from below, the tendency of the centre of gravity towards the lowest possible point (Chap. IV.), will naturally cause the body to assume that position. But the same thing happens, when the change of the direction of the axis is such as to leave the centre of gravity where it was; and it will even take place, when it cannot occur without raising the centre of gravity, provided the rotation be sufficiently rapid. Thus if we allow a circular plate to hang freely by a string tied to any part of its edge, its centre of gravity, being in the centre of the plate, will be at the distance of its radius, below the point where it is suspended. Now, if we make the plate revolve with sufficient rapidity, it will not rotate around the axis in which it was hanging (which is one of its diameters, and, therefore, one of its many equal longer axes), but about its shortest axis, which is the line that crosses its thickness, through its centre; and in order to do this, the plate must raise itself up into the horizontal position, so that its centre of gravity becomes as high as the point of suspension. The same

thing will take place, if we hang a ring by a string attached to any part of it, and make it rotate with sufficient rapidity.

228. The reason of this curious property is easily explained, when the principles of centrifugal force are understood. When a body is revolving round its *longest* axis, the parts composing its mass are disposed in such a manner, as to be at the *least* possible distance from the axis. On the other hand, when it is rotating about its *shortest* diameter, the parts are so arranged as to be at the *greatest* possible distance from the axis. Now it has been seen to be a property of centrifugal force, that it causes the parts of a revolving body to dispose themselves at the greatest possible distance from the centre; and this will be the case, therefore, when the rotation is round the shortest axis. The change from one axis to the other, when there is any opposing force, however, (as in the case last mentioned) can only take place when the centrifugal force becomes, in consequence of the quickness of the rotation, sufficiently powerful to overcome it.

229. It is very interesting to remark, that the form actually possessed by the earth and other planets, is the only one at all approaching to that of a globe, in which the axis of rotation is not liable to be disturbed. If they had been exactly globular, and all the axes consequently equal, a very slight amount of force would have changed the direction of the rotation from one axis to another, and there would have been no tendency to the recovery of the first position. If, on the other hand, they had been egg-shaped, they would have had only one longer axis, but an infinite number of short axes; since there is no limit to the number of equal lines which might be drawn through the centre, all of them the shortest that can unite the opposite sides. Hence an egg-shaped body, revolving around its long diameter, will be easily thrown out of its position, and will rotate around one of its shortest axes; but as these are all equal, it will have no tendency to remain in any one of them permanently, and will be disturbed by a very slight force. Lastly, if the body is flattened at the poles, or shaped like an orange, it has but one short diameter,—the line joining the poles; whilst the number of long diameters (crossing the equator) is unlimited. Hence,

whilst its rotation round either of these is unstable, it will tend to continue moving around the short axis, and will return to this position when it has been removed from it. The importance of this arrangement will be made evident hereafter; it is enough here to state, that the recurrence of the seasons, the length of day and night, and many other important conditions affecting the welfare of the living inhabitants of the earth, would have been rendered irregular, if the position of the axis of the earth's rotation were liable to undergo change. It is most beautiful to observe how one action of the centrifugal force is thus connected with another;—the same agency that produces the altered form of the earth, taking advantage (as it were) of this alteration, to render its motion stable and uniform.

230. If a body be suspended or supported in such a manner, that any force which affects it can act freely upon it, either in producing an onward motion, or a rotation round its axis, the kind of motion produced will depend upon the precise direction in which the force is applied. If it act equally upon all parts of the body, it will urge every one of them in its own direction with a corresponding force; and the motion of the body will be simply onwards, without any rotation. This is the case with the attraction of gravitation; and consequently, a falling body does not turn on its axis whilst descending. The same takes place when a force acts directly through the centre of gravity of the body; since this is the same thing as acting equally upon all its parts. An illustration of this we may find in the motion of the cricket-ball struck by the bat; or of the billiard-ball, when the line in which the stroke is given passes through its centre. But if the force be applied in any other way, a movement of rotation is produced, as well as an onward motion in the direction of the force; since the different parts of the body will be differently affected by it, and those which receive the greatest impulse will tend to move faster than the rest, which they can only do by imparting a rotatory movement to the body. It is remarkable that in such a case, the onward movement is precisely the same as if there were no rotation; and that the rotation takes place exactly as it would have done had the axis been fixed, and

the whole force expended in producing it. A remarkable example of this compound movement is seen in the action of *chain-shot*. These consist of two cannon-balls, united together by a strong chain, and fired from the same cannon. Now, unless the impulse happens to pass exactly through the common centre of gravity of the two balls and chain (which it can only do by a rare accident) those are not only propelled forwards, but they receive a rotatory motion about each other, which gives them a most destructive power. They fly forwards as far as if they did not revolve; and they revolve as they would do if they did not fly forwards. Chain-shot are now generally superseded by *double-headed-shot*, in which a strong iron bar is substituted for the chain. The latter act on the same principle as the former.

231. The movement of the earth in its orbit, and its rotation around its axis, may thus be accounted for, on the supposition that it was produced by a single impulse, not acting through its centre, but applied at a point about 24 miles on one side of it. The much more rapid rotation of Jupiter might have been in like manner produced, at the same time with his motion in his orbit, by an impulse acting at a distance of about 16,000 miles from his centre. It may happen that the movement of rotation thus communicated to a body may interfere with its onward motion, so as to destroy or even to reverse it. Thus, if a billiard-ball be struck sharply with a cue *below* its centre, it will receive from it at the same time a tendency to onward movement, and a tendency to rotate on its axis, in such a direction as to roll backwards if no other force were in operation. Hence it will not *roll* forwards, but will *slide*; and its friction is so great, that its onward movement is soon destroyed, whilst the tendency to rotation is only partially checked; and the very strange phenomenon is witnessed, of a ball moving forwards, gradually coming to a stop, and then rolling backwards, without receiving any fresh impulse, or meeting with any obstacle. This kind of stroke is used with great advantage by skilful billiard-players.

232. *Centre of Gyration.*—When a body is moving in a straight line, all its particles have the same velocity; and consequently the effect of its stroke or impact upon another body

is the same as if its whole weight were concentrated in the centre of gravity, and were moving with the velocity which it possesses. But when a body is moving round a centre, the motions of its different parts are not equal in velocity; for those which are furthest from the centre pass through a larger circle than those nearer to it, though the whole revolution is performed in the same time. Thus, as already remarked, the equatorial portion of a revolving globe moves much faster than the portion in the neighbourhood of the poles; and, of the different parts of the interior, those move fastest which are at the greatest distance from the axis. Now, since the *momentum* of each particle depends upon its weight multiplied by its velocity (§. 177), it is evident that the momenta of the several portions of the revolving globe will be very different. Nevertheless, it is not difficult to conceive, that the *sum* of all of them may be represented by a *single* force, equivalent to that which would be produced by the whole weight of the body, moving in a circle at a certain distance round the axis. Thus, if a circular plate be put in motion around one of its diameters,—as when we spin a piece of money on its edge,—the whole force of its rotation might be represented by a weight equal to that of the plate (if it were possible to concentrate such a weight in a single point), moving in a circle whose radius is equal to half that of the plate. Or, in other words, supposing the plate to be set in motion by a certain impulse, its velocity will be the same as that with which its whole weight, concentrated in one point, would move in a circle of half its own radius. All bodies moving round a fixed axis have a similar property; that is, there is in each one a certain point, at which, if the whole mass were concentrated, it would receive from the impulse the same velocity round the axis;—or, in other words, there is a point at which, if the whole weight of the mass were concentrated in it, the force of its motion would be precisely the same as that which the whole body actually possesses. This point is called the *centre of gyration*,* and its distance from the axis is called the *radius of gyration*. This distance varies according to the form of the body; but it is, of

* From the Latin *gyro*, which means to move round a centre.

course, greater, according as the principal mass of the body is disposed at a distance from the centre.

233. It is evident from what has been just stated, that if a body in motion about a fixed axis encounter an obstacle in the position of its centre of gyration, it will expend *all* the force of its motion upon that obstacle. If, on the other hand, the obstacle be not at that distance from the axis, the collected force will be divided between the obstacle and the axis itself, on the principle of a weight supported between two props. Hence we see that a blow with any instrument will be most advantageously given, when it strikes in the situation of its centre of gyration; and that the effect of the blow is not then felt by its axis, as it would be in any other case. Thus, a cricket-bat, and the arm of the player who strikes with it, may be regarded as a mass revolving round an axis in his shoulder-joint. If the ball be struck at the centre of gyration of this mass, the whole effect of the blow is expended upon the ball; and it has no *reaction* upon the shoulder, so that no part of the shock is felt there. Expert players soon learn to know about what point of a bat they thus strike most effectually; and in this consists a great secret of good batting.—In the same manner, a carpenter, when using a mallet, does not strike most effectually with the middle of its principal mass, which would be the position of its centre of gravity; but with a part farther from the handle, where the force of the motion is collected in the centre of gyration. If he strike with any point but this, he will not expend the whole force of the blow upon the object, but a part will return and *sting* his hand. We shall presently see another very important application of the same principle, in the case of the tilt-hammer (§. 237).

234. *Axis of spontaneous Rotation.*—Suppose a beam of wood to be floating at rest upon water, and to receive a blow near one of its extremities; that end will, of course, move forwards, but the other end will move backwards; and we should find a certain part of the beam which does not move at all, but serves as an axis, round which the other parts seem to revolve. This is called the *axis of spontaneous rotation*. Its position will depend upon the form of the body, and upon the place where it

is struck. It has been shown that, if a body at rest, but free to move in any direction, be struck by a force passing through its centre of gravity, there will be no rotation, but an onward movement of the whole mass: whilst, if the blow do *not* pass through the centre of gravity, some portions receive a greater effect from it than others, and consequently the mass is put in rotation about its centre of gravity, at the same time that it is projected forwards (§. 239). Now it is evident, that, in consequence of this double movement, some parts of the body are carried backwards by the rotation, whilst they are moving forwards with the mass; and if the velocity of the rotatory movement, carrying any part backwards, *exceed* the forward movement which it shares with the rest of the mass, it is evident that the part so acted on will really be moving backwards; as is seen in the case just mentioned. For the whole beam has an onward movement; but it has also a rotatory motion about its centre; and thus the extremity most distant from the blow moves backwards, by just as much as its rotatory movement exceeds the forward motion of the whole beam. The part of the beam which is really at rest, therefore, is that around which it tends of its own accord to revolve in the first instant of its motion,—when struck in such a manner, that the rotation of its extremities is more rapid than the onward movement of the whole. But if the blow were given nearer to its centre, the onward movement would be more rapid, and the rotation slower; so that the whole beam really moves forwards, though the side which receives the blow moves faster than the other. In such a case, we may conceive the centre of spontaneous rotation to be, not *in* the body, but at a point beyond its farther extremity. Or, we might find a point in the beam, such that a blow struck against it would make the beam revolve round one of its extremities as a centre; the backward rotatory motion of that extremity being exactly equal to the onward movement of the mass.

235. *Centre of Percussion.*—We further see that, since a blow communicates no motion to those parts of a body which lie in the axis of spontaneous rotation, a fixed axis passing through the body in that situation would receive no shock or *percussion*

from the blow. If the place of the blow were altered, however, the centre of spontaneous rotation would no longer be the same; and a shock would be communicated to the axis. The point at which a body supported upon a fixed axis may be struck, without the axis receiving any shock, is termed its centre of percussion; and it is evident that this must be in every respect the most advantageous situation for an impulse to be communicated to it; since the whole force of the blow will be expended in putting the body in motion, and none will be left to communicate a shock to the axis. There is an instrument termed the Ballistic Pendulum, used to determine the velocity of cannon-balls when first fired, which affords a very good exemplification of this principle. It consists simply of a heavy mass of wood, suspended at the end of a long iron bar. The velocity with which it begins to move, when the ball strikes it, is known by observing the height to which it ascends; and from this the force of motion and the velocity of the ball are easily calculated. Now if the ball is fired against the mass of wood, so as to strike it at its centre of percussion, it simply causes the mass to swing like a pendulum; but if the stroke take place at any other point, it will tend to tear away the axis.

236. It is a very curious fact, that, if the centre of percussion of a body moving round a fixed axis be ascertained—and the place of the axis be changed, so as to make it pass through what was before the centre of percussion—the point which was previously the axis becomes the centre of percussion to the new axis. This principle has a very important connection with the doctrine of the pendulum, as will be shown hereafter (§. 280).

237. The importance of attending to these principles in the construction of machinery, will appear from one simple illustration. The Tilt Hammer, used in the forging of iron and steel, is a mass of great weight, fixed to a strong arm (commonly a beam of wood) of considerable length. This is supported upon an axis, which turns in collars, firmly bound down to a solid mass of iron and masonry, and this is deeply imbedded in the earth. The hammer is raised by the action of a wheel, turned by steam or water power; on the circumference of which are

fixed *cogs* or projections, that strike upon the arm of the hammer, behind its axis. The hammer is thus made to rise, immediately after it has fallen (by its own weight) from the point to which it was raised just before. Now the expense of erecting and

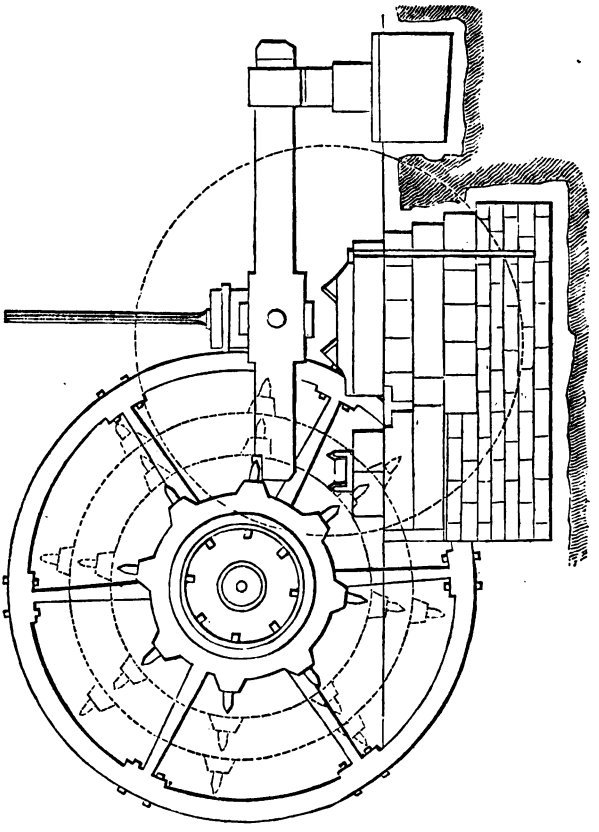


Fig. 53.

keeping at work one of these hammers is very considerable; for however securely they are fixed in the first instance, they are very liable to break their axes, and to tear away their collars. This is owing to a want of sufficient attention to

scientific principles in its construction. For if the hammer be formed and fixed in such a manner, that the blow which it receives from the wheel strikes its centre of percussion, the whole force of the stroke will be expended in lifting the hammer; and no *jar* will be experienced by the axle. And, again, if the blow given by the hammer be at its centre of gyration, the whole force of the stroke is communicated to the body beneath it; and the axis receives no shock or strain (§. 333). This seems never to be the case with those constructed upon the ordinary plan; since they all appear to expend a large part of the power which raises them, in beating about their axes, and in perpetual efforts to tear away their collars. Some practical knowledge of the kind of arrangement necessary appears to have been arrived at by the workmen employed in erecting these hammers; but this is of a very imperfect kind, and can only be rendered complete by the aid of scientific skill. This is one of the many instances, in which the benefit of science is shown (as the great Bacon long ago expressed it), "in shortening the long turnings and windings of experience."

CHAPTER VIII.

OF THE MOTION OF FALLING BODIES.

238. The fall of a body towards the earth is the most familiar and constantly-occurring example of the law of gravitation. But it does not serve as a good illustration of this law; since we *see* only one part of its action,—the descent of the body to the ground. But since the attraction of all bodies is *mutual*, there must be a movement of the earth towards the stone, as well as a motion of the stone towards the earth. This motion, however, is proportional, for each body, to the bulk of the other. Thus, suppose that the stone weighed 1 lb., and the weight of the earth were 1,000,000 lbs.;—for every foot the stone moves towards the earth, the earth would move towards it 1,000,000th of a foot,—a quantity so small as to be quite inappreciable. But the disproportion between the bulk of the earth and the largest bodies that ever fall on its surface, is really far greater than this; so that the space through which the earth moves to meet the stone is so excessively minute, when compared with the space through which the stone falls to the earth, that we may disregard it in all our calculations. But the earth itself, if its movement round the sun were checked, would fall towards *it*, just as a stone drops to the earth; for the bulk of the sun is nearly 1,400,000 times that of the earth; and if these two bodies were free to move towards each other, therefore, the sun would only move 1 mile for every 1,400,000 miles traversed by the earth.

239. The motion of a stone in its fall to the earth is of that kind which is termed *uniformly accelerated* (§. 156); and the reason why it should be so is very obvious, when the nature of

attraction and the laws of motion are considered together. The attraction of the earth for the stone is continually generating a fresh amount of force; but if this were made to cease at any time, the body would continue to move at a uniform rate, with the force it had previously acquired. Hence, when a new amount of force is being constantly added to that previously existing, the motion must necessarily be uniformly accelerated. This principle is illustrated by a familiar comparison. Every school-boy knows that a long pea-shooter will *carry* much farther than a short one. The natives of South America shoot birds with arrows impelled by the breath through a hollow cane of six or eight feet long. A long gun, moreover, will *carry* much farther than a short one; and the ball, therefore, leaves its mouth with a much greater rate of movement, than it has acquired when it leaves the mouth of a short gun. The reason is very obvious. So long as the body to be propelled continues within the tube, it is being acted on,—by the force of the breath, in the case of the pea-shooter,—by that of the powder, in the case of the gun. It is continually receiving, therefore, a fresh amount of force, whilst in motion from that already applied, and its rate of movement within the tube will therefore be constantly increasing. When it quits the tube, however, it ceases to be acted on by any new force; and its rate of motion will entirely depend upon that which it had acquired at that moment. As its subsequent motion is being continually retarded by the friction and resistance of the air, as well as by the attraction of the earth, the distance to which it will be projected will, of course, be proportional to the rate of its movement, or the force it had gained when it left the tube.

240. Now if we consider the nature of the motion of a stone allowed to fall freely towards the earth, we shall at once perceive the influence of the same cause,—the continual application of a fresh amount of force, in producing the same result,—a continually accelerated motion. For at the moment at which the stone is let go, it has no velocity at all; when the earth's attraction first puts it in motion, its rate is very slow: if that attraction, however, could be suspended at any point of time, the stone

would continue to move, by the force it had acquired with a regular velocity; but as this attraction is continually operating, it must, by constantly adding to the force with which the body was previously moving, produce a constant increase in its rate. Thus, a heavy stone, allowed to fall freely to the ground, will move in the 1st second through a little more than 16 feet; but at the end of that time it will have acquired a force which would be enough, if acting alone, to carry it through 32 feet in the next second. In the next second, however, it moves not only through these 32 feet, by the force it had acquired in the first, but through 16 feet additional, on account of the fresh impulse it is continually receiving through the earth's attraction, which is precisely the same during the 2nd second as during the 1st. Moreover, at the end of this 2nd second, it will move with the same continued rate of 32 feet per second, which it had acquired at the end of the 1st second; and with an additional velocity of 32 feet, which it has acquired during the 2nd second; and thus its rate of movement at the end of the 2nd second will be 64 feet per second. It will continue to move with this velocity during the 3rd second; but will also gain, from the constant attraction of the earth, the same increase as in the 1st and 2nd seconds,—namely, an additional *fall* of 16 feet during the second, and an additional *rate* of 32 feet at its end. The space fallen through in each of the 1st three seconds will therefore be, 16, 48, 80 feet; and the velocity acquired at the end of each will be 32, 64, 96 feet.

241. Carrying on the same mode of reckoning, we should find that the *velocity acquired* at the end of any given number of seconds, is twice that number of seconds multiplied by 16 feet, the space fallen through in the 1st second. And as the space fallen through in any given second, is determined by the velocity it had acquired at the end of the preceding second, with the fresh movement it gains in the second itself, the amount is at once ascertained by adding 16 feet to the velocity gained at the end of the preceding second. The following table will show the results of this simple calculation for the first ten seconds :—

VELOCITY ACQUIRED AT THE END OF EACH SECOND.				SPACE FALLEN THROUGH IN EACH SECOND.	
		16 feet		16 feet	I.
I.	32 feet	+ 16 feet	=	48 ...	II.
II.	64 ...	+ 16 —	=	80 ...	III.
III.	96 ...	+ 16 —	=	112 ...	IV.
IV.	128 ...	+ 16 —	=	144 ...	V.
V.	160 ...	+ 16 —	=	176 ...	VI.
VI.	192 ...	+ 16 —	=	208 ...	VII.
VII.	224 ...	+ 16 —	=	240 ...	VIII.
VIII.	256 ...	+ 16 —	=	272 ...	IX.
IX.	288 ...	+ 16 —	=	304 ...	X.

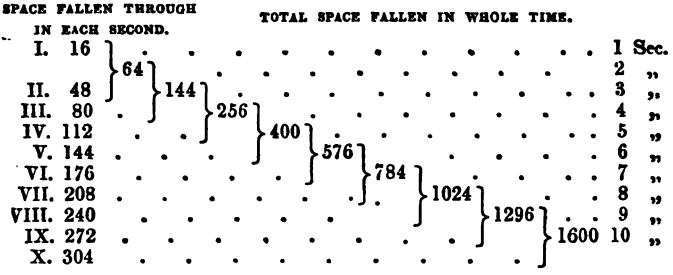
The column on the left gives the velocity acquired at the end of each second; this added to 16 feet, the additional quantity gained in each second, gives the whole amount through which the body moves in the *succeeding* second, as expressed in the right hand column. Now, when the numbers in this column be examined, they will be found to be multiples of 16 by the series of odd numbers, commencing with 1; and it is a very simple mode of reckoning the space fallen through in any given second, to multiply by 16 the odd number corresponding to that of the second in such a series as the following:

I.	II.	III.	IV.	V.	VI.	VII.	VIII.	IX.	X.
1	3	5	7	9	11	13	15	17	19.

It will be further remarked, that each odd number is *one less than double* the number of the second. Thus, if we wish to know what would be the space fallen through by a stone in the 15th second of its descent, we double 15, and take 1 less, which gives us 29, the odd number corresponding to XV. in this series; on multiplying 29 by 16, we get 464, the number required.

242. We more commonly wish, however, to know the *whole* space fallen through in any given number of seconds. This may, of course, be estimated by adding together the amount fallen through in each. Thus, in the first two seconds, the body falls through (I.) 16 + (II.) 48 = 64. In the first three, it falls through (I.) 16 + (II.) 48 + (III.) 80 = 144. And in the first four, it falls through (I.) 16 + (II.) 48 + (III.) 80 + (IV.) 112 = 256. Now these numbers, 16, 64, 144, 256, which express the total space fallen through in the 1st, 2nd, 3rd, and 4th seconds respectively, are found to

correspond exactly with the squares of these last numbers, multiplied by 16. Thus, for the 1st second, $1 \times 1 \times 16 = 16$, the number of feet fallen through in it. For the two first seconds, $2 \times 2 \times 16 = 64$, the number of feet fallen through in them. For the 3 first seconds, $3 \times 3 \times 16 = 112$, the number of feet fallen through in them. And, for the 4 first, $4 \times 4 \times 16 = 256$, the number of feet fallen through in them. Hence the rule for ascertaining the whole number of feet fallen through in any given number of seconds is simply this:—Square the number of seconds, and multiply the product by 16, which will give the amount required. Thus, if I let fall a stone from the top of a precipice, and 7 seconds elapse before I hear the sound of its arrival at the bottom, I should know that the height of the precipice would be $7 \times 7 \times 16 = 784$ feet.* The following table will show the results of the foregoing calculation up to the 10th second :—



243. The whole subject may, perhaps, be more easily comprehended by the aid of a very simple diagram. In the whole triangle, A B C, (Fig. 54) the perpendicular side, A B, expresses the time through which a body is falling; in the present instance, it is divided into 5 parts, which represent 5 seconds. The horizontal side, or base, B C, expresses the force acquired by the body at the end of that period; and the horizontal lines at IV., III., II., and I., in like manner express the force possessed by the falling body at the end of each of the

* This estimate would have to be corrected for the loss of time occasioned by the travelling of the sound from the bottom of the precipice to the top; as will be explained in the Treatise on Sound.

preceding seconds. As A represents the point from which the body is let fall, its force is *there* nothing; but it is undergoing continual increase, as represented by the constant tendency of the slanting side A C towards the right, so as to increase the lines at I., II., III., IV., and V., in regular proportion. Now, the body setting off with no force, acquires at the end of the first second, a force expressed by the horizontal line at I., and in so doing, it passes through a space represented by the small triangle 1. The force acquired at the end of the first second would *of itself* carry it, in the next second, through a space represented by the parallelogram, of which one side is represented by the *force*, and the other by the *time*; and this parallelogram contains the two triangles 2, and 3, each equal to 1. But during the 2nd second, there is an additional force gained, equal to that acquired in the first; and this causes the body to traverse the additional space represented by the triangle 4. Hence, the whole space which the body falls through in the 2nd second, is represented by the three triangles, 2, 3, 4; aud the force it has acquired at the end of the 2nd second is represented by the horizontal line at II. This force would of itself carry it in the 3rd second through a space represented by the parallelogram that contains the triangles 5, 6, 7, 8; and the additional force gained in that second is represented by the triangle 9. The same principle applies to the representation of the 4th and 5th seconds; and by prolonging the triangle downwards, any number of seconds might be represented in the same manner.

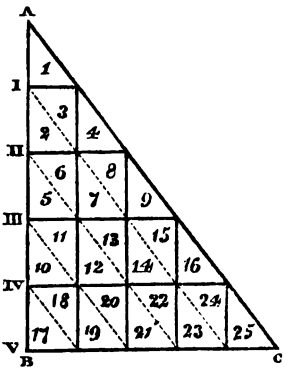


FIG. 54.

244. In this diagram there are set before the eye the rules already laid down. For it is seen that, if we call the space fallen through in the first second 1,—the space fallen through in the next second will be, by the previously acquired velocity 2, and by the additional velocity 1, making in all 3,—and the total

space fallen through in the first two seconds will be 4, the square of 2. In the same manner, the space fallen through in the 3d second is altogether represented by 5 triangles; that of the 4th second by 7 triangles; and that in the 5th second by 9 triangles; —in each case, by one less than double, as already explained. And, lastly, the whole space fallen through in any given number of seconds is represented by the whole number of triangles above its horizontal line;—being, for 1 second, 1,—for 2 seconds, 4,— for 3 seconds, 9,—for 4 seconds, 16,—and for 5 seconds, 25; in each case, the square of the number of seconds.

245. Some simple exercises may be advantageously founded upon these rules; serving to test the student's acquaintance with them. Thus, it is required to know which of two stones, A and B, will arrive first at the ground, A being let fall at a height of 1024 feet from the ground, and B, two seconds afterwards, being let fall at a height of 576 feet. It is evident, then, that B commences its fall from a point 448 feet below A, and that the first second of *its* fall will correspond to the third second of A's. Hence, in order to compare the relative situation of the two at each second, we must make such a table as the following; in which the place of A is given by the number of feet it has passed through from the commencement of its fall; whilst that of B is found by adding to the number of feet through which it has fallen at each period, the amount (448) which it was below A when the former began to fall.

	A FALLS.		B FALLS.				
I.	16						
II.	64						
III.	144		I.	16	+ 448	=	464
IV.	256		II.	64	+ 448	=	512
V.	400		III.	144	+ 448	=	592
VI.	576		IV.	256	+ 448	=	704
VII.	784		V.	400	+ 448	=	848
VIII.	1024		VI.	576	+ 448	=	1024

Hence it appears that they will both arrive at the ground at the same time; since, although B is 320 feet below A at the end of the first second of its descent, A gradually gains upon B, in consequence of the greater velocity it has acquired through its longer time of falling. In the next second, supposing the fall

to continue, A would leave B behind; for this being the 9th second of A's fall, it will have passed through 1296 feet at the end of it; whilst, being only the 7th second of B's, the latter will have only fallen through 784 feet, which, added to 448, makes its distance but 1232 feet from the point at which A started; and this difference would go on increasing with every succeeding second.

246. These laws regulating the descent of falling bodies, hold good not only when the bodies are allowed to fall freely through the air, but also when, by any artificial means, their proper velocity is diminished; and this fact has been very beautifully applied in the construction of a machine by which they may be experimentally illustrated. We will suppose two equal weights, A and B, suspended by a line that passes over a pulley; it is obvious that these weights will remain at rest, provided that they are not made to move by any force applied to them. But, if we add a small weight to one of them, that one will of course descend. It will not move, however, with a velocity of 16 feet during the first second, or a rate at all near it; for though the small weight, if left to itself, would fall at this rate, the attraction of the earth for it, when it is attached to one of the weights, causes it to set in motion both the weights, and also the string and pulley. By far the largest proportion of its force, therefore, is expended in overcoming the *inertia* (§. 147) of these bodies; and the velocity of the movement which it makes in connection with them will be proportionably less. Thus we will suppose the two weights, A and B, each to weigh 7½ oz., making together 15 oz., and the additional weight to be 1 oz. Then the attraction of the earth for this 1 oz. weight will have to put in motion 16 oz. weight;* and thus the force having to do 16 times the work, can only do it 1-16th as fast, so that the weights will move, one down and the other up, at the rate of only 1 foot per second, instead of 16. But supposing the motion to continue, it will take place according to the same laws as those which govern the ordinary descent of falling bodies;—that is, the space fallen through in the 2nd

* In this calculation, the inertia of the wheel, for the sake of simplicity, is not include

second will be 3 times that of the 1st, or 3 feet; and the total space fallen through in the first 2 seconds will therefore be 4 feet. In like manner, the spaces fallen through in the 3rd, 4th, and 5th seconds respectively, will be 5, 7, and 9 feet; and the *total* space fallen through at the end of the 3rd, 4th, and 5th seconds, will be 9, 16, and 25 feet,—that is the square of the number of seconds multiplied into the space fallen through in the first second, which, in the present instance is 1 foot.

247. This is the principle of the very beautiful instrument named Atwood's Machine, after its inventor. It consists of an upright scale of feet and inches, which may be made of any convenient length,—usually 6 or 7 feet. At the top of this is a pulley, the spindle of which turns upon friction-wheels, (§. 237), so that the loss that would be caused by its friction is prevented as much as possible. There is attached to the machine a small piece of clock-work, with a pendulum adjusted to beat seconds. Over the pulley hangs a line with a weight at each end; and the weights are so made, that, by taking off or putting on different pieces, we may give them exactly the proportion to each other which we desire. If they are equal, they will hang stationary, as in the former instance; but if we put them in motion, they will continue to move at the same rate as far as the length of the line permits,—friction being almost entirely removed, and the resistance of the air not making a perceptible difference in so short a space. But if we add a small weight to either of them, that one will descend with a uniformly-accelerated motion, as already stated. By varying the proportion between the additional weight and the sum of the two, we can exactly bring the velocity to any scale we please. The most convenient is one of 3 inches per second; and it is obtained by the following adjustment. This velocity is 1-64th of the velocity of a falling body left to itself (16 feet = 192 inches, which, divided by 3, gives 64); and therefore the attrac-

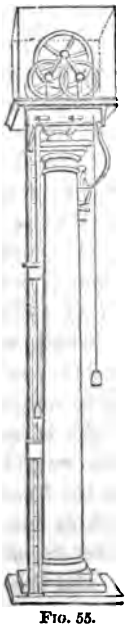


Fig. 55.

tion of the earth for a certain weight must be made to move 64 times that weight, in order to produce a movement of only 1-64th the velocity. Hence (putting out of view the inertia of the pulley, which has also to be considered), each of the weights should be $31\frac{1}{2}$ times the additional weight, so that this may be 1-64th of the whole.

248. The heavier weight, being thus adjusted to descend through 3 inches in the first second, will pass through $3 \times 3 = 9$ inches in the 2nd second, $5 \times 3 = 15$ inches in the 3d second, $7 \times 3 = 21$ inches in the 4th second, $9 \times 3 = 27$ inches in the 5th, and so on. In like manner, the space fallen through in any number of seconds, will be the square of that number, multiplied by 3 inches, the space fallen through in the first second; thus, the whole space fallen through in 5 seconds will be $5 \times 5 \times 3 = 75$ inches. This is shown on the machine, by fixing a flat stage at the several parts of the scale of inches to which the weight ought to descend in 1, 2, 3, 4 and 5 seconds respectively; the heavier weight being then drawn up to the top, and let go exactly at the same time with any beat of the pendulum, it will be heard to strike upon the stage exactly with that one of the succeeding beats for which its place has been adjusted.

249. The machine will not only thus illustrate the laws regulating the space fallen through in each second, and the total amount of the whole; but it will also show, by an ingenious contrivance, the velocity acquired at the end of any particular second, and the rate at which the body would continue to move by it alone, the force of gravity being suspended. This is accomplished by giving to the additional weight the form of a bar, and laying it on one of the weights, in such a manner as to be lifted off by a perforated stage, through which the weight itself can pass. When the additional weight has thus been lifted off, the two weights become equal, and they continue to move, therefore, solely by the force they had acquired. Now it has been shown that the velocity acquired at the end of any number of seconds, is twice that number of seconds multiplied by the space fallen through in the first second. Thus, if the perforated stage be fixed at 12 inches down the scale, and the

solid stage at 12 below this, the heavier weight will descend through the first 12 inches in two seconds; but it will then have acquired a velocity which will enable it to go through the next 12 inches in another second, even when the additional weight is removed. If the solid stage were fixed at 24 inches below the first, that space would be traversed in 2 seconds; if at 36 inches, in 3 seconds, and so on;—the velocity of 12 inches per second, acquired during the first two seconds, having a uniform continuance until it is checked. In like manner, if the perforated stage be placed at 27 inches down the scale, and the solid one be placed at 18 inches below this, the loaded weight will descend to the first in 3 seconds ($3 \times 3 \times 3 = 27$); and it will pass through the next 18 inches, after its additional weight has been removed, in the succeeding second,—since its acquired velocity at the end of the 3d second is 18 inches per second ($2 \times 3 \times 3$), continuing, if allowed to do so, at the same rate, for any number of seconds.

250. The influence of gravitation upon a body which is thrown up from the earth's surface with a certain force, has the effect of progressively *retarding* its movement, until it is at last completely checked; this retardation of a *rising* body is uniform, and is governed by the same law as the *acceleration* of a *falling* body. For it is obvious that the earth's attraction will make a certain diminution, during the first second, in the rate of the body's motion, so that it will commence the next second with a retarded velocity; the continued attraction downwards will make a further diminution during the succeeding second; and thus its rate will be gradually diminished by the end of each second, in exactly the same proportion as it would have increased if the body had been moving towards the earth,—namely, 32 feet for each second. Thus, we will suppose a body projected upwards from the earth's surface at the rate of 160 feet per second; during the first second of its movement, it will be gradually retarded, so that it will only move through 144 ($160 - 16$) feet; and by the beginning of the next second its velocity will have been reduced to 128 feet per second. In the next second it will only move upwards to the amount of 112 feet; and its velocity at the end of that time will only be 96

feet. By the same diminution, in the succeeding seconds, its rate of motion at the beginning of the 5th second will be only 32 feet; and, owing to the constantly-retarding force of the earth's attraction, the whole of this force will be spent in carrying up the body through 16 feet more, after which it will cease to rise, having been altogether carried up to the height of 400 feet: The moment it ceases to rise, it must begin to fall again; and in 5 seconds more it will reach the ground, having fallen 400 feet, and acquired the velocity of 160 feet per second, the very same as that with which it was first projected.

251. Hence the law with reference to *rising* bodies is exactly the converse of that which has been stated in regard to their *falling*; for the movement of the former is uniformly retarded by the earth's attraction, to the same amount that the velocity of the latter is increased by it; and the height which a body thrown up with a given force will reach, is exactly the same as the height through which a body must fall in order to acquire that force, and may therefore be determined by it. Thus if we wish to know how high a body will rise, that is thrown up with a velocity of 224 feet per second, and how many seconds will elapse before it reaches the ground again, we ascertain it by considering how many seconds would be required by a falling body to acquire that velocity; this number is 7, for $2 \times 7 \times 16 = 224$. The body will continue to rise, therefore, for 7 seconds, and will gain a height of 784 feet ($7 \times 7 \times 16$); after which it will occupy 7 seconds in its fall, and will come down with the velocity with which it was projected upwards.*

252. The phenomena of retarded motion may be beautifully shown by Atwood's Machine. For this purpose, the two weights A and B are so adjusted, that one of them, A, shall exceed the other by a certain quantity, say half an ounce; which difference shall cause it to descend at the rate of 4 inches per second. But we lay upon B an additional weight equal to double this difference; so that B is now the heavier by half an ounce. This additional weight, being in the form of a bar, may be removed

* In this statement, it is not thought worth while to take into account the retarding influence of the resistance of the air, which will slightly alter the result.

by the perforated stage at any point we choose, so that A again becomes the heavier. Now if we fix the perforated stage at 36 inches down the scale, and draw up B, loaded with the bar, to the top, it will fall through this space in 3 seconds ($3 \times 3 \times 4$), and will have gained a velocity of 24 inches per second ($2 \times 3 \times 4$); but the bar is then removed by the perforated stage, and A becomes the heavier. The velocity which B had gained, however, occasions the continuance of its movement in the same direction, for a certain time; but the movement is continually retarded by the superior weight of A, and will be gradually brought to a close, and B will begin to ascend. This will not happen until B has descended to 36 inches below the perforated stage, the limit to its extent being exactly determined by the space through which it had previously fallen;—and this movement will occupy the same time, namely 3 seconds. The return of B to the perforated stage will occupy 3 seconds more; and it will have acquired, by that time, the same velocity as at first,—namely 24 inches per second. In passing through the perforated stage, it will again take up the bar, and thus be made as much heavier than A, as it before was lighter. It will continue to rise, however, by the velocity it had acquired, and will reach the top of the scale in 3 seconds more, after which it will begin to descend again, and the whole process will be repeated as at first. This alternate rise and fall of the weights would continue forever, like the swinging of a pendulum, if it were not for the friction of the pulley and the resistance of the air; these opposing causes make each movement somewhat less than the preceding one, and at last bring them to a close.

253. It is through the resistance of the air, that the rates of falling are so different in different cases. Thus, a stone or a piece of metal fall at once to the ground; whilst a feather or a shred of thin cloth are comparatively long in reaching it. Both are resisted by the air in proportion to the surface they expose to it; but whilst the surface of the stone or metal is very small in proportion to its weight, that of the feather or cloth is very large. A piece of gold, extended into a thin leaf, is buoyed up by the air just as much as a feather or piece of paper; and if it

were possible to compress a feather into such a small compass, that it should only have the same surface with a stone of equal weight, it would fall just as fast. The truth of this statement may be shown by making a piece of metal and a feather fall together through a tall receiver, from which the air has been exhausted; they will reach the bottom in precisely the same time. This is commonly termed the *guinea and feather* experiment. It was one of the most remarkable errors of ancient philosophers, that they imagined the time occupied by a body in falling, to be in contrary proportion to its weight,—that is, that a stone of ten pounds would fall ten times as fast as a stone of one pound, or in one-tenth part of the time. This error probably originated in the observation of the slow fall of what are commonly termed *light* bodies (§. 97). The attraction of the earth would produce the same effect upon all; and the difference is due only to the varying resistance of the air. Between a stone of one pound, and another of ten pounds, however, the difference of the resistance of the air would be very slight; and it seems wonderful that so easy an experiment, as that of comparing their times of fall

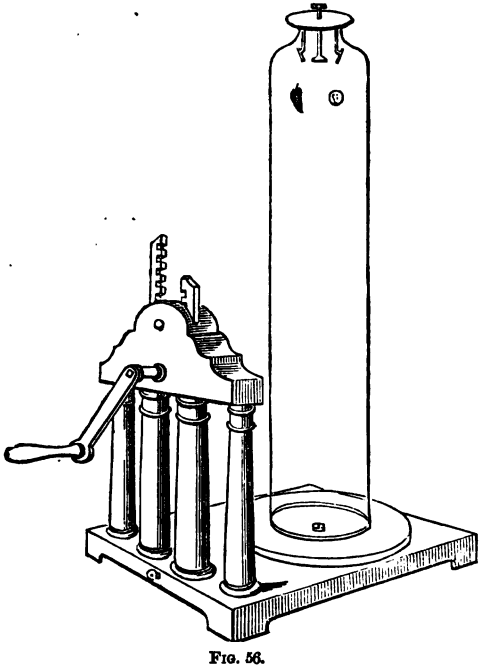


FIG. 56.

through the same height, was never made. This ridiculous error was long kept up by the authority of Aristotle, whose supremacy in matters of philosophy no one dared to question during the middle ages; and it was first overthrown by Galileo at the end of the sixteenth century.—The degree in which the resistance of the air acts on what are ordinarily termed *heavy* bodies, is shown by the fact, that a bullet descending freely through the atmosphere does not ever gain a greater velocity than about 260 feet per second, whatever may be the height through which it falls;—the resistance of the air being then so great as to resist any further acceleration. (See §. 151.)

CHAPTER IX.

THE PENDULUM.

254. By a pendulum we are to understand a heavy body suspended from a fixed point by a rod or cord, and made to swing backwards and forwards by an impulse which draws it out of the perpendicular direction. This simple instrument is of the greatest importance to us in the measurement of time, and therefore in the determination of a vast number of natural phenomena; thus in astronomy, the exact determination of time is necessary for almost every observation of the heavenly bodies; and it was by accurately observing the differences in the number of beats of the same pendulum during a given time, at different parts of the earth's surface, that the difference between the polar and equatorial diameters of our globe was first ascertained (§. 95).

255. There is little difficulty in comprehending certain obvious principles, on which the action of the pendulum depends. We shall in the first instance, for the sake of simplicity, consider the weight or bob as spherical, and the line that suspends it as having itself no weight; so that the centre of gravity of the whole is in the centre of the bob. Let A B be such a pendulum, hanging at rest from the point A; it is obvious, from what has been formerly stated regarding the properties of the centre of gravity, that this will be in a state of rest only when it is in the lowest position that it can assume, which is, of course, when the line A B is perpendicular. But suppose that the ball B is drawn or pushed out of the perpendicular, as to C; then it will tend to return to the point B, precisely as would a rounded body set free to roll down

an inclined plane; since in so doing it will descend through the height E B. But its motion, at first slow, will gradually increase in rapidity, on the principles explained in the preceding chapter; and by the time that it has arrived at B, it has acquired a considerable velocity. This acquired velocity is sufficient to carry the body onward in the same circular line, even though it is no longer descending, but, having passed the lowest point B, it begins to rise again. For it will be evident, from the law of retarded motion (§. 250), that the velocity gradually acquired by the ball in its accelerating descent from C to B, is exactly that which, if applied in the opposite direction to the ball when at B, would move it from B to C with a velocity gradually retarded by the influence of gravity; and which, acting on it in the direction B C, will carry it onwards to the point D at an equal distance on the other side. At D it would come to a stand, the moving power being then expended. Having arrived at D, it recommences its descent towards B, and gathers, as before, enough of force to carry it beyond B, to C, the point from which it started, and from which it will again return in the same manner.

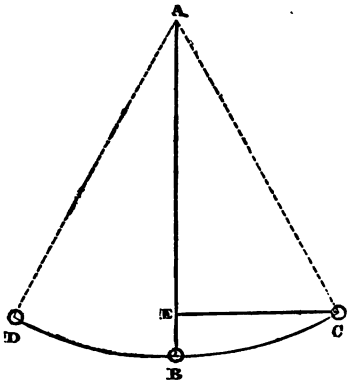


FIG. 87.

256. The vibrations of such a pendulum, even if friction could be done away with at the point of support, must come to an end in no very long time; since the extent of every one is rendered less than that of the last, by the resistance of the air. A pendulum made to vibrate in a perfect vacuum, however, will continue for many hours.

257. The length of time which is required for each vibration is but little influenced by the degree of *swing* which it has; that is, the vibrations occupy nearly the same time whether they are

long or short. Indeed, for all practical purposes, the times of vibrations of different lengths may be regarded as equal, at least while they are not *very* long; as will be presently shown. This property is termed *isochronism*, from two Greek words signifying equal times; and the vibrations are said to be *isochronous*, when long and short vibrations are performed in the same time. The isochronism of the pendulum was not known before the time of Galileo; and is said to have been first observed by him when very young, in the swinging of a lamp hung from the roof of the cathedral at Pisa. He was struck with the equality of the times in which the lamp continued to perform its vibrations as its motion subsided; "and this observation of a *child* became in the mind of the *man* a principle of philosophy, on which some of the greatest discoveries of science have been founded." But the time of the vibrations depends upon the length of the pendulum; and for every measure of *length*, there is a certain determinate *time*. Thus, the length of a pendulum which should occupy one second in each vibration (such as that of a common eight-day clock), would be found to be about 39$\frac{1}{7}$ inches,—if we could measure the distance from the point of suspension to the point in which its whole weight might be regarded as concentrated. Now the length of a pendulum vibrating half-seconds, such as that of a small table-clock, would be about 9$\frac{3}{4}$ inches, or one-fourth of the last, reckoned in the same way; whilst the length of a pendulum whose beat occupies two seconds, is about 156$\frac{1}{2}$ inches, or four times the last. Thus we see that, when the time is doubled, the length is quadrupled; and that, when the time is only half, the length is a quarter. The law expressing the proportion between the time of vibration and the length of the pendulum is, therefore, simply this; —that the time varies in proportion to the square of the length. Thus a pendulum whose vibrations are required to be 3 seconds in length, must be 9 times the length of that vibrating single seconds, or about 352$\frac{1}{4}$ inches, almost the height of a common three-story house; whilst a pendulum to vibrate only thirds of a second, must be only 1-9th the length of a seconds' pendulum, or about 4$\frac{1}{9}$ inches.

258. In order that the laws regulating the action of the pendulum may be properly understood, it will be necessary to consider the mode in which the force of gravity will act upon bodies, in causing their descent along inclined surfaces, whether these surfaces are *plane* or curved.

259. Every one knows that a body left to itself will roll or slide down a hill, provided that its friction be not too great to prevent its being set in motion. We are continually in the habit of trying the level of a table, by putting a marble, a ruler, a pencil-case, or any round body, upon it; and observing if it has a tendency to roll in either direction, or remains at rest in all positions. We are also familiar with the fact, that the rapidity of the movement depends on the degree of steepness or inclination of the surface,—that is, upon the amount of its *fall* in a given length,—being greater as this is increased. And it is further a matter of common observation, that the body commences its movement slowly, and increases in its rate, until it arrives at the bottom. The modification in the law of gravity, which is applicable to this case, is very easily understood. Putting aside the effects of friction and resistance of the air, a body moving down an inclined plane will have acquired, by the time it reaches the bottom, a velocity exactly equal to that which it would have acquired by falling directly through a height equal to that of the plane. Thus, suppose an inclined surface to be 144 feet higher at one end than at the other; a body, in falling directly through that height, will occupy 3 seconds; and at the end of that period, it will be moving with a velocity of 96 feet per second (§. 242). This velocity, then, will be precisely that which the same or any other body would acquire in rolling down an inclined plane having the same height, *whatever be the length of the plane.* Thus, the plane might be so long—a mile for instance—that its inclination would be scarcely perceptible; or it might be so short—200 feet for example—as to be steeper than the steepest accessible hill: in both instances, the velocity acquired at the bottom will be the same, so that the body would move over the same space of level ground in the next second; but the length of time

necessary for the acquirement of that velocity will be very much greater in the former case than in the latter, and the rate of movement will therefore have been much slower.

260. It is evident, then, that we can so proportion the length of an inclined plane to its height, as to make the rate of movement what we please. Thus, if we gradually diminish the height, the descent will be slower and slower, until the plane becomes level, in which case there will be no movement at all; or we may gradually increase it, until the plane becomes perpendicular, and then the body upon it will fall freely, according to the regular law. In any case, there will be precisely the same proportional acceleration of the movement during the descent as when the body is falling freely; but the rate per second will be diminished according to the amount of inclination, just as it is in Atwood's machine (§. 247), by the partial counterpoise of the weight. As the *rate* of movement which the body has acquired at the end of its descent, is always that which it would have gained when falling perpendicularly through the same height, whilst the *time* which is occupied in its fall depends upon the *length* of the plane, it is obvious that the rate per second at which the body commences its movement will depend upon the proportion between the height of the plane and its length. Thus, supposing that the plane be 144 feet high, and 288 feet long; a body would require only 3 seconds to fall through its height perpendicularly, but would occupy 6 seconds in rolling down the incline, commencing with a velocity of only 8 feet per second, instead of 16. Its rate of movement will increase in the regular proportion; thus falling 8 feet in the first second, it will fall (4×8) 32 feet in 2 seconds, (9×8) 72 feet in 3 seconds, (16×8) 128 feet in 4 seconds, (25×8) 200 feet in 5 seconds, and (36×8) 228 feet or the whole length of the plane in 6 seconds. At the end of this time it will have acquired the velocity of $(6 \times 2 \times 8)$ 96 feet per second, which would have been acquired by a body falling perpendicularly in 3 seconds. But suppose that the plane had a length equal to 16 times its height; the body would then require 16 times 3 seconds for its descent, and

would only commence at the rate of 1 foot per second. The whole space passed through in 48 seconds is found by multiplying the square of 48 by 1 foot (the space passed through in the 1st second), as in the instances formerly stated (§. 246) ; and this gives 2304 feet, the length which the plane has been supposed to be (16 × 144). The velocity acquired at the end of the descent is found, by the same rule, to be (48 × 2 × 1) 96 feet, the same as that of a body falling through a steeper plane, or through the perpendicular.

261. Hence we see that, when a body descends along an inclined plane, its rate of motion is diminished in the exact proportion in which its length exceeds its height; but that the law of acceleration, owing to the continual action of the force of gravity, is exactly the same as if the body were falling freely. It is easy to understand why this diminution of rate should take place. For when a body is resting on a level surface, the whole of its weight is resting on that surface, and its gravity cannot, therefore, put it in motion. On the other hand, if the plane be raised into the perpendicular position, none of the weight is supported by it, and the force of gravity acts freely on the body. In all intermediate positions, a larger or smaller part of the weight is supported, in proportion as the plane approaches the horizontal or the perpendicular position. Now the action of the force of gravity upon a part of the body has to put in motion the whole of it, just as in Atwood's machine; and thus the rate of its movement must be diminished, according to the proportion between the moving power and the amount to be moved. How this proportion is to be estimated, will be explained when the Inclined Plane is considered as one of the mechanical powers (§. 352).

262. When a body moves down a *curved* surface, however, the case is materially altered. In the inclined plane, the amount of descent or fall is the same in every part of the plane; and consequently the acceleration of the body's motion is *uniform*. But this is not the case in a curved surface, as will be evident from a little consideration. In the accompanying figure, the straight line A B represents an inclined *plane*, which is

divided into eight portions by horizontal lines drawn at equal distances; these portions are equal, so that the *length* of the plane, everywhere, corresponds with its *height*. But A C B and A D B are two portions of circles, beginning and ending at the same points; and the movement of a body along surfaces having these curvatures will be very different from that of a body descending the plane A B. For it is shown by the figure that, at the beginning of its course, the curve A C B departs but little from the level; so that for a body to descend through 1-8th of its height, it must pass through much more than 1-8th of its length; whilst, in the latter part of its course, its rate of descent is much more rapid, since it approaches more nearly to the perpendicular. Precisely the opposite is the case in regard to the curve A D B. The rate of movement of a body descending through the curve A C B will be at first slow, therefore, but will afterwards increase more rapidly; whilst, if descending in the curve A D B, its rate will be at first much more rapid than on the inclined plane, but it will afterwards be increased in a much less degree. The precise rate of a body descending through either of these curves, or any others, can be determined by mathematical investigations, which would be here out of place; but the curious result is obtained from all,—that, whatever be the nature of the curve, the velocity which the body has acquired at the end of its descent, is exactly that which it would have acquired in its descent along an inclined plane of any length but having the same height, or in its perpendicular fall through a space equal to the height of the plane.

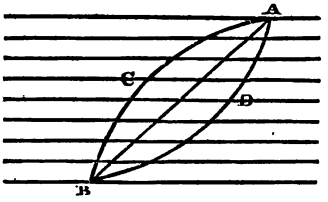


FIG. 58.

263. It is not difficult to understand that a curve might be so formed, that the amount of its inclination at every point shall be in proportion to the distance (along the curve) of that point from the bottom; as for instance that, half way up the ascent, the curve shall rise 1 inch for a foot of its length;

whilst, at twice the distance from the bottom, the curve shall rise 2 inches for a foot of its length. Such a curve will have this remarkable property,—that, from whatever part of it a body be allowed to descend, it will reach the bottom in the same time. This will be easily understood by reference to the subjoined figure. Let A B be the curve; and Q and P two bodies so placed, that the distance P A (measured along the curve) is equal to twice Q A; then, by the nature of the curve, its inclination at the point P is twice as great as at Q, and a body which begins its descent at P will have twice the rate of the body which starts from Q. In the first second, therefore, P will have fallen twice the distance, reaching p 1; whilst the body Q reaches only q 1: and as the distance P p 1 is twice Q q 1, the remaining distance A p 1 is equal to twice the remaining distance A q 1; hence the acceleration during the next second of the motion of the body P, will be twice that of the body Q; and P will move through the space between p 1 and p 2, whilst Q moves only half the distance, that between q 1 and q 2. In the same manner, the distances which remain to be traversed by P and Q during the third second are still as 2 to 1; and the rate of P being all along double that of Q, these distances will be performed in equal times, so that the two bodies will arrive at A together.

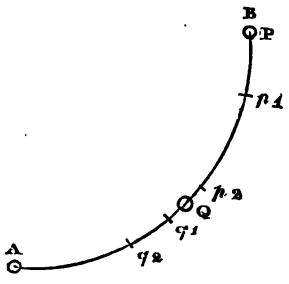


FIG. 59.

264. The same principle holds good in regard to bodies commencing their descent at *any* part of the curve. Thus, suppose Q to commence near the bottom, and P at a distance (measured along the curve) equal to six times that of Q; then the inclination or *fall* of the curve at P being six times that of the curve at Q, the rate of a body starting at P will be six times that of a body starting at Q; and as, from the nature of the curve, the acceleration of motion, during each succeeding second, bears the same proportion in both cases to the rate with which they

started, the body P will in each second move six times as fast as Q in the corresponding second, and will therefore arrive at the bottom at the same time with it. Such a curve is called an *isochronous* curve; and it is that known to mathematicians as the *cycloid*. It is precisely the curve which any point in the circumference of a carriage-wheel describes, when its motion round the axle is combined with its forward movement along the road; and it may be marked along a board, by fixing a pencil or piece of chalk on the circumference of a wheel, and making the latter roll on any straight edge.

265. In all the preceding statements, the effect of *friction* has been left out of the question; it would, however, greatly influence the result; since in the case of two bodies descending along unequal lengths of the cycloid, the one which traverses the longest space is more retarded by friction than the other. The most effectual way of getting rid of this friction, is to support the body by a string hanging from a fixed point, in the manner of a pendulum. But a body thus supported would obviously vibrate in a portion of a circle; and some peculiar means must be taken to make it vibrate in a cycloid. The very simple means to be adopted, depend upon a peculiar property of that curve. If two surfaces be cut out of the form of half cycloids (represented by A B and A C) and they be placed together, so that their extremities join in A whilst their other ends B and C are in the same horizontal line,—a body P suspended from A by a string equal in length to A B or A C, will oscillate from B to C along a curve which is not a circle but a cycloid. From the peculiar nature of this curve it results, that, whether the oscillations be long or short, whether they commence from B, P, or E, they will occupy the same time; since the body will descend along the cycloid in equal times, wherever it commences its descent; and the same law holds good regarding its

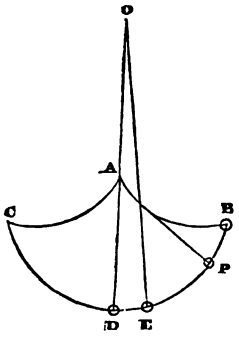


FIG. 60.

ascent on the other side. By such a contrivance, then, we might obtain a pendulum which should be perfectly isochronous, —that is, which should oscillate in precisely the same time, whether its movements be long or short.

266. But though the plan appears so simple, it cannot be accurately put in practice; for it is impossible to find any material fit to support a pendulum, which would not require some force (however small) to bend it against the two cycloidal cheeks, and which would not be in some degree attracted by them when brought into close contact. Hence the operation of such a pendulum would be sufficiently disturbed, for its movements to be less regular, than those of a simple pendulum oscillating through small spaces. For if a circle be drawn from the point O, which is at twice the distance of A from D, this circle will so nearly coincide with the cycloid for a short distance on either side of the point D (say to E) that the movement of a pendulum along that portion of the circle will be isochronous, whether they be longer or shorter. If, however, the oscillations be much longer, they will cease to be isochronous; for they will occupy a longer time, in consequence of the greater space to be passed through; since this greater length is not compensated in the circle, as it is in the cycloid, by a proportionably greater inclination. Hence, in the construction of clocks of the best kind, the simple pendulum is employed; and it is connected with the clock-work in such a manner as to oscillate through small spaces, so that its beats always occupy the same time, whether they are a little longer or a little shorter than the average.

267. But the slightest variation in the length of the pendulum, that is, the distance of the weight from the point of suspension, influences the time of its oscillation; and this may be readily explained according to the law of the descent of bodies along inclined surfaces, already stated. For it is obvious that, since the ball of a short pendulum has to move down a much *steeper* curve than the ball of a long one, it must therefore descend much faster; so that a very slight diminution in the length of the pendulum, by increasing the steepness of the curve even in a very trifling degree, will diminish the time of its oscillations;

and though this diminution may not be perceptible when two or three, or even twenty or thirty are counted, it becomes very evident when a large number are registered, as they are in a clock. In regulating a clock, therefore, we shorten the pendulum (by turning a screw at the bottom, which slightly raises the weight) when we desire it to go faster; and lengthen it, by letting down the weight a little when we desire it to go slower. An alteration of no more than 3-10ths of an inch will make a difference of 5 minutes a day in the going of a clock.

268. The proportion already stated (§ 257) to exist between the lengths of pendulums oscillating in different times,—that the lengths vary as the squares of the times,—follows naturally from the law of the descent of falling bodies,—that the heights through which they fall vary as the squares of the times. Thus, let A C be a pendulum of any given length, whose point of suspension is A; if the weight be let go from B, it will descend through the quarter of the circle whose centre is A, until it reaches its lowest point C, after which it will ascend as high as B on the other side, if permitted to do so. In descending to C it falls through the perpendicular height B D, which is equal to A C,—say in two seconds. In one second it will only fall through a quarter of that height (§ 242); and therefore the length of a pendulum A *c*, adapted to describe the curve *b c* in half the time that is required by A C to traverse B C, must be only one fourth of A C. The same thing holds good with respect to smaller portions of the circle. The pendulum A C in moving from E to C, falls through the space E F; and the pendulum A *c*, of one-fourth the length, in moving from *e* to *c* falls through *e f*, which is equal to one-fourth E F, in half the time. In the same manner, a pendulum must be four times the length of A C, to occupy twice the time in describing any portion of its curve, that is

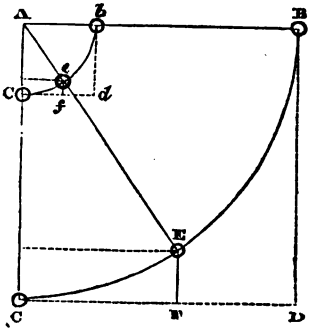


FIG. 61.

required by A C to describe a corresponding portion of *its* curve.

269. The time of oscillation of a pendulum is liable to be affected by changes of temperature; since almost all substances expand by heat and contract by cold; and, as already remarked, a very slight alteration is sufficient to produce a decided difference in the going of a clock. In order to prevent this, several ingenious contrivances have been devised, in which the effect of a change of temperature on one part is made to counterbalance itself in another. These are termed *compensation pendulums;* and those forms in most general use will be described in the treatise on Heat. The simplest is that which is termed the *mercurial* pendulum; and its construction is very easily understood. Instead of a solid weight, it carries at the bottom a jar containing mercury. The expansion of mercury is much greater, in proportion to the length of the column, than the expansion of the steel or iron rod by which it is suspended; and whilst the latter expands downwards, so as to increase the length of the pendulum, the former expands upwards, from the bottom of the jar towards the point of suspension. In consequence of the greater proportionate expansion of the mercury, the rise of the surface of the short column in the jar is enough to compensate for the lowering of the jar by the expansion of the much longer steel rod which carries it; so that the centre of gravity in the jar is always kept at the same point, and the *acting* length of the pendulum remains the same, therefore, at all temperatures. A piece of dry wood, however, has its length so little affected by heat or cold, that a pendulum whose weight is supported by such a rod, is almost as good as one in which there is a contrivance for compensation; and in many large public clocks this is the plan adopted.

270. We have hitherto considered the action of the pendulum as if its rod were without weight; and as if the whole weight which it carried were concentrated into one point, the centre of gravity. But this is not actually the case; and the difference comes to be of much importance. For if we consider the whole weight to be made up of a great number of separate particles,

and these to be all suspended from the same centre, it is obvious that they will not all oscillate in the same length of time, since some of the particles are much further from the point of suspension than others. Suppose the weight to be a round disc like that usually employed (which is made thin at its edges, in order to be as little obstructed as possible by the resistance of the air); then if that weight were cut across into three parts, and these parts were suspended by strings of such a length that they would hang at the same distance from the point of suspension as before, —the middle piece would oscillate about in the same time with the whole weight, whilst the upper piece would oscillate faster, and the lower piece slower. Thus we see that the connexion of these parts into one whole, makes the particles near the point of suspension oscillate more slowly than they would otherwise do, or retards them; whilst those more remote are urged forward in their oscillations, by the tendency of the nearer particles to more rapid movement. Or suppose that a straight rod were suspended at one end, and set to swing like a pendulum; the oscillation of every separate part of that rod would be performed, if suspended by itself, in a different time, depending upon its distance from the centre of suspension; but by their union into one solid mass, those that would oscillate slowly on account of their distance from the centre are hastened, and those that would oscillate more quickly are retarded, by the rest. It is easy to understand, therefore, that the time of movement of the whole rod will be a kind of average of the times in which its several particles would oscillate, if hung separately at their respective distances from the centre of suspension; and that there is some point of its length in which the several effects will be all balanced, all the particles above it having just the same tendency to move faster, as the particles below it have to move more slowly. This point is termed the *centre of oscillation;* and its distance from the centre of suspension is the virtual or acting length of the pendulum.

271. The place of this centre of oscillation cannot in general be found except by intricate mathematical calculation, or by very careful experiments. It may be very near the centre of

gravity; or it may be at a considerable distance from it,—in fact even far below the extremity of the pendulum. It is very near the centre of gravity, when the weight of the pendulum is heavy, and the rod light; so that the quicker oscillations of the part of the weight above its centre of gravity are nearly counterbalanced by the slower oscillations of the part below. And if the whole of the weight could be *really* brought together in the centre of gravity, this would be also the centre of oscillation. But in the case of a uniform straight rod, whose centre of gravity is in the middle of its length, the centre of oscillation is above this; for if the bar were cut across at this point, and each half were suspended separately at its previous distance from the centre of suspension, the upper half would oscillate considerably faster, whilst the lower half would not oscillate much slower than the entire rod. Hence, when the whole rod is united together, the upper parts have a greater tendency to push on the lower than the lower have to drag behind the latter, and the time of oscillation is shorter than it would be, if the pendulum had the virtual length of half the rod.

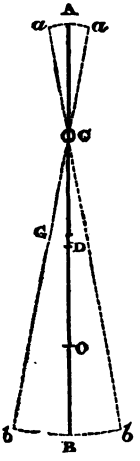


FIG. 62.

272. On the other hand, suppose the rod to be prolonged *above* the centre of suspension, then the centre of gravity is raised, but the centre of oscillation is lowered. For let A B be such a rod, and C its centre of suspension; then the part A C above the centre *descends*, whilst the part C B below the centre *ascends*, and therefore exactly counterbalances the tendency of the portion C D (equal to A C) to move with the rest of the rod. (This is evident, because if A D were the whole length of a rod, supported upon the centre C, it would have no tendency to move either in one direction or the other, its two ends being equal.) The *acting* portion of the pendulum is, therefore, the part D B; and the centre of oscillation O will be that point in it at which the tendency to quicker movement in the particles in the upper portion D O, counterbalances the slower

movement of those in the lower portion O B. Hence the centre of oscillation is much below the centre of gravity G; and if we increase the length A C, of the portion of the rod above the centre of suspension, we shall bring the centre of gravity nearer and nearer to that centre, whilst it throws the centre of oscillation further and further off towards the lower end of the rod.

273. If, instead of a rod, we employ a wire with two weights upon it, the lower one being fixed, and the upper one sliding on the wire, we have a pendulum which, within a very small compass, may be made to represent pendulums of many different lengths. For if the upper weight A be placed at such a height that the centre of gravity is but little below C, then the preponderance of B will be all that there is to put the mass in motion. The case then closely resembles that of Atwood's machine; since the greater part of B's tendency to descend is expended in causing A to ascend; and the remainder, which would oscillate in very short times if left to itself, has to move both A and B along with it; so that its rate of motion is diminished, just in proportion as their united bulk exceeds their difference. The more nearly the two weights are made to balance each other, the less will the difference be, and the slower will be the movement. Thus, if A and B be each at the distance of $9\frac{3}{4}$ inches, and B exceed A by one-eighth of their combined weight, then the tendency of that small quantity to oscillate in half-seconds will have to put in motion eight times its weight, and its rate will be eight times as slow; so that such a pendulum will really be 4 seconds in going through each oscillation, to perform which would require a simple pendulum of $(16 \times 39\frac{1}{7})$ $626\frac{2}{7}$ inches in length. Thus a pendulum not more than a foot from end to end, may be made to oscillate in times equal to those of any simple pendulum from a few inches to many feet in length. Such pendulums are for many reasons not so *certain* in their oscillations as those of the ordinary kind, being more influenced by slight causes, such as a difference in the resistance of the air at different times; and they cannot therefore be advantageously employed for time-pieces. But they are very

usefully employed in instruments termed *metronomes*, which are designed to beat time in the performance of music; and they are extremely convenient for this purpose, since a small case contains the pendulum and the clock-work that keeps it in motion; and this pendulum may be readily adjusted, by altering the place of the upper weight, so as to make any number of beats in a minute; and all modern music has the time, in which the composer intended that it should be played, marked at its commencement.

274. From the laws of the action of the pendulum which have now been explained, it is obvious that the material of which the weight is composed has no other influence on the time of the oscillations, or on the length of time they will continue without assistance, than that which results from the resistance which each experiences in proportion to its weight. Thus, suppose two balls of the same weight, one of lead, and the other of cork, to be suspended by strings of the same length, and to be put in vibration,—the resistance of the air would be experienced by the cork ball in a much greater degree than by the lead, on account of its much greater size: this resistance will slightly diminish the length of each oscillation, and at last it will bring the pendulum to rest. Or suppose that the two balls are equal in size, so that the resistance of the air is the same; then the cork ball will be retarded most, because the force with which it descends is less, and the resistance bears a larger proportion to it. Suppose the weight to be diminished, whilst the surface remains large—as when a feather is hung by a thread and made to vibrate;—the resistance is then so great, in proportion to the tendency of the feather to descend, as almost to overpower it. Yet if a feather suspended by a thread were made to vibrate in a space completely exhausted of air, along with a ball of cork, and a mass of lead of any weight, suspended by strings of the same length, they would all vibrate in the same times, and would continue their movements for as long a period.

275. The vibrations of a pendulum may be made to continue for any length of time, by giving it, at each oscillation, such

a slight additional impulse, as may serve to make up for the loss occasioned by friction and the resistance of the air. The object of the weight and wheel-work of a clock is to communicate such an impulse; and also to register the number of oscillations which the pendulum makes. The pendulum is connected with the wheel-work by a peculiar contrivance termed the *escapement:* this is so arranged with reference to the highest wheel, that each oscillation of the pendulum shall allow the wheel to move onward by a space equal to half of one of its teeth; and that the wheel, which is made to turn by the power of the weight communicated to it through other wheels, shall, at the same time, give a very slight additional impulse to the pendulum. This wheel (termed the scape-wheel) is the one on whose axis the seconds-hand of the clock is placed, each revolution being accomplished in one minute; it is connected with another wheel which is made to occupy 60 times as long in revolving, and this carries the minute-hand; and this is connected with another wheel, which revolves in twelve times the period, and carries the hour-hand. Thus the scape-wheel registers, by the hand which it carries, the oscillations of the pendulum up to 60, or one minute; the minute-hand registers the number of revolutions of the second-hand up to 60, or one hour; and the hour-hand registers the number of revolutions of the minute-hand up to 12, or in some clocks 24. In some clocks, there is an additional index of the days; which makes one revolution in a month.

276. If the pendulum and escapement were removed from the clock, there would be nothing to prevent the train of wheels from being turned round with great rapidity, by the weight or spring acting on it; and the weight would speedily *run down.* On the other hand, if the weight or spring cease to act, the oscillations of the pendulum soon come to an end. The rate of movement of the wheels is entirely controlled by the pendulum; thus to the same clock we might attach a pendulum vibrating seconds, or one vibrating half-seconds; and its rate, the weight being the same, would be twice as great in the latter case as in the former, since the teeth of the scape-wheel are allowed to

pass twice as fast. An addition to the weight will not make the clock go faster, but slower; for it will give a slight additional impulse to the pendulum at each oscillation; and this, making its swing greater, will increase the time which it occupies.—The application of the pendulum to clocks, as the regulator of the movement of the wheel-work, was first made by Huyghens, about the year 1657. Previously to that date, the pendulum had been employed in astronomical observations, to measure small periods of time, such as those in which the sun and moon traverse a space equal to their own diameters; but no means had been devised for keeping it in continued and regular action.

277. It is a matter of great importance to determine with perfect accuracy the length of a pendulum vibrating seconds; that is, to ascertain the distance between its centres of suspension and oscillation. This is different, at different parts of the earth's surface, in accordance with their varying distances from its centre (§ 95); and if the length of the pendulum vibrating seconds in each place can be ascertained, it gives very important assistance in the determination of the figure of the earth. Even a comparatively small distance between two places will make a decided difference in the length of the pendulum vibrating seconds at each. Thus, at London, which is in lat. $51\frac{1}{2}°$, the length of the seconds' pendulum is estimated at 39·13929 inches; whilst at Unst, in the Orkneys, in lat. $60\frac{3}{4}°$, the length (owing to the greater attraction of the earth as we approach the Poles) must be increased to 39·17146 inches, for the vibrations to be performed in the same time. The difference may be more easily understood by comparing the number of vibrations which the same pendulum will make in a given time at the two places; for a pendulum which vibrates 2,390 times within a certain period in London, will vibrate 2,391 times in the same period at Unst, so that it would gain about a second and a half in the 3,600 oscillations which the seconds' pendulum makes in an hour.

278. In fixing our standard of measure, it is of great importance to be able to connect it with some known length; which, if

the standard should be lost or injured, may enable us to replace it. Thus, in Britain, all our measures of length were determined by a standard kept in the Houses of Parliament, which was destroyed when they were burned; and, if an accurate copy of this had not existed elsewhere, the standard would have been altogether lost. The only way of replacing such a loss would be, by having previously ascertained the proportion which the standard bore to some *natural* quantity, which could be accurately measured, and which always remains the same; so that, by a reference to this, the standard might be reconstructed.

279. The French Government have taken, as their *natural standard*, the distance from the pole of the earth to the equator, or a quarter of the whole circumference measured on the meridian line; and of this, the ten-millionth part constitutes the mètre, the standard from which all the French weights and measures are computed. The length of the mètre is 39·37079 inches. The length of the quarter-circumference is determined by astronomical observation; and although it is scarcely to be supposed that this determination is perfectly free from error, yet it is easy to see that an error of some considerable amount in the whole, will be very minute when divided into ten million parts; and also that, supposing it were necessary to replace the standard, the same set of observations, repeated in the same manner, would give a result almost exactly the same.

280. The English Government have taken as their standard the length of the pendulum vibrating seconds in the latitude of London, at the level of the sea, and in a perfect vacuum; and to ascertain this, a series of very ingenious experiments has been made by Captain Kater. By those who had previously endeavoured to determine the length of the pendulum, the place of the centre of oscillation had been calculated from the form of the body employed, and the density of its different parts; but a different method, and one which appears much more accurate, was adopted by Captain Kater. He made use of the curious discovery of Huyghens (who published a work on the theory of the pendulum in 1673, in which the remarkable property of the cycloidal pendulum was explained), that in any pendulum, the centres of

suspension and of oscillation are mutually convertible;—that is, a pendulum being suspended from a certain point, and its centre of oscillation being known, if we then hang it from that centre of oscillation, it will vibrate in the same time as before, and the previous centre of suspension will become the centre of oscillation. Now it is obvious that, supposing the centre of oscillation not to be exactly known, it may be determined from this property of the pendulum. For if the pendulum be suspended from one centre, and its number of beats be observed, we have only to adjust the other centre in such a manner, that the number of beats shall be the same when the pendulum is suspended from it; since in no other point than the centre of oscillation can this equality exist.

281. It is found more convenient in practice, however, to have the centres fixed in the bar, and to make the weight (or a small part of it) movable; so that by a change in its position, the times of oscillation may be made equal for each point. The form of the pendulum employed by Captain Kater is represented in the accompanying figure. It consisted of a bar, into which two axes were fixed at *c* and *o*; and when either of these axes was made to rest upon a hard plane surface, the pendulum would swing from it as from the centre of suspension. In the left-hand figure, the pendulum is represented as suspended from the point *c*; and in the right-hand figure from the point *o*. On the bar is fixed a principal weight *d*; a smaller weight *e* is movable, but may be fixed at any point; and there is a still smaller weight, *f*, which is also movable, but capable of being fixed. By a change in the place of *e*, the oscillations of the pendulum in the two positions may be brought to nearly the same rate; and they may be finally adjusted to an exact equality, by changing the place of the smallest weight *f*. Thus, suppose the number of beats, when the pendulum is suspended from *c*, to be 606 in a given time; and the number, when it is suspended from *o* to be 601;—by slightly

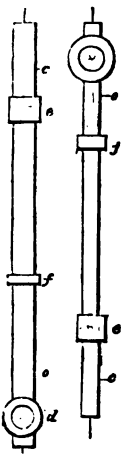


FIG. 63.

moving the weight *f* towards *o*, the time of the vibrations of the pendulum will be increased when the pendulum is suspended from *c*, and their number in a given time consequently diminished. On the other hand, when the pendulum is suspended from *o*, the alteration in the position of *f* which has brought it nearer to *o* renders the vibrations faster, and therefore more numerous in a given time; and thus by the increase of the one, and the diminution of the other, the number of vibrations of the pendulum, when suspended from either centre, is brought to precisely the same number. When this is the case, we know that the point *o* is the centre of oscillation when *c* is the centre of suspension, and that *c* is the centre of oscillation when *o* is the centre of suspension; so that, by measuring the distance between the two centres, we obtain the virtual length of the pendulum, which vibrates at that particular rate.

282. It is not at all necessary that such a pendulum should be adjusted so as to vibrate in seconds; since, the precise length of *any* pendulum being known, and the number of its vibrations in a given time, the length of the seconds' pendulum is ascertained by the simple rule already given. As in such experiments it is not permitted to connect the pendulum with clock-work, by which its motion might be kept up,—since the additional impulses thus communicated would produce a considerable error in the result,—the observation of its vibrations can only be continued for a short time; and they are compared with those of a pendulum of an ordinary clock, accurately adjusted to vibrate in seconds, by the following method. It is advantageous for the two pendulums to be nearly, but not quite, the same length; and one is hung exactly in front of the other, so that it can be seen whether they pass the same parts of their beat together or separately. Let us call the two pendulums A and B, and suppose that A is vibrating seconds, whilst B is oscillating rather faster. If now both pendulums be set in motion at the same instant, and be looked at in front, B will be seen to gain a little upon A at each oscillation, and at last it will be moving one way whilst A is moving the other, having gained half a beat; after as many more beats, it will have gained as

much more, and will re-commence at precisely the same time with A, having performed one more oscillation in that interval. That *coincidence* will be only for a moment; since in the very next beat B will have gained a little upon A: but it will be repeated again and again, after every similar number of beats. It is not requisite to count the number which intervenes, but merely to ascertain how many *coincidences* take place in a given time as shown by the clock. Thus, suppose that 10 coincidences are observed in one hour, during which A (the seconds' pendulum) makes 3600 oscillations; then for every 360 oscillations there will have been one coincidence, indicating that B has performed one oscillation *more* than A in that period. If B be longer than A, and its oscillations be slower, precisely the same principle applies; but the number of its vibrations is then one *less* than that of A, for every coincidence.

283. When the rate of a pendulum and its virtual length have thus been ascertained, the length of the seconds' pendulum is easily calculated by the following proportion:—as the square of the time of the one pendulum is to the square of the time of the seconds' pendulum, so is the ascertained length of the one pendulum to the length of the seconds' pendulum. When the latter has been determined by a common rule-of-three sum, there are still many corrections to be applied to it, in order to rectify it for the purpose of serving as a standard. Thus, the height of the place of observation above the level of the sea may make a sensible difference in the result; and in order that observations made in different places may be compared with each other, it is desirable to reduce them all to an uniform standard,—the sea-level. Again, as the density of the air is so different at different times, and as its resistance acts so differently on pendulums of different forms and materials, a correction must be made for it; and here, too, it is most convenient for the comparison of observations made under different circumstances, that they should be all corrected so as to represent the length of a pendulum oscillating in a perfect vacuum. This correction can be made by calculation, founded on the form and density of the pendulum. The result of these calculations has

been to make the length of the seconds' pendulum, in the latitude of London, 39·13929 inches; and this has been declared by Act of Parliament to be the standard of measure. It was intended that, in case of the standard being lost, destroyed, or injured, the new one should be constructed by the natural standard thus obtained;—that is, if the length of a pendulum vibrating seconds were divided into 3,913,929 parts, the standard yard should be 3,000,000 of those parts. It is doubtful, however, whether this determination of the length of the pendulum is accurate; and it has been stated on high authority, that if a new standard were constructed from it, this would differ sensibly from the old one. It appears that there are sources of error which had not been suspected; and that these should impair our reliance on the perfect accuracy of the experiments hitherto made in this country. The latest, and probably the most accurate, series of such experiments, is that made at Königsberg by the celebrated astronomer Bessel; the result of these gives as the length of the seconds' pendulum, in 54° 13′ north latitude, 440·8179 French lines, which is equivalent to about 39·1593 English inches. There is not much difference, however, between this estimate and that of Captain Kater, for a corresponding latitude.

CHAPTER X.

OF THE SIMPLE MACHINES, OR MECHANICAL POWERS.—THE LEVER, WHEEL AND AXLE, AND PULLEY.

284. In the preceding chapters, we have been occupied with the considerations of *forces* acting *directly* upon bodies, or their particles. Thus we traced, in the first instance, the operation of the forces of cohesive and adhesive attraction, in uniting together the particles of the same or different kinds. We then considered the attraction of masses of matter for each other; and especially that form of it, so important to us, in which it acts on the earth's surface,—the attraction of gravitation. The principal consequences of this attraction were then pointed out; the laws of the centre of gravity, and the mode in which they affect the stability of structures, were explained; and the laws governing the fall of bodies towards the earth were stated, and their application shown. And under the head of the Laws of Motion, the manner in which bodies are affected by forces of other kinds, whether acting together or in different directions, was fully discussed.—There are a great number of instances, however, in which force is *indirectly* applied to the body on which it is to be exercised; some machine or instrument being employed, by which the force may be applied more conveniently as to its amount or direction, or by which its character may be so changed, that it can accomplish what it never could have done in its original form. Thus, to take the simplest possible illustration, we use a poker to stir the fire, because it would not be convenient, but would on the contrary be extremely injurious, to place the coals in the desired position by the direct use of our hands, though we certainly *could* do so; we therefore apply our

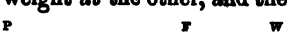
force to the poker, and the poker acts on the coals. Again, in the case of a water-mill, the force of running or falling water is made, through the intervention of machinery, to grind corn, to roll out bars of iron, or to work a massive hammer,—operations which it *could* not perform, except when indirectly applied in such a mode as this.

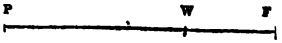
285. Now there is no power, in any machinery, of *creating* force; it can only apply, in a more advantageous manner, the force by which it is itself moved. This will be easily understood, from what has been formerly stated (Chap. VI.) of the tendency of all matter to remain in the state in which it may be at any given time. Thus, a machine at rest remains at rest until it is moved by a power applied to it; and when in motion, it would remain in motion, if it were not for friction and the resistance of the air, until stopped by a force equal or superior to that first applied. Any inferior force will be overcome by it; and thus the power first communicated may be applied to any operations which only require this. A steam-engine, for instance, is a machine contrived for the purpose of advantageously applying to use, the power communicated by the expansive force of steam; but it can itself create no power, and remains inactive until steam is forced into it from the boiler. Now, in machines of some kinds, a great power slowly applied is employed to effect a number of operations which must be performed very quickly; this we see, for example, in a mill for spinning cotton or silk, of which the spinning-reels are turned round many hundred times, whilst the axle worked by the steam-engine turns only once. But we should find that we might stop any one of these reels by the touch of the finger, so little is the power applied to it; whilst no force that we could employ could offer the least resistance to the motion of the steam-engine. Or, to take a more familiar instance, in the movement of a watch, the power communicated by the main-spring is applied to a train of wheels, and produces a much more rapid movement in the balance and the wheel which acts upon it; but we find the power with which they move to be far less than that of the main-spring, since the slightest touch of the finger will check their revolution,

whilst the winding-up of the main-spring requires a far greater force. Hence we see that what is gained in velocity is lost in power. The converse—that what is gained in power is lost in velocity—is also true; and of this we may find numerous instances, in which machines are so constructed as to concentrate (so to speak) the power applied to them, so that it is rendered sufficient to overcome a far greater obstacle than before, but does this much less rapidly. Thus, by a system of pullies or a windlass, we see a man raising a weight many times greater than he could lift without their assistance; but whilst his arms move through a considerable space in pulling the rope of the pullies, or in turning the windlass, the weight rises in a far smaller degree.

285.* All machines, however large and complex, are made up of parts which are very simple in themselves; and these may be termed *simple machines* or *mechanical powers*. They are usually thus enumerated. The Lever, Wheel and Axle, Pulley, Inclined Plane, Wedge, and Screw. The laws governing the action of each will now be separately considered.

The Lever.

286. By a *lever* is understood an inflexible straight bar, supported, at some part of its length by a prop, or *fulcrum;* it is employed to move a *weight*, or overcome a *resistance*, bearing on one point, by a *power* applied to another. In our idea of a lever, therefore, we have simply three things to consider,—the power, weight, and fulcrum; the relative distances and positions of these govern the action of the lever. There are two principal varieties of the lever; in the first, the power is at one end, the weight at the other, and the fulcrum between them, in this mode: 

; such a lever is spoken of as belonging to the *first* order. In the other kind, the fulcrum is at one end, so that the weight and power are both on the same side of it. In this case, the weight may be nearest to the fulcrum, and the power at the other end of the lever, the arrangement being thus: 

; such a lever is said to be of

the *second* order. Or the power may be nearest to the fulcrum, and the weight at the opposite extremity, so that the arrangement will be

; this lever is said to be of the *third* order.

287. The fundamental rule governing the action of all these forms of lever is precisely the same. When the lever with its power and weight are in equilibrium, the power and the weight will be to each other in the inverse proportion of their respective distances from the centre; which is the same as saying that the power is to the weight, as the distance of the weight is to the distance of the power. Thus, suppose that we have a piece of meat exactly balanced, when hung from a steel-yard (§ 318), by the weight on the opposite side of the point on which it turns (the fulcrum); now if the weight be 2 lbs., and its distance from the fulcrum be six times that of the point from which the meat is hung, the latter will weigh 12 lbs. Or, to put the same general statement into a different form, the power multiplied by its distance from the centre will always be equal, when the lever is in equilibrium, to the weight multiplied by *its* distance from the centre. Thus, in the case of the steel-yard, suppose the meat hung at a distance of three inches from the fulcrum, the weight must be moved to a distance of 18 inches, in order to balance it; because $3 \times 12 = 18 \times 2$. Hence, by knowing any three of these terms, we may find the fourth, as in a common Rule-of-Three sum. For we have only to multiply the two terms belonging to the same side of the lever, and divide by the term belonging to the other side, to get the fourth term. Thus, we wish to know what power, applied at a distance of 15 inches from the fulcrum, will balance a weight of 18 lbs. at $2\frac{1}{2}$ inches from the fulcrum. By multiplying together $2\frac{1}{2}$ and 18 we have the product 45; and on dividing this by 15, we get 3 lbs., the number required. Or, if it be required to know at what distance a weight of 80 lbs. must be placed, to balance a power of 16 lbs. applied at a distance of 25 inches; by multiplying 16 and 25 we obtain as the product 400; and this, divided by 80, gives 5 inches, the distance required.

288. The preceding statements apply to the state of equilibrium of a lever; when it is used, however, for mechanical purposes, it has to overcome resistance or to put a weight in motion; and the power applied, therefore, must be something more than is exactly equivalent to the weight or resistance. In general, a lever is employed for the purpose of overcoming a greater resistance by a less, or of raising a greater weight by a smaller one; and in such a case, the power must be applied at a greater distance from the fulcrum, than that at which the weight bears. But, being applied at a greater distance from the fulcrum, it will have to move through a preportionally large space, in order to raise the weight to a small amount: this will be at once seen by holding a stick or rod between the finger and thumb, in such a manner that the parts projecting on each side shall be unequal, and then causing it to swing upon the part held, as upon a centre; the two ends will describe unequal spaces, the length of which will be exactly proportional to their respective distances from the centre. Hence, if the power have to move through a space as much greater than that through which the weight is raised, as its own distance from the centre is greater than that of the weight, it follows that the power multiplied by the space through which it moves, will be equal to the weight multiplied by the space through which *it* moves. Hence, *what is gained in power is lost in velocity*, a principle of very extensive application in mechanics. In no instance can we apply power in such a manner, as to raise a weight many times as large as itself, without at the same time being obliged to move the power just so many times faster; or, on the other hand, if we desire to produce a great velocity, by any mechanical contrivances, we must sacrifice power in the very same proportion. By the lever, and the other mechanical powers which are modifications of it, we can only exchange power for velocity, or velocity for power: we cannot create either.

289. As examples of the *first* order of levers, the following may be mentioned among familiar objects. The handle of a pump is a lever of this nature, the fulcrum being the point on which it turns,—the water to be raised, and the friction to be

overcome being the resistance,—and the force expended in pumping being the power. Now here, the hand in pumping moves through a much larger space than the sucker of the pump; but the power which it applies is small, in comparison to the resistance to be overcome. A common poker, when used to raise the coals, is another example of a lever of the first order; the point at which it rests on the bars of the grate being the fulcrum, the coals the weight, and the force of the hand the power. All those instruments for cutting or holding, composed of two parts, which cross each other in the middle,—such as scissors, pincers, pliers, garden-shears, &c., are levers of the first order. In the elbow-joint of the human body, there is another example of it. The muscle which straightens the elbow, is a thick mass of flesh, lying on the back of the upper arm; and this is attached to a portion of one of the bones of the fore-arm, which can be felt projecting behind the elbow-joint. When this bony projection is drawn upwards by the action of the muscle, even in a very slight degree, it causes the hand to move through a considerable space; for the point of attachment of the power is so near to the centre on which the fore-arm moves, as to cause this to be acting under great disadvantage in regard to power, whilst velocity is gained by it. For suppose that the fore-arm, from the elbow-joint to the palm of the hand, is 16 inches in length, whilst the projection to which the muscle is attached is only 4-5ths of an inch long on the other side, the relative lengths of the two sides or arms of the lever would be as 1 to 20. Hence, a contraction of the muscle to the amount of 1-20th of an inch, would cause the hand to move backwards an inch; but, in order to push down a pound weight with the hand, by straightening the elbow-joint, a power equivalent to 20 pounds must be exerted by the muscle.

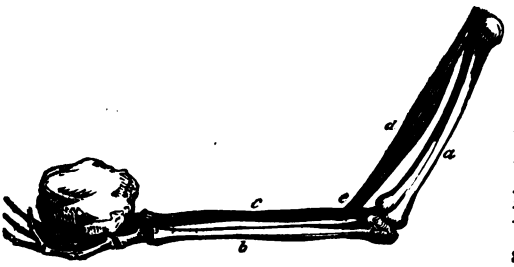
290. Of levers of the *second* kind, also, numerous examples may be found in common things. Thus, when we raise or force forwards a heavy stone by means of a crow-bar, of which one end is made to rest against the ground, whilst the other is moved by the hand, the implement becomes a lever of the second order. In carrying a weight in a wheel-barrow, again, we make use of

the same advantageous application of force; for the axle of the wheel being the fulcrum, the handles at which the power is applied are about twice as far as from it, the hollow part in which the weight is laid; and thus the strain upon the hands is only half the actual weight. A pair of nut-crackers is another instance of the same kind of lever; for the hinge is the fulcrum, the resistance of the shell is the weight, and the hand applied at the extremity is the power. In opening a door, again, we do in reality employ a lever of the second order, although the weight is distributed along the whole space between the fulcrum and the power; for, according to the principle formerly explained (Chap. IV.), we may regard the weight as really concentrated in a point midway between the hinge and the handle; so that the power required to move it will be only half what it would be, if the weight were at the point where the power is applied. The oar urges forwards the boat by its power as a lever of the second kind; for the blade presses against the water in such a manner that the latter serves as its fulcrum; the hand of the rower is the power; and the boat is the resistance or weight, against which the oar acts where it passes through the row-lock.

291. It has been seen that levers of the first kind may or may not act at a mechanical disadvantage; for either arm—that which bears the power, or that which acts on the resistance—may be the longer. The lever of the second kind, however, must always act a mechanical advantage,—that is, the power required will always be less than the resistance or weight,—because the distance of the power from the fulcrum is always greater than that of the weight. There is this difference in the action of levers of the first and second order;—that in the former, the motion of the power causes the weight to move in the contrary direction, as when we raise the sucker of the pump by pressing down the handle: whilst in levers of the second order, the power and the weight move in the same direction, as when we raise or force forwards a stone with a crow-bar. The question which lever we shall use, therefore, is a mere matter of convenience, since we can gain the same power by both.

292. But in a lever of the third order, there is *always* a

disadvantageous application of the power with respect to the weight, since its distance from the fulcrum is less; but, on the other hand, there is a proportional gain in velocity. Examples of the application of this lever are very frequent in the construction of animals; for the tendons which move a limb are almost always inserted near the joint, so that, by a small amount of contraction on their part, a considerable motion is produced. Of this, again, the elbow-joint affords a good illustration. The fleshy mass that forms the front of the upper arm is chiefly concerned in bending the joint; its tendon is attached at a point a little in front of the elbow, so as to constitute the power; whilst the hand, with anything it may contain, acts as the resistance. There will be, therefore, the same mechanical disadvantage in the action of this muscle, as in that of the muscle already described as attached at the back of the elbow-joint for the purpose of straightening it; and the same proportional increase in the rapidity of the movement. Hence a very powerful muscle is required to raise a moderate weight in the hand; but its contraction need not be considerable, in order to carry the hand through the whole range of its movement. With the same object, the muscle which raises the lower jaw is attached at a point, *a*, very little in front of the joint. The force which it must sometimes exert, even in man,



a, bone of the arm; *b*, *c*, bones of the fore-arm; *d*, muscle; *e*, its point of attachment.

FIG. 64.

FIG. 65.

cannot be less than three or four hundred pounds; whilst in the tiger and other large carnivorous animals it must be frequently as much as two thousand pounds, or even more. Other examples of a lever of this kind are to be found in the shears used for shearing sheep, and in a pair of tongs. The treadle of a lathe is a lever of the second order, when the crank is in the middle of it, and the hinge or fulcrum behind it, the foot being applied in front. But in some lathes the hinge is in front of the treadle, the crank at the farther end, and the foot applied to the middle; and the treadle is then a lever of the third kind.

293. The process of raising a ladder is a good illustration of the variation produced in the character of the lever, by an alteration in the position of the power. When it is first lifted from the ground, by a force applied at one end, it is a lever of the second order; since (as just explained in regard to the door) its centre of gravity, in which its whole weight may be regarded as acting, is in the middle of its length, so that a power applied at its end will have an advantage of 2 to 1. But when the ladder has been partly elevated, the hands of the person raising it are made gradually to approach its foot; when they are moved to the position of the centre of gravity, they are acting neither at an advantage nor disadvantage; for though the top of the ladder will then be moving twice as fast as the hands, the bottom will not move at all, so that the average movement of the whole will be just the same as that of the hands: and when, in order to raise the ladder nearly to the upright position, the hands are made to act upon it still nearer its bottom, it becomes a lever of the third order,—the weight being now more distant from the fulcrum than the power.

294. It is evident that, in levers of the first and second orders, we may gain any amount of power we please, by simply making the distance between the power and the fulcrum just so many times greater than the distance between the fulcrum and the weight. It was by Archimedes that this truth was first perceived; and he expressed it by saying: "Give me but a place where I may stand, and I will move the world." The principle upon which this assertion was made is perfectly correct; but he seems to

have omitted in his calculation one important element,—the *time* necessary to produce the effect. It has been already shown, that the relative amounts of movement of the power and the weight will be proportional to their respective distances from the fulcrum; and if the arm to which the power is applied, be increased to such an enormous length as would be necessary to gain the mechanical advantage required for such a purpose, the time that would be occupied in moving, even to the amount of an inch, and with the whole force of a man, a body of the weight of the globe would be quite inconceivable. This may be readily shown by a simple calculation. Taking the diameter of the earth at 7,930 miles, the number of cubic feet in it may be calculated to be 38,434,476,263,828,705,280,000; and assuming each cubic foot to weigh 300 lbs., we shall have for the weight of the earth in pounds the number 11,530,342,879,148,611,584,000,000. Now, supposing Archimedes to act at the end of his lever with a force of 30 lbs., one arm of it must be 384,344,762,638,287,052,800,000 times longer than the other, that he may move this mass with it. And one arm of the lever being this number of times longer than the other, the end of that longer arm, when made to turn round its fulcrum, must move exactly that number of times faster, or farther, than the end of the other; so that, whilst the end of the shorter arm was moving 1 inch, the end of the longer arm must move 384,344,762,638,287,052,800,000 inches; or, on the other hand, when Archimedes had made the end of the lever to which he had applied his arm move this immense number of inches, he would only have raised up the earth, which was resting on the other end, to the amount of one inch.—Now it is estimated that a man pulling with a force of 30 lbs., and moving the object which he pulls at the rate of 10,000 feet an hour, can work continually for 8 or 10 hours a day; this is the extreme of the power which a single man can apply. Each day, then, Archimedes could, at the utmost, move his end of the lever 100,000 feet, or 1,200,000 inches; and hence it may thus be readily calculated, that to move it through 384,344,762,638,287,052,800,000 inches, or to move the other end—that is, the earth—1 inch, would require the continued labour of Archimedes for 8,774,994,580,737 CENTURIES.

295. It is to be remembered, however, that in this calculation the weight of the earth is estimated, by comparing its size with that of bodies on its surface having a certain known weight. Now it has been explained that the weight of such bodies is nothing else than their downward pressure, produced by the attraction of the earth for them. In this sense, therefore, the earth itself cannot be said to have any weight. It is attracted towards the sun with a certain force, which, if acting alone, would cause it to press towards that centre, in the same manner as a stone presses towards the centre of the earth. But this force is in fact completely balanced by the centrifugal force, which tends to keep the earth at a distance from the sun. In reality, therefore, Archimedes would have had no difficulty in moving the world, could he have brought his lever to bear upon it. It rests upon nothing, and is suspended by nothing; it is perfectly free to move in any direction. Had Archimedes a place on which to stand, a kick with his foot would have pushed the earth out of its situation; and the lever would have been a superfluous force. This, however, was evidently not the idea of Archimedes; who considered the earth as fixed in some way by its own weight, and as capable of being moved from its place by a certain mechanical force, in the same way as we should move a heavy stone; and the calculation in the preceding paragraph has been made to follow out this idea.

296. If it were necessary to construct a lever having a very great mechanical power, it would be found much more convenient to employ a system of levers bearing on each other, than a single lever. Thus, suppose that we desired by a pressure of 1 lb. to balance 1000 lbs., we should be obliged, in a single lever, to make the distance of the power from the fulcrum 1000 times that of the weight. But the same end may be obtained by a

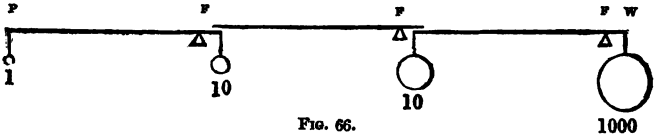


Fig. 66.

combination of three levers, in each of which the relative length of the arms is as 10 to 1. For if these levers be arranged, so as

to bear upon one another in the manner here represented, the power of 1 lb. applied to the first will balance a weight of 10 lbs. at 1-10th the distance from the fulcrum. But the weight-end of the first lever is applied under the power-end of the second, and will thus tend to raise it with a force of 10 lbs. As the second lever has a mechanical advantage equal to the first, a power of 10 lbs. applied to raise that end will press down the weight-end with a force of 100 lbs. This force being applied over the power-end of the third lever, will tend to raise its weight-end with a force of 1000 lbs. Such a compound lever might be comprised within a narrower compass, by an arrangement like that here represented. The highest lever, which is of the first order, gives a power of 10; the other two, which are of the second order, have each a power of 11, the relative distances of the power and the weight being as 11 to 1; and the combined power of the three, being ascertained by multiplying their respective powers together, will be $(10 \times 11 \times 11)$ 1210. In this, as in every other similar case, however, what is gained in power is lost in velocity; and the combination of levers has no advantage above a long single lever of equivalent power, except in its greater convenience. Neither, however, are commonly employed in the mechanical arts,—at least in the forms just described; since, when great power is required, there are other more convenient modes of obtaining it.

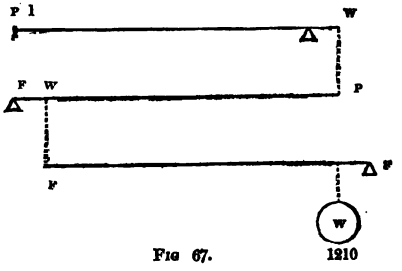


Fig 67. 1210

297. There is an application of the principle of the lever in a circumstance of common occurrence, which should not be overlooked. Two men, A and B, carry a weight W upon a pole between them, —as in the case of a sedan chair. If the weight be at the same distance from each end of the pole, it will be equally divided between them; but not other-

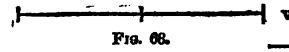


Fig. 68.

wise. For we may consider the pole as a lever, to which each man applies the power, the fulcrum being at the opposite end. Thus, the power being applied by A, the end resting on B is the fulcrum. Hence the lever is one of the second order; and, as the distance of the power from the fulcrum is twice the distance of the weight, A will be required to raise his end of the lever with a force equivalent to only half the weight. The same may be said of B; and thus the weight is equally distributed between them. But if the weight were nearer to B than to A, the case would be different; for B would then have a larger proportion to bear. Suppose that its place were such that the whole length of the lever being 3, its distance from A is 2, and from B only 1. Then the power of A is applied, B being the fulcrum, at a distance from it equal to three times that at which the weight is suspended; and consequently the pressure on him is only one-third of the weight. On the other hand, the distance of B's power from the fulcrum, at A, is to the distance of the weight from the fulcrum, as 3 to 2; and therefore B has to bear two-thirds of the weight.

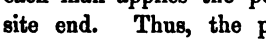


FIG. 69.

298. By a proper arrangement, a very heavy weight may be equally distributed amongst a considerable number of men. The following method is said to have been used in Constantinople for raising and carrying the heaviest burdens, such as cannons, mortars, and enormous stones; and the rapidity with which they were thus transported from one place to another is stated to have been truly surprising. A B is a bar of sufficient strength to sustain the whole weight of the load P, which is attached to its middle point; C D and E F are cross-bars fixed to this, near its extremities; and to the extremities of these cross-bars are again fixed others *a b*, *c d*, *e f*, *g h*; and to these last again, in like manner, others, whose extremities

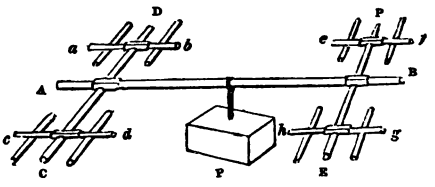


FIG. 70.

are borne upon the shoulders of the men who are to carry the load. Supposing one man's shoulder to support each extremity of these last-mentioned bars, the whole number, whose effort will be combined to lift and carry the weight, will be 16. If other cross-bars had been fixed in like manner to the extremities of *these*, the united effort of 32 men might have been applied. If other cross-bars had been fixed yet again to the extremities of the last, 64 men might have united their strength to the task; and so on for any number. If the bar A B, which supports the weight, carry it suspended accurately from its middle point; and if the point where each cross-bar is fixed to the one preceding it in the series, be exactly half-way between the points where the next two are affixed to *it*,—then the weight will be equally distributed between all the bearers. A small inaccuracy in this respect will produce great inequality in the amount of the pressure on the several bearers. The principal inconvenience of the method is the weight of the apparatus itself, which will be increased by that of the additional cross pieces, whenever an increase of power is to be thus gained.

299. Hitherto, we have supposed that the power and the weight press directly upon the arms of the lever, or hang in perpendicular lines from it, as they would do if left to themselves. But it may happen that one or both of them may have a different direction. Thus, suppose that, in order to raise the weight W, by the lever A B, of which C is the fulcrum, we apply our power, not in the downward line A D, but in the direction A E. It is obvious that power thus applied will not act so advantageously as if it pulled in the downward direction A D; and that more must be thus applied, in order to do the same work. It is also evident that the loss will be greater, in proportion as the line A E departs from the perpendicular, and

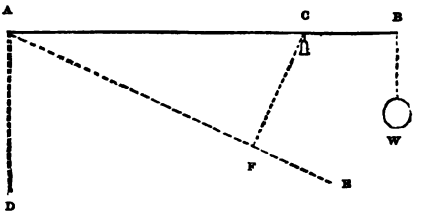


FIG. 71.

approaches A B; in which last direction, no force whatever would have the effect of pulling down the end A. The real operating power is ascertained by drawing a line from the fulcrum C, perpendicular to A E; and the length of the line C F is that at which the power is really acting.

300. The same principle applies to the operation of the bent lever, in which the power and the weight may be both acting perpendicularly to the earth's surface, but not perpendicularly to the arms of the lever. Thus, in the bent lever A C B, of which C is the fulcrum, the lengths of the arms A C and C B do not represent the real working length of the lever; for weights inversely proportional to those lengths would not balance each other. The real action of such a lever is ascertained by drawing the horizontal line D E through the fulcrum, and drawing A D and B E perpendicular to this. The lines D C and C E, then, represent the real proportions between the acting lengths of the arms A C and C B, in this position of the lever. But it is a peculiarity of the bent lever, that its power is very different in different positions; for if the same lever be placed in the position represented in Fig. 73, the acting length of the arm A B is now even less than that of B C,—that is, with regard to any forces (such as suspended weights) which may be pulling in the directions A D and C E. If the forces act perpendicularly to the arms of the

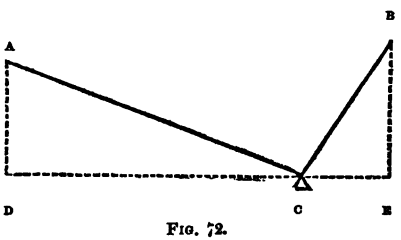


Fig. 72.

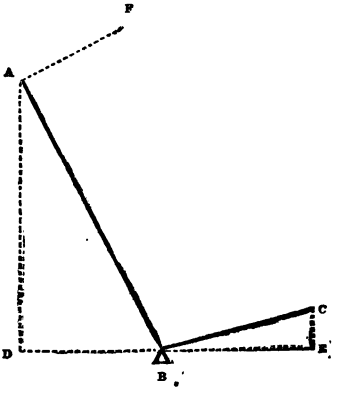


Fig. 73.

lever,—as, for instance, when the arm A B is moved by the hand pressing in the direction A F, the action of the bent lever is the same as that of the straight one.

301. Various modifications of the bent-lever principle are employed in machinery; among the most interesting are those which are introduced into the various forms of the Printing - Press. The advantage they confer is principally this; —that the lever is much more powerful in its action in one position than in another; and that, when least powerful, the motion of the weight more nearly approaches that of the power in rapidity. Now this is precisely what is required in a machine that shall apply and take off a great pressure quickly; since, when the pressure is first being applied, the resistance is small, less power is required, and the press-board may be advantageously caused to move fast; but, in proportion to the amount of pressure that has been made, will be the resistance to any additional pressure, and greater power will then be needed.

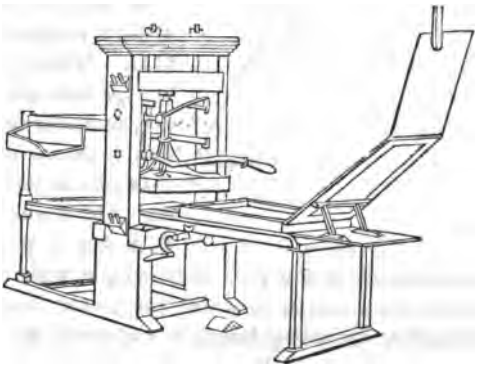


FIG. 74.

302. In the Stanhope Press (as it is termed after Lord Stanhope, its inventor) the press-board or *platten* is forced downwards by a powerful screw, which bears on its upper surface. So long as this screw is moved with an equal force and rapidity, its pressure will be equal; but, by the combination of levers that moves it, the screw is made to revolve more rapidly and with less force during the first part of the *pull*, but very slowly and with greatly increased force during the last part of it. A short lever

S A is fixed into the screw, and this is turned round by the connecting rod A B. This rod is united to the end B of the bent lever B C D, of which C is the centre. The power—the hand of the pressman—is applied at D in a direction always perpendicular to the arm C D. When the *pull* is commencing, the connecting rod A B is nearly perpendicular to the short arm B C; and therefore the acting length of that arm is just what it appears to be. On the other hand, the lever S A has a position, at the commencement of the pull, very oblique with reference to the line A B, the direction in which the power is operating on it; and therefore its acting length is expressed by the line S F drawn perpendicularly to the direction of the force. This line represents the long arm of a lever of the second order, of which the fulcrum is the centre of the screw, the weight or resistance being applied at its circumference, whose distance S H from the centre is therefore the short arm. Of course, the more prolonged the long arm, the greater force will be exerted by the same power in turning the screw; whilst the shorter the arm, the greater will be the amount of motion given to the screw by the same motion of the power.

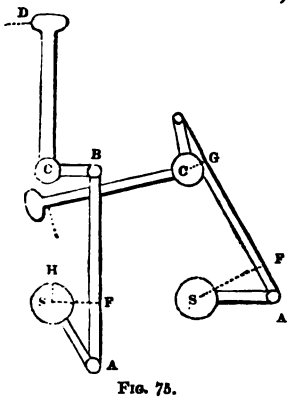


FIG. 75.

303. Now on looking at the situation of the levers at the conclusion of the pull, we shall see a great change in their efficiency. The bent lever is now thrown into a position in which the acting length of its weight-end C B is much diminished, in consequence of the oblique direction in which its force is applied; for instead of being B C it is really no more than C G, the line perpendicular to the direction of the force. By this shortening of the weight-end, the power of this lever is greatly increased; and it is obvious that, by turning the arm C D a little further round, B A and B C will be brought still more nearly

into the same direction, thus diminishing the distance C G to an extremely small amount, and increasing the power to an enormous extent. On the other hand, the change of the position of the arm S A has increased its acting length to that of the perpendicular S F; but this, too, gives an increase of power, since S A is the power-arm of the lever. By the combined increase in the power of these two levers, which takes place gradually during the pull, the pressure which the same force applied at D would exert is immensely increased; so that, by a proper adjustment, it may be rendered several hundred times greater at the end than it was at the beginning, the motion of the platten being diminished in the same proportion.

304. Even an ordinary straight lever may be made to act upon this principle, so as to gain much greater power in one position than it has at another. Thus let A C be a lever of the second order, of which C is the fulcrum, and W a weight suspended from the point B. In the horizontal position of the lever, the arm A C is 4 times the length of the arm B C; and a power of 1, applied perpendicularly to the surface of the lever, will therefore balance a weight of 4. But if the lever be raised into the position D E C, and the same power be applied at D, perpendicularly to the surface of the lever, whilst the weight hangs freely suspended from E, it would be able to sustain a much greater weight; for, whilst the acting length of the power-arm remains the same, that of the weight-arm has been diminished from A C to C F, which last is only 1-9th of A C, so that the power of the lever is increased from 4 to 9.

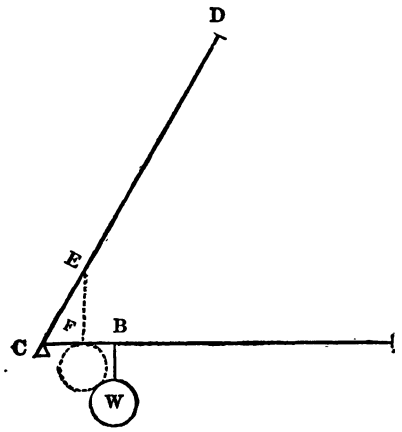


Fig. 76.

305. This principle is applied in another printing-press, which is known as the Columbian. The platten is not here forced down by a very strong lever ABC, of which A is the fulcrum, B the point of resistance, and C the point at which the power is applied. The end C is connected by the rod CE with the lever DF, the fulcrum of which is at D. Motion is given to this lever by means of the connecting-rod FG, which unites the end F with the end G of the bent lever GHI, whose fulcrum is at H. Now it is obvious that a power applied in the direction IK, will tend to draw the connecting-rod GF towards G, with a force continually increasing, and a rapidity proportionally diminishing, just as in the Stanhope press. But as the end F of the lever DEF is moved towards G, it gradually comes to form a right angle with GF; so that the power of the latter is applied to it in the most advantageous manner. On the other hand, by the same movement, the lever DEF and the connecting rod CE are brought more nearly into a line; so that the distance D *e*, which is the acting weight-arm of the lever, is gradually diminished, and reduced almost to nothing. In this manner, during the pull, the power of the lever DF is enormously increased by its change of position; whilst its rate of action on the rod CE is proportionally diminished. A further power is gained by the leverage of ABC, which is a lever of the second order, having a proportion of about 3 to 1 between the distances of the power and of the weight from the fulcrum. By the combined action of all these levers, any power that may be required is easily obtained; and, as in the former case, the levers may be so arranged as to give this power only when it is most required, —at the end of the pull; and to move the platten with much greater quickness at its commencement, when power is not required, but time is valuable.

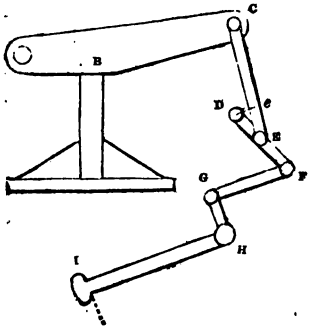


FIG. 77.

306. The action of the *Crank*, which is used to convert a straight motion into a circular one, or a circular movement into a straight one, is dependent on the same principle. Any one may watch its operation in a common lathe, the wheel of the itinerant knife-grinder, or the steam-engine; but a simple inspection of this kind will not lead to the full comprehension of its action, which it is desirable here to explain. The crank is nothing else than a lever A B fixed upon an axle or shaft at A, and moved by the connecting-rod B C. In this position, the power of the connecting-rod B C is applied almost as advantageously as possible; since it acts in a line nearly perpendicular to A B. Suppose the rod to be drawn upwards, however, until it has brought the crank into the position A *b*, in which it is in a straight line with *b c*,—it is evident that the latter then has no power of giving it any further movement, but that any further pull in the upward direction will only exert a direct strain upon the axle A; and it is also evident that, whilst the end B of the crank has been moving to *b*, the power has been gradually acting on it more and more disadvantageously, until it is altogether lost. But if the crank be made to turn a little further, so as to be no longer in the same line with the connecting-rod, and the latter then descend with a certain force, that force will act upon the crank more and more advantageously, until it brings it again into the *most* advantageous position A B′. Having passed this, the continued downward motion of the rod will cause the crank to turn until it reaches the position A *b*′, when the two are again acting in the same straight line, so that no pull exercised by the rod in either an upward or downward direction can turn the crank further. But if it be moved ever so little out of the straight

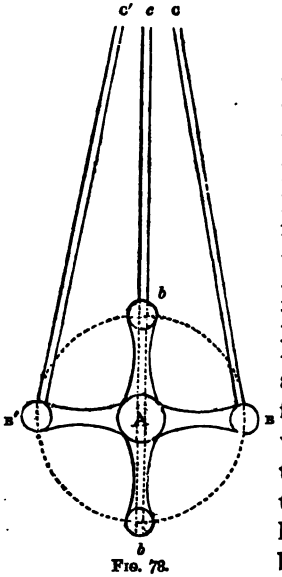


FIG. 78.

line, and the connecting-rod be again moved upwards, it will apply a continually-increasing power to the crank, until it brings it back to the position A B.

307. Hence we see that, supposing the connecting-rod to be moved alternately upwards and downwards, as it is by the beam of a steam-engine, its action upon the crank is continually varying; and that there are two points in the revolution of the crank, at which no power that the connecting-rod can exert, will continue its circular movement. It is necessary, therefore, that some means should be adopted for equalizing the application of the power; and for giving to the crank an independent movement, which shall carry it on through those parts of its rotation, at which the power cannot act directly upon it. This is accomplished by a very simple contrivance. The axle turned by the crank carries a large and heavy fly-wheel, which, when it is once set in motion, will continue its revolution for some little time without any additional power. The inertia of this fly-wheel of course requires, in the first instance, a certain expenditure of power for putting it in rotation; but, when this is once accomplished, the same inertia keeps up its movement (§.143). As its momentum, in consequence of its weight and velocity, is very great, it is able to keep up the action of the machinery during the time when the crank is in its most disadvantageous positions; and it also equalizes the application of the power,—storing it up (as it were) when in greatest abundance, and imparting it when needed. By this action, therefore, the axle is made to carry the crank out of the positions A b and A b', into those in which the power can act upon it. In the common lathe and grinder's wheel, the action of the fly-wheel is even more important; for here the power produced by the pressure of the foot is applied only in the downward direction, or whilst the crank is moving through the semicircle b B' b'; and the rotation is continued by the momentum of the wheel alone, whilst the crank is ascending through the semicircle b' B b. In the steam-engine used to propel vessels, the weight of a fly-wheel renders it inadmissible; and though the paddle-wheel serves, to a certain degree, the same purpose, it cannot produce an equable motion.

A single engine, therefore, is rarely used for this purpose; but two are employed, of which the cranks are so fixed to the same axle, that, whilst one is acting most disadvantageously, the other shall be acting to the greatest advantage. The same plan is adopted in the locomotive engines used on railways.

308. The crank is employed, however, not merely to convert straight motion into a rotatory one, but also to convert a rotatory movement into a straight one; and it is here, also, of advantage to apply the power to several cranks fixed in different directions, rather than to one only, or to any number fixed in the same direction. For in the latter case, the power with which the axle is being turned would only come into advantageous operation at two parts of its revolution, those in which the crank might be in the positions A B and A B′ (Fig. 78); but if the cranks be numerous, and be so arranged that some of them are in these positions, whilst others are in the positions A *b* and A *b′*, the power with which they are turned by the axle, will be much more equably applied. In the accompanying figure of a rice-mill, *a a a a* are four sieves, in which the rice is placed, for the purpose of having the husks beaten off, by pestles which move up and down within them. These pestles are at the lower end of the rods *c c c c*, which are fixed within a framework that slides up and down between the guides *b b b b*. They are set in motion by the cranks *d d d d*, which are so arranged, that whilst one is at its highest point, another is at its lowest; and, of the two others, one is in the middle of its ascent, and the other at the same part of its descent.

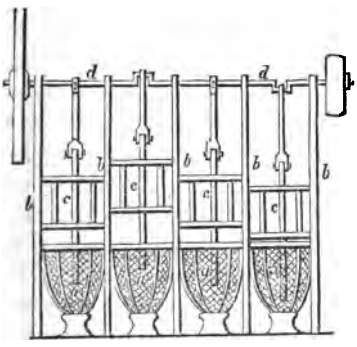


FIG. 79.

Balance.

309. The use of the balance, in its various forms of scales,

steel-yard, weighing-engine, &c., is entirely dependent upon the principles of lever-action. We employ it to determine the weight of a substance, by balancing it against another whose weight we know. The ordinary balance, or pair of scales, is a lever of the first order, with the fulcrum (which is the point of suspension) in the centre, and the two arms exactly equal. From the extremities of these arms, the scales are freely suspended, so that a weight placed in either of them hangs perpendicularly from the point at which it acts on the lever, when the latter is in a horizontal position. This, as we have seen, is essential to the correct action of the lever; since, if one of the scales were drawn inwards, the acting length of its arm would be diminished, because the force is applied in a less advantageous mode (§.299); and the same would occur, if the scale were drawn outwards. The weight of the balance itself must be so adjusted, that its centre of gravity shall be at a greater or less distance *below* the fulcrum, for the beam will then have a tendency to hang at rest in the horizontal position. If the centre of gravity were *at* the fulcrum, the balance would hang at rest in any position. And if it were *above* the point of support, the beam might be brought to rest in a position exactly horizontal; but it would be upset by the slightest disturbance, and would have no tendency to return to it again (§.116). In a properly constructed balance, the centre of gravity, G, being below the fulcrum, will oscillate to one side like a pendulum, when one end of the lever is pressed down by a force suddenly applied to it.

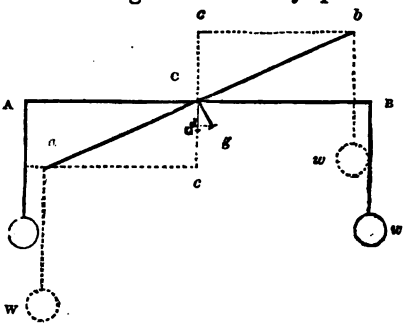


FIG. 80.

310. Now, such a displacement of the beam does not occasion any alteration in its leverage, so far as the scales are concerned; for the loss of power occasioned by the different

direction of its application is the same for both. This will be readily seen from the subjoined figure. Let the beam A B be thrown into the position *a b*; then, according to the principle formerly stated (§.299), the real acting length of the arm A C will be expressed by the line *a c*; and that of B C by the line *b c*. But it is easily shown that, in any position of the balance, *a c* will be always exactly equal to *b c*. The tendency of the balance to regain its equilibrium, when this has been disturbed, is due, therefore, to the displacement of its centre of gravity, which has been thrown from the position G, in which it hung perpendicularly below the point of support, to the inclined position *g*. The more it is displaced, the greater is its tendency to return to the perpendicular; and at last, therefore, this tendency is strong enough to resist the force which set the beam in motion, so that *g* begins to move back to G, and the beam returns to the horizontal position. But, just as the pendulum, when drawn out of the perpendicular, and then let go, has a tendency to swing as far on the other side, through the force it has acquired in its descent, so has the centre of gravity *g* a tendency to pass G, and to move to the same distance on the other side; thus occasioning the beam to become inclined in the opposite direction. Like the pendulum, it will again return, when the force which carried *g* onwards has been destroyed; and *g* will again return to G, pass it, and swing to the other side. The balance will thus perform a series of oscillations, which, like those of the pendulum, would continue for an indefinite period, were it not for the effects of friction and resistance of the air, which cause each oscillation to become less than the preceding, and at last bring the beam to rest in the horizontal position, so that G shall be in the line drawn perpendicularly downwards from C.

311. If the foregoing explanation have been rightly understood, there will be no difficulty in comprehending the action of the balance when used for weighing. Let us suppose, in the first instance, that equal weights are put into the two scales; it is obvious that there will be no disturbance in the equilibrium of the balance, since the product of each weight by the length

of its arm will be the same. And further, if the balance be set in motion, the oscillation will continue as before, until the centre of gravity regains that position in which alone the whole machine can remain at rest. But if a small additional weight be placed in one of the scales, so that W (the weight in one scale) is made greater than *w* (the weight in the other), then the balance will not come to rest in the horizontal position, but in one inclined to it. For the difference of W and *w* will obviously be a force tending to draw down the arm A C (Fig. 80) into an inclined position *a* C, in which it will act with a leverage represented by *a c*. This force is resisted by that which is produced by the displacement of the centre of gravity from G to *g*; and this tends to restore the beam to its level, with a lever power also represented by the distance to the perpendicular, *g d*. Now as *a* C is more and more pressed down, it is obvious that the acting length *a c* of its arm will be more and more diminished; and at the same time, *g d* will be increased; so that there will be a position in which the difference of W and *w* multiplied by *a c*, shall become equal to G multiplied by *g d*. This position will be that in which the balance will come to a state of rest. We know, by the depression of the arm A C, that the weight W is greater than *w*; and accordingly we add to *w*, or take away from W, until we find that they are equal, by the beam being brought to an equilibrium in the position A B. It is obvious that, the smaller the difference between W and *w*, the less inclined will be the position of the beam; since the required alteration in the place of G will also be less. But, on the other hand, the difference of W and *w* may be so great, that it cannot be counterbalanced by any alteration in the place of G; and the beam will then be turned into the perpendicular direction, if so permitted.

312. It is obvious that the correctness of a balance will depend upon the equality in the length of its arms; for, if these be of unequal length, equal weights in the two scales will not be in equilibrium; and, in order to bring the balance into equilibrium, the longer arm must have a less weight. Hence, as such a balance *appears* to indicate that the bodies in the two scales are

of equal weights, when they are really not so, it is a *false* balance. It may be made to hang in equilibrium when empty; for, by making the shorter arm, with its scale, rather heavier than that of the opposite side, the weight of each multiplied by its length, will give the same product. But, if weights known to be equal are placed in the opposite scales, the deception is at once shown by the preponderance of the longer arm. If the unfairness of a balance be suspected, it is very easily put to the test; for we have only to change the weights, and the bodies weighed, from one scale to the other, and the deception will be manifested by the want of equilibrium which will then show itself. For it is evident that, in an accurate balance, we may do this without any alteration in the equilibrium; since equal weights, being multiplied by equal lengths of arm, will always give the same product. But we will suppose that we have to do with a balance of which the two arms A and B have a proportional length of 8 and 7: such a balance cannot be brought to an equilibrium unless the weight in A be to that in B as 7 to 8, since $8 \times 7 = 7 \times 8$; but supposing the weights changed to the opposite scales, we shall have on the side of A, $8 \times 8 = 64$, whilst on the side of B we shall have $7 \times 7 = 49$.

313. It would be possible, however, to weigh truly, even with a balance so false as this; and we may do this in either of two modes. If we place the body in either scale, and ascertain the weight necessary to balance it in the other,—and then repeat the same process on the other side,—by multiplying together the two false weights thus obtained, and taking the square root of the product, we shall obtain the true weight. Thus, suppose that a body weighs 14 ounces in one scale, and 16 ounces in the other; the product of 14 and 16 is 224, the square root of which is $14\frac{2}{3}$, the true weight of the body in ounces. A simpler mode, however, is the following. Let the body to be weighed be placed in either scale of the balance; and any heavy but minutely-divided substance (leaden filings for example) be deposited in the other, until the body is precisely counterbalanced. Now if the article to be weighed be removed from the scale-pan, and weights be

put in its place, until the equilibrium is again restored, the weight thus substituted will exactly represent that of the body, since it balances precisely the same load on the opposite side of the beam. This mode, which is known as Borda's method of double weighing, is the most accurate that can be devised for any purpose, even if we have reason to believe the balance to be correct, since it is altogether independent of any errors arising from its construction.

314. By the *sensibility* of a balance is understood the readiness with which it will be turned by a very small weight; and, consequently, the amount to which it will be pressed down by this before its equilibrium is attained. This is chiefly determined by the position of its centre of gravity; for it is evident that, the further the point G is below C (Fig. 80), the greater will be the space $g\,d$ through which it is moved by any given alteration in the position of the beam, and the greater will be its power of counterbalancing the effect of a weight placed in the scale A. Hence, for a very delicate balance, the weight of its parts must be so arranged, that the centre of gravity G shall be but a very little below C. Such a balance, however, will have the disadvantage of remaining a long time in a state of oscillation, and not readily coming to an equilibrium; since, the nearer G approaches to C, the less will be the tendency of the system to arrange itself in such a position that the beam shall be horizontal. For ordinary purposes, in which great nicety is not required, it is of advantage that the balance shall not be long in coming to rest; and this is effected by placing its centre of gravity at a sufficient distance below C.

315. The perfection of a balance depends, however, upon many other circumstances, which are quite independent of the adjustment of its centre of gravity. Among these, the principal one is freedom from friction in the points at which the beam is suspended, and at which the scales are hung from the beam. In order to attain this, the fulcrum of the best balances is a piece of steel, resembling the blade of a knife, but thicker at the back,—technically termed a knife-edge. This is fixed into the

beam, with its edge directed downwards, and rests upon two flat pieces of hardened steel or agate. The scale-pans are suspended in a similar manner from knife-edges placed at the extremities of the beam, with their edges directed upwards. In order that no injury may arise from the constant bearing of the beam upon its points of support, it is customary for good balances to be provided with a frame, so contrived as to lift up the beam, by the motion of a lever at the bottom of the stand. This frame has another use. It has been already stated that a very sensible balance is long in coming to a state of equilibrium, when loaded with weights which are nearly or quite equal. By observing its vibrations, however, we may form a tolerably accurate guess when the weights are equal; for if the oscillations are the same to each side, it is known that the balance, when it does settle, will take the horizontal direction. The amount of these oscillations is judged of by a long index or pointer, which is fixed at right angles to the beam (like the *tongue* of an ordinary balance) and points to a divided scale at the bottom of the stand. Now when we have reason to believe that the weights are nearly or quite equal, the process of the beam's return to equilibrium may be much shortened, by moving the lever and frame already mentioned, in such a manner that the beam may be brought into the horizontal position, though not lifted off its bearing. If the weights are equal it will remain there, and the operation is concluded; if not yet equalized, the addition of a very minute weight will maintain the equilibrium.

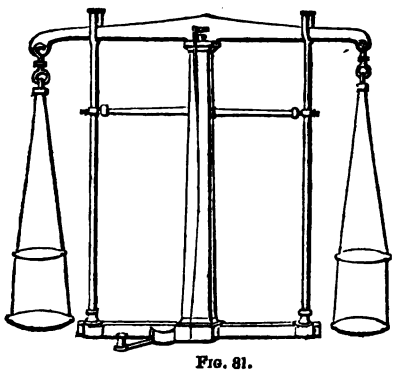


FIG. 81.

316. The most perfect balance yet constructed, is probably the one which was employed for determining the weight of water contained in the national standard bushel. Its beam is

made of a piece of mahogany, about 70 inches long, 22 inches wide in the middle, but tapering off towards its extremities, and $2\frac{1}{4}$ inches thick. The reason for preferring wood is that, in a beam of this size, it combines lightness and strength more than does any other material; and also that is not liable to be suddenly affected in its length by a change of temperature. This last is a consideration of much importance; for if, during the use of a delicate balance with a metal beam, a draught of cold or hot air were to blow unequally on the two sides of it, the length of the arms would be altered by the change of temperature in an unequal degree, and the performance of the instrument would be erroneous. The points of support of the beam are two planes of hard polished steel, on which a very strong knife-edge rests; and the scales are hung from similar planes also resting on knife-edges.* By a very ingenious contrivance, however, the beam and scale-pans may be made to rest upon round pivots, by a motion of the handle at the bottom. The balance is then rendered less sensible; and an opportunity is thus given of bringing the weights in the scale-pans nearly to an equality, before that extreme sensibility is given to the balance (by bringing the knife-edge supports into action) which renders the adjustment difficult. This contrivance has the further advantage of diminishing the wear of the knife-edges and of the planes on which they rest, by greatly shortening the time in which they are bearing on one another.

317. As the place of the centre of gravity is influenced by the amount of weight in the scales, a fresh adjustment of it should be made, in a balance in which the greatest sensibility is required, for every different weight; otherwise it will be much more sensible when the scales are but lightly loaded, than when they are heavily weighted, since in the last case the centre of gravity is lowered too far below the fulcrum. In the balance now described, this adjustment is made by means of a ball, which could be moved by means of a screw, nearer to the

* It is not requisite that these edges should be sharp, as was formerly imagined to be necessary; the sharper they are, the more liable to injury they will obviously be.

fulcrum or farther from it, in the perpendicular line; so as to raise or lower the position of the centre of gravity. The sensibility of the balance, when thus adjusted, is extreme. When loaded with a weight of 250 lbs. (avoirdupois) in each scale, it was very sensibly turned by the addition of a grain on either side. As each pound contains 7000 grains, the weight in each scale was 1,750,000 grains; and thus the balance turned with a weight not much greater than one *two-millionth* part of that to be determined.

318. The *steel-yard* is another form of balance, in which the principle is exactly the same, but the mode of applying it is different. Its value consists in the simplicity of its construction, a single weight being adapted to determine the weights of substances differing considerably in amount. This is accomplished by making a variation in the acting length of the arm from which the weight is suspended; that on which the body is hung remaining the same. The principle of its action is easily understood. Let A B be the beam of the steel-yard, and C its centre of support very near the extremity A, to which the body P is hung. The weight of the arm A C being made equal to that of B C, so that the lever itself shall be in equilibrium, it is evident that, if a weight equal to P be hung on the side B C at a distance equal to A C, the equilibrium will still be retained. But the same effect will be produced by hanging a weight equal to $\frac{1}{2}$ P at twice the distance A C, or a weight equal to $\frac{1}{3}$ P at three times the distance A C; since the weight multiplied by the length of its arm will thus always give the same product. The steel-yard being provided with a single weight, W, of a certain amount, the amount which this will balance at P will depend entirely on the distance of W from C. Thus, suppose the weight to be 2 lbs., if it be placed at 1 (the distance C 1 being equal to A C), it will balance a weight of

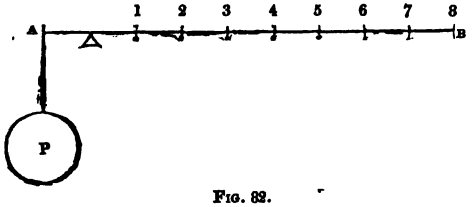


FIG. 82.

2 lbs at P. If placed at 2, which makes the distance C 2 equal to twice A C, it is able to balance a weight of 4 lbs. at P; if placed at 3, a weight of 6 lbs; and so on,—the amount it may thus be made to counterpoise being only limited by the length of the arm B C. In weighing with the steel-yard, therefore, we move the weight backwards and forwards, until the body at P is counterpoised; and its actual weight is known by a scale marked on the beam. In general, steel-yards are so constructed as to have two different scales, one adapted to weigh smaller, and the other greater weights. This is accomplished merely by an alteration in the position of the fulcrum. In the lever represented in Fig. 82, the weight W (2 lbs.) if removed to 8, its extreme end, will balance 16 lbs.; but suppose that, the whole length of the lever remaining the same, the fulcrum were removed nearer A, so that the distance B C should then be equal to 25 times A C; the weight W placed at B would then support 50 lbs. at P, and would balance proportional weights at the intermediate points.

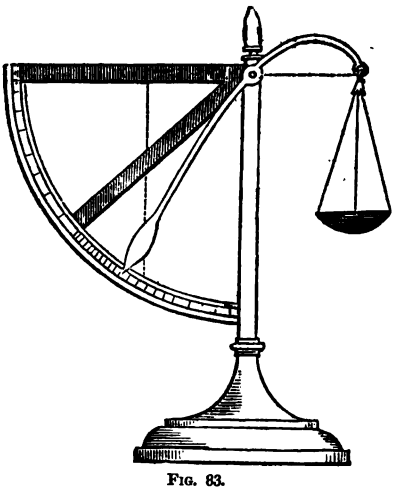


Fig. 83.

319. The bent lever-balance is somewhat similar in its construction to the steel-yard; for it acts by a fixed weight, the distance of which from the fulcrum increases or diminishes. Its operation will be understood from the accompanying diagram, when the principle of action of the bent lever is kept in mind (§.299) The acting length of the arm which carries the scale, is represented by the horizontal line drawn from the fulcrum to its line of suspension; and the acting length of the arm that carries the weight is shown by the space inter-

vening on the other side, between the fulcrum and the dotted line drawn perpendicularly through the weight. In the present position of the balance, these distances are equal; and if the weight in the scale be equal to that fixed to the balance, the beam will be in equilibrium at that point. But if an additional weight were placed in the scale, the weight on the opposite side would be raised; and the acting length of *its* arm will be much increased, whilst that of the scale arm will be but little changed, or partly diminished. The position in which equilibrium will be obtained, under this new force, will be that in which the weight P in the scale, multiplied by the acting length of its arm, shall be equal to the constant weight W, multiplied by the variable length of *its* arm. The heavier the weight in the scale, therefore, the higher must the weight be raised on the other side; and by the height at which it settles itself, the weight placed in the scale may be known, from the divisions marked on the quarter-circle through which its point may move. Such balances are in common use at the present time as letter-weighers; but though convenient, they are not very accurate.

320. In determining very heavy weights, such as those of loaded carts, a machine is employed, on the principle of the compound lever. The cart is made to stand on a platform, which rests at its four corners on four hard steel studs, which project upwards at C, C′, E, E′, (Fig. 84), from the levers D B, D′ B′ &c. The fulcrum of each of these levers is at the extremity (B, B′ &c.) near the studs; and the two levers on each side are united, near the centre of the apparatus, by short cross pieces, which are brought to an edge beneath. These levers are therefore of the second order; the fulcrum being at B, the weight at C, and the power (as will presently be seen) being applied at D.

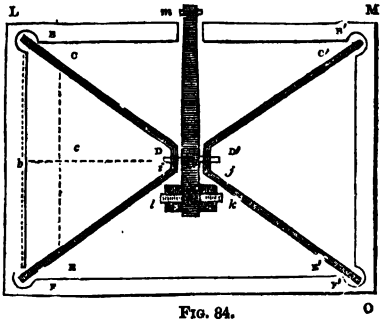


FIG. 84.

The other lever in this compound system is F P m', in which F is the fulcrum, composed of an edge-piece of hard steel resting on solid supports; through P is another edge-piece, on which rest on either side the edged pieces of the levers D B, D′ B′ &c.; and there is another at m n, which is connected with a rod passing upwards. This rod is connected with a balance or steel-yard of any kind, having some convenient situation. Now it is evident that, if the end m n be drawn up with a certain force, this force may be considered as the power in the lever m P F; and it will tend to raise the cross piece i j on which the weight is resting, with a force as much greater, as the distance F P of the latter from the fulcrum is less than the distance F m. In like manner, the force applied to raise i j, becomes the power in the lever D C B; and is as much greater at C, the point on which the weight bears, as D C B is less than D B.

321. The amount of force thus gained, is estimated by multiplying the power of one lever by that of the other (§.296);* and we must of course include in this estimate the power of the steel-yard or lever to which the connecting rod from m n is attached. Now let us suppose the arms B C and F G to equal 1 foot, whilst the arms B D and F m are each 10 feet; then the power of each of these levers will be as 10 to 1, and their combined power will be as 100 to 1. Further, if the connecting rod from m n be attached to the short arm of a lever on the steel-yard principle, so as to be at a distance of only 1 inch from the fulcrum, whilst the longer arm to which the weight is hung is 12 inches, the additional power will then be as 12 to 1; and the power of the whole combination will be as 1200 to 1, or a weight of 1 lb. at the end of the steel-yard lever will balance a weight of 1200 lbs. on the platform. By a proper adjustment of the levers we may obtain any proportion we please between the small and the large weights; thus an ounce may be made to balance an hundred-weight. Small portable weighing-machines, for taking

* It is evident that, although the weight of the cart is distributed, by means of the platform, upon *four* levers, the combined effect of these in pressing down i j is the same as if only *one* were employed.

the weight of the human body, are constructed on the same principle; but the arrangement of the levers in them is usually such, that a pound is made to counterpoise a hundred-weight.

Wheel and Axle.

322. The space through which a man can raise a heavy weight by an ordinary straight lever is small; because, when he has pressed down the end to which he applies the power as far as he can advantageously do, he has no means of raising the weight further, except by altering the position of the fulcrum, and supporting the weight in some other mode whilst he is doing so. When a *continuous* motion is required, as when we desire to raise a stone to the top of a building, or water from a well, an ordinary lever could not be made use of, except in a very inconvenient mode. For such purposes, however, the *wheel and axle* offers every facility. This is only another form of the lever, so arranged as to have the continuous action required. We will suppose two ropes to be coiled in contrary directions, round a cylinder or drum supported on pivots, and a weight hung from the end of each. If the weights be equal, the cylinder will have no tendency to turn on either side, but will remain in equilibrium, like a balance. For suppose *a d b e* to represent such a cylinder cut across, C to be the centre on which it turns, and *a w*, *b* W, to be the lines in which the weights hang. Then *a* C and *b* C, the lines drawn from those points to the centre of the circle, may be considered as representing the two arms of the lever; and as these lines will be always equal, in any position of the cylinder, (since they will always be radii of the circle), the equal weights, which hang in lines perpendicularly to their extremities, will remain in equilibrium.

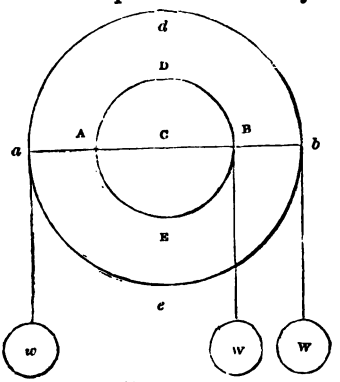


FIG. 85.

323. But suppose that, on the same spindle, there be two cylinders, one larger than the other; and that equal weights be hung from these in the same manner as before;—these weights will then be no longer in equilibrium, but the one that is hung to the larger cylinder will have a tendency to descend. The reason of this will be obvious on again referring to the figure. For let A D B E be the section of the smaller cylinder, from which hangs the weight W in the direction B W, opposite to the weight *w* on the larger cylinder. It is obvious that the weight *w* must have a greater power of turning the cylinder in its direction, than the weight W possesses for *its* direction; since the former is acting at the distance C *a*, whilst the latter is acting at the distance C B; and the same inequality will exist in any position of the cylinders. Such an apparatus may be regarded, therefore, as a lever having a continuous action,—C being the fulcrum, C B the weight-arm, and C *a* the power-arm; and the mechanical advantage gained by it will be exactly proportional to the number of times that C *a* exceeds C B. By extending C *a*, therefore,—that is, by enlarging the circle to which the power is applied,—we may gain any amount of advantage we desire. But here, as in the case of the simple lever, what is gained in power is lost in velocity; since the hand applied to the large cylinder or wheel will move so many times more than the weight suspended from the small cylinder or axle, as the diameter (and therefore the circumference) of the former exceeds that of the latter.

324. In the various forms of the wheel and axle, the weight hangs from a rope coiled round the axle; but the power is applied in various modes. The wheel may have projecting-pins upon its surface, to which the hands may be applied, as in that employed for steering a ship; or it may have teeth cut upon its edges, into which another wheel or pinion (§. 336) may work. But it is not necessary for the efficiency of the machine, that the power should be applied to a complete *wheel;* since a single spoke of that wheel will answer the purpose just as well. This is, in fact, the principle of the common windlass used for raising water out of a well, earth and stones out of a pit, and various other such

purposes. The winch constitutes the power-arm of the lever; whilst the distance between the centre of the axle and the point at which the cord pulls upon it (or in other words the radius of the axle) is the weight-arm. In estimating the power of a windlass, therefore, we divide the length of the winch, reckoned from the centre to the handle, by the radius of the axle; since weights inversely proportional to these, hung in the situations of the power and the resistance, would be in equilibrium. Thus, suppose the radius of the axle to be 2 inches, and the length of the winch 2 feet, the machine will have a power of 12; and a force of 1 lb. applied perpendicularly to the handle will balance 12 lbs. hanging from the axle. Sometimes the axle has no fixed handle or lever, but bars are inserted into holes cut in it, and these are shifted as occasion requires; this is the construction of the windlass, which is fixed in the fore part of a ship for the purpose of heaving the anchor. In order to gain sufficient power, the bars must be very long, and these can only be worked through a small part of a circle, without coming in the way of the deck; when they have been moved as far as they will go, therefore, they are shifted into a fresh set of holes and again drawn down. The axle is sometimes placed in a vertical position, so that the bars may go round horizontally. This is the case with the capstan, which is usually placed near the middle of a ship's deck, and is used for drawing up goods from the hold, &c., being turned by men who walk round and round upon the deck, pressing the bars before them.

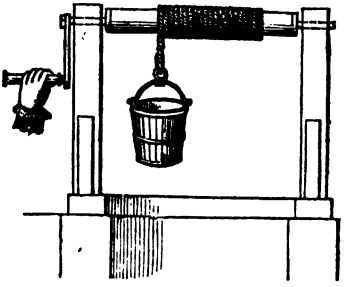


Fig. 86.

325. Although we may regard the power of the wheel and axle as capable of being increased, like that of the lever, to any extent, yet the same objections apply in practice to the carrying of this increase beyond a limited amount. If the radius of the

axle be diminished beyond a certain size, it will become too weak to bear the weight; and the radius of the wheel or winch cannot be increased beyond moderate dimensions, without inconveniences so great as altogether to prevent its employment. Hence, in order to gain a very large amount of power, it is necessary to make some alteration in the construction of the apparatus. We might, for instance, combine two of these machines together, upon the principle of the compound lever,—the cord passing from the axle of the first over the wheel of the second, and the resistance overcome by the first thus serving as the power to the second. Or we may apply some other mechanical power, such as a pinion (§. 337), or endless screw (§. 374), to turn the wheel. But the same end may also be answered by a very simple modification in the form of the axle, and in the mode in which the weight is suspended from it. The axle, instead of having the same diameter along its whole length, is smaller at one part than at another, so as really to consist of two cylinders of unequal sizes. The weight is suspended to a pulley, which plays on a cord that is coiled round both axles in contrary directions. When the winch is turned in such a manner as to wind the rope upon the larger part of the axle, it will be unwound from the smaller, but not to the same quantity; so that the rope will be shortened by the amount of the difference between the quantities coiled *on* the *larger* cylinder and *off* the *smaller* one. This shortening will raise the pulley and weight through half its amount, since it is divided between the two cords on which the pulley bears.

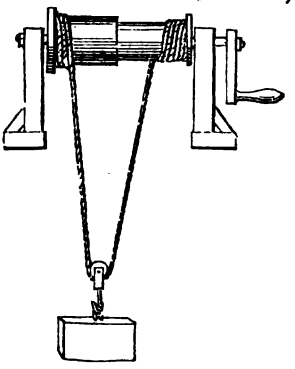


FIG. 87.

326. Now it is evident that, if the axle were of the same size throughout, and the two ends of the rope were coiled round it in contrary directions,—the pressure of the weight, being equally divided between the two strings, and acting in each case

at the same distance from the centre of the axle, will not give to this any tendency to turn round in either direction; and further, that, if the axle were turned round by the winch, the weight would not be moved, since it would uncoil at one part as fast as it would be coiled up at the other. On the other hand, suppose the diameter of the larger part of the axle to be twice that of the other; the force which the weight exerts upon that part, by means of the string coiled round it, will be twice as great as that exerted upon the other by *its* string in the opposite direction; and the difference of the two forces will be the real amount to be overcome by the power applied to the handle. The more nearly the two parts of the axle approach each other in size, the more nearly will the effect of the one string upon the smaller axle counterbalance that of the other string upon the larger one; and as this difference is all that the force applied has to overcome, the mechanical advantage gained is very great, and may be increased to any amount, merely by diminishing the difference in diameter between the small and the large parts of the axle. This difference is in fact the real working diameter of the axle; and the power of the machine is estimated, as before, by ascertaining how often this diameter is contained in that of the wheel, or of the circle which the winch describes. But as the weight is hung on a movable pulley, in such a manner that it bears equally on both strings, and as only one string is concerned in raising it, an additional power is gained, equal to double that just mentioned (§. 343). But, for the reasons already mentioned, every increase of power is accompanied by a corresponding decrease in the velocity of the weight, when it is compared with that of the power. We have thus seen that the principle of this machine,—which is usually termed the Chinese wheel and axle,—is that of making the weight or resistance partly counterbalance itself, so that the power may act only against the remainder or unbalanced portion. This principle is introduced into some other machines, with equal success (§. 351 and §. 376).

327. The applications of the principle of the wheel and axle in the construction of various machines are very numerous. Besides those already mentioned (§. 324), which are only

varieties in the form of the simple wheel and axle, we may notice the different kinds of water-wheels which are employed as moving powers. In these, the weight of water, or the impulse from a stream, being applied to the circumference of the wheel, exerts a power, which, when communicated to the axle, is multiplied in proportion as the diameter or radius of the wheel exceeds that of the axle. Where there is a small stream having a considerable fall, the kind of wheel employed is that which is termed an *overshot* wheel; the float-boards of which are so formed, as to receive the water as it falls, and to deliver it, by the turning of the wheel, into the stream at the bottom, so that its weight may act on the wheel during the whole time that it is making the descent. As the wheel moves round, its successive float-boards receive the successive quantities of water which are brought by the streamlet; and thus the whole quantity that falls is made to exert its power on the wheel. Of course, the higher the fall, the larger may the wheel be made, and the greater will its power be. On the other hand, when the stream has little or no fall, but a rapid current, it is made to turn a wheel merely by its impulse on the float-boards; which are made, not to hold the water, but to be struck by it. Such a wheel is termed an *undershot* wheel. Where a stream has a rapid current, however, it is seldom that we are unable, by damming it up, to produce *some* fall; and the water may then

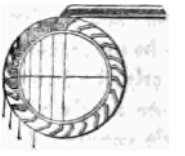
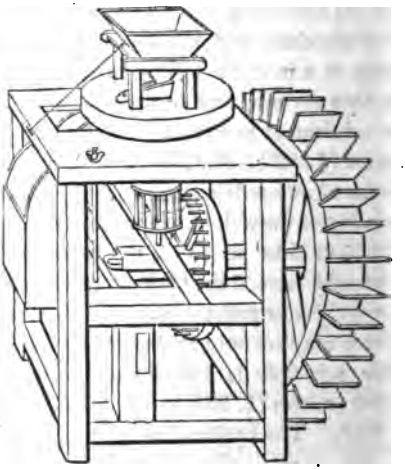


Fig. 88.



Corn-mill, turned by undershot wheel.

Fig. 89.

be advantageously brought, not to the top or bottom of the wheel, but to its side, so as to be received by the float-boards, and to act upon them, during its descent through about a quarter of the whole circle, to the bottom. Such a wheel, which is the most common of all water-wheels, is termed a *breast*-wheel.

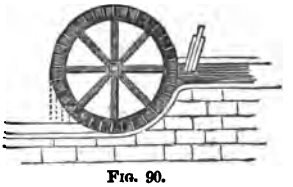


FIG. 90.

328. It is obvious that we may make the diameter of an undershot wheel as great as we please; and that, the greater the diameter, the larger will be the gain of power. But in this, as in all previous instances, what is gained in power is lost in velocity; since a given amount of movement in the water, which would carry a wheel of 12 feet in diameter through a whole revolution, will only carry a wheel of 24 feet through half a revolution. The most advantageous diameter of a breast-wheel will depend, like that of the overshot wheel, upon the height of the fall. It is obvious that the water does not act equally in moving the wheel, during the whole of its descent; for, as the power produced by its weight always acts in lines perpendicular to the earth's surface, its action at A is in the direction A *a*, and therefore the length of its leverage (according to the principle formerly stated, §. 304), is only C *c*. By the time that the wheel has moved round, so that the weight is at B, it will act in the direction B *b*, and therefore with the lever-power C *c'*; and when it has arrived at D, it will act with the power of the full radius D C. It is obvious that no weight of water at E will have any influence in turning the wheel, since its pressure is in the downward direction E F; but as soon as it is acting, to the least degree, on one side of this, it will begin to exert a power which continually increases until it reaches D, after which it will diminish in the same proportion.

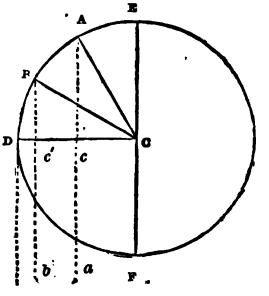


FIG. 91.

329. In the paddle-wheel of a steam-boat, also, the principle of the wheel and axle is applied; but in a contrary mode. The power is here applied to the axle, in order to produce a certain effect at the circumference of the wheel; and there is consequently a great loss of power, but a corresponding gain in velocity. If the wheel were *fixed*, and made thus to revolve by a power applied to the axle, a rapid current would of course be produced in the water, by the action of the float-boards; but, in the present instance, the water is the fixed point; so that, by the movement of the wheel, the axle and the vessel which carries it are forced onwards. In all the paddle-wheels of ordinary construction, there is a great loss of mechanical power, arising from the oblique mode in which the float-boards act upon the water, during the greatest part of their movement in it. This will be readily understood from the subjoined figure, representing the acting portion of a paddle-wheel, of which C is the centre, and A B D E F are five of the float-boards. The direction of their movement is from A to F; and A, therefore, is just entering the water (whose surface is represented by the line *a b*) whilst F is leaving it. By the action of these float-boards, the axle has to be moved onwards in the direction C *c*; and they ought to press upon the water, therefore, in the direction precisely opposite, namely, *a b*. But none of the float-boards except D will do this; for A and B will be pressing upon the water in a downward direction, so that a portion of their force is thus uselessly expended; whilst E and F will press upwards, and, in leaving the water, will carry up some of it upon their surface, so that its weight creates the necessity for an additional power to turn the wheel. Various ingenious contrivances have

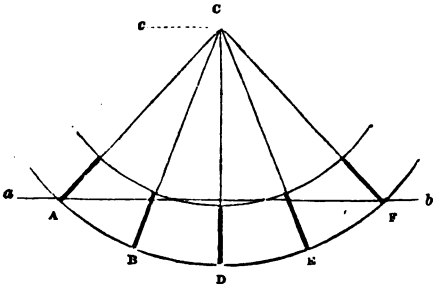


FIG. 92.

been devised, for causing the float-boards to dip into the water, and to leave it, in the perpendicular direction; so as to be always acting upon the water in the line *a b*, and to avoid the carrying up of the *back-water*, as it is termed, upon the floats that are leaving the water. But notwithstanding the great advantage thus gained, it is found that the complexity of the machinery required, and the risk of its getting out of order, more than counterbalance this; and it is not probable that any substitute for the ordinary paddle-wheel will ever be introduced.

330. We have now to speak of those machines, in which the principle of a succession of wheels and axles, acting alternately on one another, is introduced. It has been shown (§. 325) that this principle is exactly the same as that of a compound lever; the resistance overcome by the first axle, becoming the power of the second wheel; so that the power of the whole combination is found, by multiplying the power gained by the first wheel and axle by that gained by the second. Now, in practice, such combinations, or series of combinations, are extremely common, and they constitute, in fact, the principal part of mill-work of all kinds. The chief variation is in the mode of communicating the power from one revolving surface to another. This *might* of course be effected by a simple cord, like that by which the weight ordinarily hangs from the axle, passing from it to the wheel, and coiled round its circumference; so that, by the power applied to the first wheel, this cord might be wound round its axle, and motion be thus given to the wheel. But such a mode would be attended with this great inconvenience,—that, as soon as the cord is entirely wound off the wheel on to the axle, the action of the machine must stop. The mode which is adopted

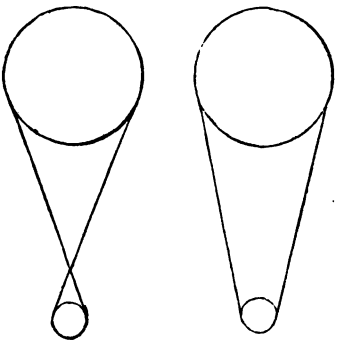


Fig. 93.

in practice, therefore, is to make an *endless* cord or band pass rather tightly over the axle and the wheel which it is to turn; and it is advantageous to cross this, if the sizes of the two be very different, in order to give the cord or band a greater hold upon their surface; as it is by the friction of this cord or band, that the motion of the one is communicated to the other. (Fig. 93.) When the band is crossed, however, the motion of the wheel and axle will be in contrary directions.

331. This kind of connection is used for various purposes. Sometimes it is employed simply to communicate motion from one revolving spindle to another at some distance; and if, as frequently happens, the second spindle be of the same size with the first, no mechanical advantage is either lost or gained; but the moving power may be thus conveyed from one end of a long room to another, or from the top to the bottom of a high building. If the revolving spindle be smaller, however, than the one which is to be thus put in motion, it is obvious, from the explanations already given, that power will be gained, in proportion as the diameter of the wheel moved exceeds that of the axle that moves it; but in the same proportion velocity will be lost, since the axle will have to turn just as many times round to produce one revolution in the wheel. The contrary will necessarily be the case, when the diameter of the moving wheel exceeds that of the axle to be moved; for the velocity of the latter will be multiplied, whilst its power is diminished. This is the principle of the common turning-lathe; where a large wheel, put in motion by a crank, is made to act upon a small pulley, and the substance to be turned is thus whirled round with great velocity: and it is much employed in mill-work, in which a small machine has often to be turned with great velocity, but with very little power, by means of a great power moving at a slow rate.

332. The great advantage, then, of this mode of connecting the spindles in revolution, is that the motion may be communicated, by merely lengthening the band or cord, from one to another at almost any distance,—until, in fact, the weight of the band itself comes to be a serious objection; and even this will

not interfere, if the motion is being communicated in a vertical direction, as from the bottom to the top of a mill. There is another advantage attending it, also; which is the great simplicity of the means by which its action may be suspended for a time, and again set on. This is an object which must always be kept in view in the construction of any machinery; since the change is very frequently required. In the various machines employed for spinning, weaving, &c., continual stoppages are required, on account of the breaking of threads or other accidents; these must be made by a momentary action, and in such a manner as not to interfere with the action of other machines turned by the same power. In mills for such purposes, there is commonly a large cylinder or drum, running from one end of each principal room to the other, and put in motion by the steam-engine, water-wheel, or other moving power. This drum carries a number of bands, that communicate motion to the spindles of the various machines placed along the room.

333. Every one of these machines ought to be capable of being checked at once, without the motion of the drum being suspended or retarded for an instant. This is accomplished by a very simple contrivance, known under the name of the *fast and loose pulley*. On the spindle of every machine there are two pulleys, which we shall call A and B; one or other of which is always being carried round by the band from the main spindle. One of the pulleys, A, is *fixed* on the spindle of the machine; and consequently when it is turned, the spindle must be turned with it. The other, B, which is close to A, turns loosely on the spindle; and consequently it may go on revolving with any velocity, without communicating its movement to that spindle. By the simple application of a lever, the band may be pushed from either pulley upon the other. When it is upon A, it will cause the revolution of the spindle and the action of the whole machine; but, if a stoppage be required, the band has merely to be pushed by the lever, off the pulley A, on to B; and as the latter goes on revolving, without in the least affecting the axis on which it turns, the machine remains at rest, until the strap is pushed back to A. In some machines, it is so contrived, that this change

shall be produced by themselves, whenever it is necessary; thus rendering them independent of the constant attention of their overlooker.

334. But it is often required that the surfaces of the wheel and axle should act immediately upon each other; and this could not be accomplished if they were both smooth, as their friction would not be sufficient to turn the machine. But if they are both covered with minute roughnesses, these will create sufficient friction, to cause the revolution of one to put the other in motion, provided the resistance to be overcome is not too great. This plan, also, is much adopted in mill-work, especially for the smaller parts of spinning-machines; here the requisite amount of roughness is given by covering the surfaces of the spindles with buff leather, or with wood cut across the grain. The latter mode has even been adopted with success for large machinery; and it has the advantage of producing a motion which is extremely smooth and even, and is consequently accompanied with but little noise; but it could not be depended upon for overcoming any very considerable resistance. The most usual method of directly transmitting motion from one wheel to another, larger or smaller than itself, is by means of *teeth*, or regularly-formed projections upon the surface of each, which lock into the intervals between the teeth on the surface of the other. In this manner, the action of one upon the other is insured, so long as the strain is not so great as to break off the tooth.

335. In order that these teeth may act in the best manner upon one another, it is necessary that they should have a certain rounded form, which can be determined upon mathematical principles. The ill consequences of giving them a square form will be made evident, when their action upon one another is considered in detail. For, in the subjoined figure, when the tooth A comes into contact with B, it acts obliquely upon it; and, as it moves, the corner of B slides upon the flat side of A, in such a manner as to pro-

FIG. 94.

duce much friction, and to grind away the side of A and the end of B. As they approach C D, their sides come suddenly into contact; and a jerk is thus sustained; and after they have passed C D, the same scraping and grinding effect is produced, until the teeth become disengaged, whilst a new set are coming into action against each other. These effects are avoided, by giving to the teeth the curved forms represented in Fig. 95; these will allow the surfaces to roll on one another with very little friction; and the direction of the pressure is always that of the line M N, which touches both wheels, and is therefore perpendicular to the radius of each; so that the power is always applied at the same leverage, and consequently acts equably.

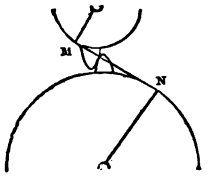


FIG. 95.

336. The various purposes for which wheel-work is applied, require numerous varieties in the mode of constructing it. When we desire simply to communicate a given power from one wheel to another, the wheels must obviously be of the same size. When we wish to gain power at the expense of velocity, however, we cause a small wheel to work into a large one; and, if we desire to increase the speed with a sacrifice of power, we cause a large wheel to turn a small one. Now, in all forms of wheel-work, it is necessary that the teeth should be of the same size in the two wheels; and therefore the space which any given number will occupy on the circumference, will always be the same, whether that circumference be small or large. Hence we may estimate the effects of a train of wheel-work as well, by ascertaining the proportion between the teeth of the several *turning* and *turned* wheels, as by comparing their diameters; and this is, in fact, by far the most convenient mode of doing so. When we wish to make the disproportion between the sizes of the two as great as possible, a very small axle with teeth raised upon it is made to work into a large wheel. Such an axle is termed a *pinion*; and it is usually made with 6, 8, 10, or 12 teeth, which are called *leaves*. Now if a pinion of 6 teeth be made to drive a wheel of

60 teeth, it is evident that the pinion must turn 10 times for 1 revolution of the wheel; but the power applied to the pinion need be no more than 1-10th of the weight to be raised by the wheel.

337. A simple combination of this kind is employed in a machine which is much used for lifting heavy stones to the tops of buildings. It consists of an ordinary wheel and axle; the wheel being cut into teeth at its edge, and being turned by a small pinion, or by an *endless screw* (§.375), which is moved by a winch. The winch thus acts on the pinion, or screw, as a wheel upon its axle; and thus its power, when conveyed to the wheel, is increased in the proportion of *its* radius to that of the pinion. The wheel and its axle have the ordinary power of that simple machine; and thus the whole power is increased by the product of the two. Thus, suppose the winch to have a radius of 18 inches and the pinion a radius of $1\frac{1}{2}$ inch,—the power gained by its lever-action is thus 12. But suppose that the pinion has 8 teeth, working into a wheel having a radius of 18 inches, with 96 teeth upon its surface; then 12 revolutions of the pinion or of the winch will only turn the wheel once. Let the axle of this wheel, from which the cord hangs, have a radius of 2 inches; then its power will be augmented in the proportion of 9 to 1; but the amount which the weight will rise, will be diminished in the same proportion. Thus the power of the winch upon the toothed wheel is as 12 to 1, and the power of the latter upon its axle is as 9 to 1; so that the combined power of the whole is 108 to 1, or a force of 1 lb. applied to the winch will sustain a weight of 108 lbs. hanging from the axle. But the velocity

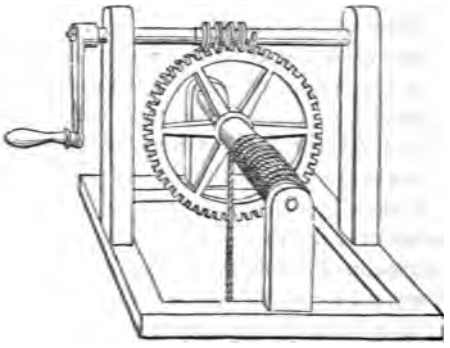


FIG. 96.

is diminished in the same proportion; since 12 turns of the winch only move the toothed wheel round once; and one revolution of this wheel only raises the weight through a height equal to 1-9th of its circumference. We might have obtained the same result by multiplying the diameters of the two wheels together, and the diameters of the two axles together; and then dividing the first product by the second. Thus, $18 \times 18 = 324$; and $2 \times 1\frac{1}{2} = 3$. If we divide 324 by 3, we obtain 108, the power gained, and the velocity lost, by this combination. This principle of computation is the same as that employed in the case of the compound lever (§. 296); and it is applicable, as we shall presently see, to a train of wheel-work to any extent.

338. Where it is desired to obtain a *very* large amount of power at the sacrifice of velocity, or a *very* large amount of velocity at a sacrifice of power, it is advantageous to employ several wheels and pinions, united into a *train*, in such a manner that each wheel shall act on a pinion fixed on the same spindle with the next wheel,—and so on. In such a combination, the mechanical advantage or loss will depend upon the situation to which the power is applied. If it be applied to the wheel which *carries* the pinion, then, as in the ordinary wheel and axle, velocity will be lost, but power gained; and this effect is increased by making the pinion work into a wheel, which shall carry another pinion,—this pinion to work into another wheel,—and so on. Thus, in the accompanying combination, the hand acts upon the circumference of a wheel, *d*, which we shall suppose to have a diameter of 18 inches; as this wheel carries a pinion, *c*, whose diameter is 6 inches, a power of 3 is thus gained. But the pinion has 8 teeth, and turns the wheel *e*, having 24 teeth; and this wheel will therefore make 1 revolution, whilst the wheel *d* and pinion *c* turn 3 times. But the wheel *e* carries a pinion *b* of one-third its diameter and

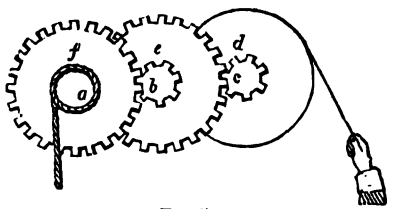


FIG. 97.

number of teeth, so that a further power of 3 is obtained; and this pinion works into another wheel *f*, which carries the axle *a* of one-third of its diameter. The wheel *f* and its axle *a* have only one-third the velocity of the wheel *e* and pinion *b*; but the power is increased in the same proportion. We have thus a series of wheels and axles acting on one another; and to obtain the combined result of all, we may either compute the advantage of each separately, or we divide the product of the diameters of all the wheels by the product of the diameters of all the pinions or axles. As the number of teeth in wheels, or of leaves in pinions, however, must be in exact proportion to their respective diameters, it is convenient to employ these as the data for calculation. Thus, in the present instance, we have seen that each wheel and pinion (or axle) gains a power of 3; and therefore, calculating the combined power of the whole on the same principle with that of the compound lever (§. 296), we find it to be $(3 \times 3 \times 3 =)$ 27. Or, following the second method, we multiply the diameter of the wheel *d* (18 inches), by the number of teeth (24) in the wheel *e*, and this product by the number of teeth in the wheel *f* (24), the product of the whole is 10,368. This we divide by the number of teeth (8) in the pinion *c*, multiplied by the number (8) in the pinion *b*, and again by the diameter of the axle *a* (6 inches), which gives a product of 384; and the quotient is 27, as before. Where the number of teeth in the pinions is contained an even number of times in that of the wheels, the former of these plans is of course the simplest; but this will be presently shown (§. 340) to be seldom the case; and the latter must be therefore generally adopted.

339. If, on the other hand, we apply the power to a wheel which works into a pinion, and this pinion carry round upon its axis another wheel, which in its turn drives a pinion, and so on, there will be a gain of velocity and a loss of power; since each wheel will be turned so many times faster than the preceding one, as the pinion carrying it had fewer teeth than the wheel which drives it. It is in this manner that the wheel-work of clocks and watches is constructed. The first wheel, *b*, is turned

by a heavy weight, *a*, or by a spring coiled up in a barrel; and this acts upon a pinion, *c*, which carries another wheel, *d*, along with it. Thus any extremely slow motion of the weight or spring will produce a comparatively rapid movement of the wheel at the other end of the train, which acts on the pendulum or balance-spring; but the power applied is diminished in the same proportion. The vast increase of velocity is best seen in the striking train of the clock; in which a series of wheels and pinions is made to give a rapid whirling motion to a small flat plate, in order that the resistance of the air against this may produce a greater equality in the movement of the train, than would otherwise occur. It is also well seen in the machine used for spinning glass; in which a cylinder or drum is made to revolve with vast rapidity, by a slow motion applied to a small winch. In this case the gain of velocity and the loss of power, are calculated in a mode precisely the reverse of that employed in the preceding instance.

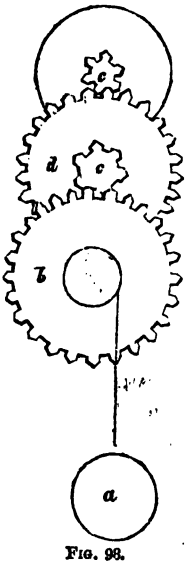


FIG. 98.

340. It is a matter of great importance, in the construction of machinery, to produce equal wear of all the teeth of the wheels, and of all the leaves of the pinions. In order to obtain this, the number of leaves must not be contained an exact number of times in that of the teeth. For if it were, the wheel would perform one revolution whilst the pinion moved round a certain number of times, and then would begin afresh; so that the same leaves of the pinion would come against the same set of teeth of the wheel. Thus, suppose the pinion to have 8 leaves, which we shall represent by I, II, III, IV, &c.; then, in the 8 revolutions of the pinion which would occur whilst the wheel turns once round, the leaf I would have come against the 1st, 9th, 17th, 25th, 33rd, 41st, 49th, and 57th, teeth of the wheel; and, when the wheel begins its next revolution, would again be brought

against the first tooth. In the same manner the leaf II would have come against the 2nd, 10th, 18th, 26th, 34th, 42nd, 50th, and 58th, teeth; and would come in contact with the same set in the next revolution of the wheel. The same will of course occur with regard to the remaining leaves. Such a system will have the disadvantage of producing a very unequal wear of the teeth of the wheel, if the leaves of the pinion have any irregularities which occasion them to act with unequal friction. Such irregularities can scarcely be avoided; but they are made to act equally upon all the teeth of the wheel, by the simple expedient of making the number of these *one more* than an even multiple of the number of leaves in the pinion. For, supposing that leaf I has acted upon the teeth 1, 9, 17, 25, &c., in the first revolution of the wheel, it will be brought to teeth 2, 10, 18, 24, &c., during the second revolution, to teeth 3, 11, 19, 27, &c., during the third; and so on. So that, as may easily be shown by a simple computation, the wheel will have revolved 65 times before leaf I again bears upon the the teeth 1, 9, 17, &c.; and in this interval, every one of the leaves of the pinion will have been engaged with each of the teeth of the wheel, so that the effect of any irregularities in the former will be equally distributed over all the latter. This additional tooth is termed by millwrights the *hunting-cog;* the term *cog* being used for all large teeth.

341. It is not requisite that the wheels that work together should have the same direction; since their axes may be inclined to each other at an angle, and may even be perpendicular one to the other. This change of direction may be obtained by inclining or bevelling the edges of both wheels, as seen in Fig. 99; or by raising the teeth of one wheel, not (as usually) upon its edge, but

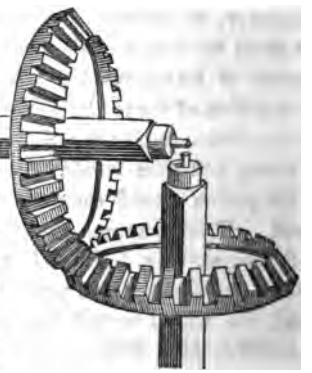


FIG. 99.

parallel to its axis, as in Fig. 100. Such a wheel is termed a *crown* wheel; and it works into an ordinary pinion, whose axis is at right angles to its own. The principle of its action is precisely the same as that of ordinary wheels. By the knowledge of these general principles, we may understand the operation of any kind of wheel-work, and compute its advantage as to power or velocity.

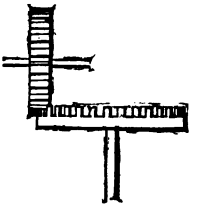


Fig. 100.

The Pulley.

342. The pulley may in itself be considered as a simple modification of the lever; and, according to the mode in which it is employed, it may be considered as a lever of the first or of the second order. A single fixed pulley, over which a rope is passed, having a weight suspended at one end, and the power applied at the other, affords no mechanical advantage; but it serves to change the direction in which the power is applied, which is often a means of applying power with greater facility. Thus, a man can much more easily raise a weight from the bottom to the top of a building, standing at the bottom, and by pulling downwards at a cord passing over a pulley at the top, than if he were at the top and had to pull the weight directly up to him. When a pulley is thus used, it may be regarded as a lever of the first order, with equal arms (Fig. 101, 1): for the spindle at C is the fulcrum; the radius B C,

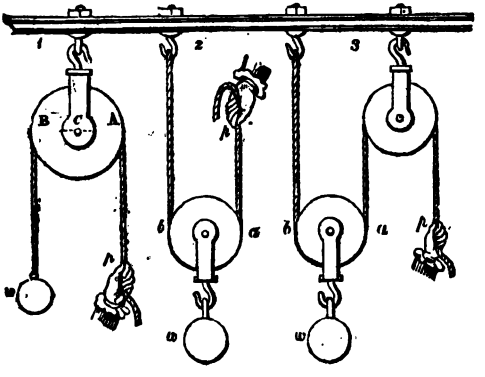


Fig. 101.

from the end of which the weight hangs, acts as one arm, and the radius A C, at the end of which the power is applied, is the other. Now, although the wheel is continually turning, as the line passes over it, the distance from the centre of the points at which the power and the weight are applied, remains constantly the same; and thus no direct mechanical advantage will be either gained or lost by it, since two bodies, P and W, if of equal weight, will remain in equilibrium when freely suspended over it.

343. But the case is different, when the weight is hung to the axle of the pulley, and one end of the string is fixed, as in fig. 2; so that, when the end to which the power is applied is drawn up, the pulley rises, carrying the weight with it. Here the diameter of the wheel *a b* is a lever of the second order; for the fulcrum is no longer the centre, but is at that side of the wheel, which rests against the fixed line. The power is applied at the side where the line is drawn up; and the weight hangs from the centre. Thus the power is applied at a distance from the fulcrum, which is double that at which the weight hangs; and therefore this pulley gives a mechanical advantage of 2 to 1. It is to be remembered that, though, as in the former case, the particular points of the wheel that bear against the two cords (here the fulcrum at *b*, and the power at *a*) are always being changed by its revolution, their distance from the centre will always remain the same, so that the properties of the lever are not affected. By this pulley, then, a power applied in drawing up the line *p a* will raise twice the weight suspended from *c*. But whilst a power acting over the single fixed pulley will raise the weight through exactly the same space that *it* descends, the power which is here applied in drawing up a weight by means of the movable pulley, will only raise that weight through half the space that itself traverses. For it is obvious that, if the cord *p b* be drawn up one foot, each of the two strings will be shortened by six inches; and the pulley will be only raised, therefore, to that amount. Hence we see that here, as in all other instances, what is gained in power is lost in velocity. The

case is not altered by the addition of a fixed pulley, which serves to change the direction of the cord, as in Fig. 101, 3.

344. But although the wheel of the pulley may thus be considered as a lever, it would be wrong to state that the power of this simple machine depends upon it; for, as a matter of fact, the pulley is only a contrivance for getting rid as much as possible of the obstacles which are opposed, by the rigidity or stiffness of cords, and by their friction when made to slide over surfaces, to the advantageous employment of certain modes of arranging these in order to gain power. For if it were possible to conceive of a cord that should be perfectly flexible, and that should move without friction through a fixed ring, we should obtain exactly the same effect as if it passed over a fixed pulley; since the direction would be changed without any gain or loss of mechanical power, the weight pulling on the cord with an equal strain throughout, and being counterbalanced by the power. In like manner, we may suppose a perfectly flexible cord, fixed at one end, to pass through a ring attached to the weight, and to be drawn up by a power applied to its other end; this simple apparatus, if friction did not exist, would answer the same purpose as the movable pulley, in saving power, but in causing a loss of velocity. For it will be easily understood that, in such a case, the weight is equally supported upon the two strings, and consequently its downward pressure is equally divided between them. The hook to which one end of the string is fixed will sustain half the pressure, and the power will only have to raise the other half. Moreover, for the reason just stated, there will be a loss of velocity to the same amount; the shortening of the string to which the power is applied being equally divided between the two.

345. By the combination of several pulleys into a system, it is easy in theory to gain mechanical advantage to any amount; but there are certain limits in practice, resulting from the great loss sustained by friction and by the want of perfect flexibility in the cords, which render it desirable to substitute other methods for gaining power, rather than multiply the number of pulleys beyond a certain amount. The numerous kinds of systems of

pulleys that have been invented at different times may be reduced to two classes,—the first comprehending those with several ropes,—and the second those with a single rope.

346. In a system of the former kind, an immense power may be gained by a small number of pulleys; but its use is not generally convenient. It is a combination of several movable pulleys, in such a manner that the power of each shall serve as the weight to the next; and thus, as in the compound lever, the combined power of the whole is expressed by the powers of the single pulleys multiplied into each other. In the adjoining figure, W represents a weight of 16 lbs. hung from the spindle of the pulley A; this weight, therefore, will draw down the two strings on which that pulley rests, with a strain of 8 lbs. on each. One of those strings is fixed at its end; whilst the other is attached to the spindle of a second pulley B, which it will tend to pull down with a force of 8 lbs. In like manner it will bear with a force of only 4 lbs. on each of the strings that support B; and one of these, being attached to the pulley C, will communicate a strain of 2 pounds to each of the strings on which *it* rests. Lastly, one of these cords being suspended to the pulley D, will pull it down with a force that will be equally distributed upon the two cords that pass under it, and will consequently press upon each with a force of only 1 lb. If one of these cords be carried over the fixed pulley E (which will simply change its direction), a power of 1 lb. hung at P will balance the weight of 16 lbs. at W. But it will be readily understood that the loss of velocity is exactly equal to the gain of power; since, for every 16 feet that P moves downwards, the pulley D will be drawn up only 8 feet, the pulley C only 4 feet, the pulley B only 2 feet, and the pulley A only 1 foot. It is further evident that if we

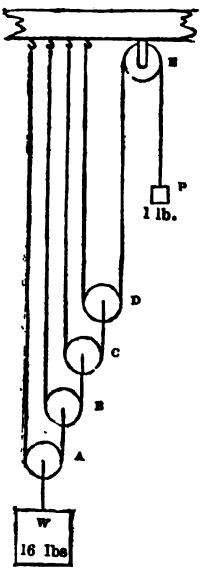


Fig. 102.

were to extend the system by adding another pulley below A, the power of the system would be doubled, and would be increased in the same proportion for every additional pulley; thus for seven movable pulleys thus connected, it would be ascertained by multiplying 2 by 2 to 7 times, the product of which would be 128. Of course the loss of velocity would be in exactly the same proportion.

347. Among the inconveniences to which such a system is liable, there is one which entirely prevents it from being advantageously employed in practice. If all the pulleys were at first at their lowest possible point, they will be separated from each other, the moment that the power begins to act; for, as just stated, the pulley D will move upwards half as fast as the power; the pulley C a quarter as fast; the pulley B only one-eighth as fast; and the pulley A, from which the weight is suspended, only one-sixteenth as fast. Hence the pulley D will have been drawn up to the top of the apparatus before the weight has been lifted far from the bottom.

348. It is on this account chiefly, that the systems commonly used are of the second of the two kinds just alluded to,—that having a continuous rope. In this arrangement we are not able to procure nearly the same amount of power from the same number of pulleys; and if the number of pulleys be greatly multiplied, the friction is enormously increased. In such a system, two, three, or more pulleys are fixed in a frame termed a block; and two of these blocks are employed, the upper one being fixed, and the lower one, to which the weight is attached, being movable. (Fig. 103.) The cord commencing from a fixed point in the upper block, passes beneath one of the pulleys in the lower block; it is then carried over a pulley in the upper block, and beneath another pulley in the lower block; again over another pulley in the upper block, and again beneath a third pulley in the under block; and if it be wished that the power should be applied by pulling downwards rather than upwards, the cord must again pass over a pulley in the upper block. It is evident, therefore, that by shortening the cord, the lower block will be drawn up so as to approach the upper one; and that the

length of the cord is the only limit to the space through which it can be made to move. The power of such a combination is to be estimated in a mode very different from that employed in the former instance. In the first place, it is obvious that the pulleys of the upper block serve only to change the direction of the cord, and give no mechanical advantage. Again, it is easy to understand that the strain of the weight is equally distributed on all the strings which pass under the lower block; and as there are six of these strings, the weight sustained by each will only be one-sixth of the whole. But the power is applied to the last string; and, in overcoming the pressure upon it, raises the whole block by shortening all the strings. The power required to counterpoise the weight will therefore be one-sixth of the latter; and the mechanical advantage of such a system is estimated by doubling the number of pulleys in the lower block, by which we ascertain the number of strings on which the pressure is distributed.

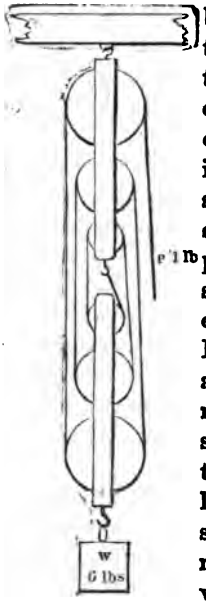


FIG. 103.

349. It is found convenient in practice to arrange the pulleys side by side, instead of one beneath the other; since in this manner we can raise the weight much nearer to the point from which the blocks are suspended. Such blocks are seen in the rigging of a ship. But this mode has great disadvantages. The rope cannot always pass from each pulley to one directly above or below it; and consequently it must then pull in a slanting direction, which will greatly increase the friction, and wear the spindles of the pulleys. Hence it follows, that when a block contains a large number of pulleys, the losses occasioned by friction and by the want of flexibility in the cord are altogether very considerable, and require a large additional power to overcome them; so that the real advantage of such a combination

is much less than we might suppose it to be from our calculations.

350. A very ingenious plan of diminishing the friction was devised by Mr. White, who proposed to cause all the pulleys to turn on the same axis or spindle. This could not be accomplished, however, if all the pulleys were of the same size, or if the cord passed over a broad drum instead of several pulleys; for a little consideration will show that the amount of movement of the part of the cord that passes under each of the lower pulleys is different. The string to which the power is attached, in such a system as that shown in Fig. 103, passes through 6 feet, whilst the string that first passes from the upper block to the lower is shortened by only 1 foot; and the proportional rates of movement of the three pulleys in the lower block will be 2, 4, and 6. Now, in White's system, the size of each pulley is adapted to its rate of movement, in such a manner that, when the whole system is turned round, the space passed through by any point on the circumference of each pulley shall be proportional to the length of cord that has to pass over it. Let the two blocks A and B be supposed to be drawn towards each other to the amount of 1 foot; then the string C C 1 will be shortened by the same amount, and this length of string will pass over the pulley C 1, and also over the pulley C 2. But there will further pass over the pulley C 2 the length of cord by which the string C 1 C 2 is shortened, which is also a foot; so that there will pass over C 2 twice the length of string that passes over C 1. In like manner, there will pass over C 3 the additional foot of cord by which the string C 2 C 3 is shortened, making in all 3 feet; and over C 4 will pass 4 feet. The same applies to any number of strings; the lengths which will pass over the pulleys

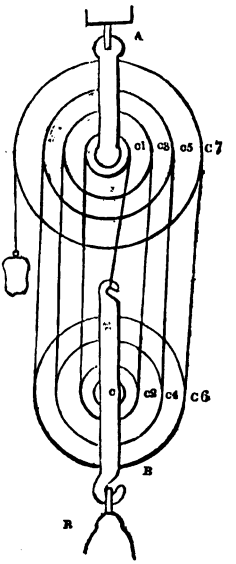


FIG. 104.

in the upper block being as 1, 3, 5, 7, &c.; and those that will pass over those of the lower block being as 2, 4, 6, 8, &c. If the sizes of these pulleys be made to bear these proportions to each other, one turn of each will give off the precise quantity of cord required to cause the blocks to approach each other; and by making those of each block form a solid piece turning on a single spindle, an immense saving of friction is obtained. These pulleys are not, however, in common use.

351. An extremely simple and ingenious application to the pulley, of the principle which has been mentioned under the head of the Chinese Wheel and Axle (§. 325) has been devised by Mr. Moore, of Bristol. Its beauty consists in permitting an increase of power to any extent, by the employment of only two pulleys. Its construction will be readily understood from the accompanying diagram. A is a fixed pulley with two grooves, of which one is a little larger than the other. An endless cord passes over the larger groove from F to E; then beneath a movable pulley B; after which it returns over the smaller groove from D to G, and hangs down, so as to become continuous with the first line. Now if a power be applied at P, so as to draw down the line 1, the line 2 will be raised to the same amount, and the circumference of the pulley A will move through the same space. Supposing that the line 3 were fixed at its extremity, the pulley B would be raised by half the amount to which the line 2 is shortened (§. 343). But the revolution of the pulley A, whilst it draws up the line 2 over one groove, lets down the line 3 from the other. If the two grooves were of the same size, therefore, the pulley B would not be raised; since the line 3 would descend as much as 2 ascends. But, in consequence of the different size of the grooves, the line 3 does not descend as fast as 2 ascends; and the rise of the pulley B will therefore be equal to half the difference in the amount.

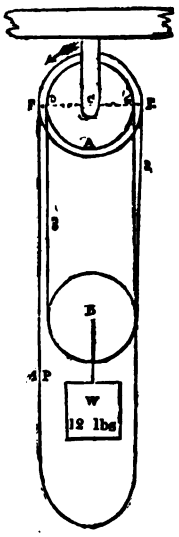


Fig. 105

But if the larger groove have a circumference of 18 inches, and the smaller of 15 inches, the line 2 will be drawn up, by one revolution of the pulley A, through 18 inches, whilst the line B will descend through 15. The movable pulley B with the weight attached to it will consequently be made to ascend through $1\frac{1}{2}$ inch, whilst P descends 18 inches; and the power gained will be 12.

352. The reason of this gain of power is at once seen, by considering that the action of the weight upon the string 3 tends to turn round the pulley in the direction of the arrow, nearly as much as the action of the string 2 tends to turn it in a contrary direction; and that a small force applied to P will, therefore, overcome the difference. If, as in the present instance, the distance from C to D be 18, and from C to E 15, a weight bearing on the string 3 would cause the pulley A to move in the direction of the arrow with a power of 15; whilst the same pressure on the line 2 would make it turn in the contrary direction with a power of 18. The difference between the two strains, therefore, will be the real amount of resistance to be overcome by a power applied to the circumference of the larger groove of the pulley A; and this will be 1-6th of the strain upon either of the strings. But this strain is only half the total weight suspended to B; and the power of the combination will hence be 12. It may be increased to any amount, by diminishing the difference between the two grooves. Thus, if the larger one have a diameter of 100 parts, and the smaller one of 99, the resistance to the motion of the upper pulley will only be 100th part of the weight bearing on each string, or 200th of the whole weight suspended to B. As the cord has no fixed extremity, and as the action of the pulleys would be altogether destroyed if it had the power of *sliding* over them, it is necessary to take some means of preventing this. The simplest is the employment of a chain instead of a cord, the links of which are laid hold of by pins projecting from the surface of the wheels. The author may express his surprise that this ingenious invention has not come into more general use; since it enables any amount of power to be obtained, without any corresponding enlargement of the apparatus, or increase of friction.

CHAPTER XI.

OF THE MECHANICAL POWERS (CONTINUED).—THE INCLINED PLANE, WEDGE, AND SCREW.

Inclined Plane.

353. THE Inclined Plane is the most simple of all machines; being nothing else than a hard inflexible surface, which may have any degree of inclination to the horizontal plane,—from a very gentle degree of ascent, to one so steep as to be almost perpendicular. When a weight is placed on such a plane, a twofold effect is produced; a part of the pressure is resisted by the plane; whilst the remainder tends to cause the weight to roll or slide down the incline, and to overcome any obstacle that may resist it. Let A C be a plane surface, inclined to the horizontal A B at the angle B A C; then the perpendicular line B C drawn at the highest end of the plane is termed its elevation. Now let W be any weight, kept at rest, (as we will suppose in the first instance) by a force W *c* acting parallel to A C. Now this weight presses upon the inclined plane in the perpendicular line W *a*; and this force we may resolve, by the construction of the parallelogram W *b a d*,

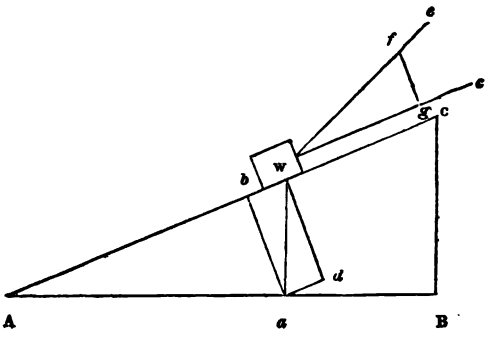


FIG. 106.

into the two forces W *b* and *b a*, the first acting in the direction of the plane surface, the second perpendicularly to it. This last is completely resisted by the plane; whilst the former can only be resisted by some force acting in the direction W C. But suppose this force to be acting in some other direction, as W *e*; to ascertain its action upon the weight, we must resolve it as before, into two forces, of which one, W *g*, shall act in the direction of the plane, and the other *g f* in a line perpendicular to it. The former, therefore, will act against the force W *b* which impels the body down the plane; and, if superior to it, will cause the body to ascend the plane; whilst the latter, acting upwards perpendicularly *from* the surface of the plane, will partly counterbalance the downward pressure W *d* or *b a*. It is evident that, the steeper the plane, the larger is the proportion which W *b* will bear to *b a*; that is, the greater will be the tendency of the body to roll down the plane, and the less will its weight be supported by the surface.

354. The mode in which the power of the inclined plane is estimated, may be deduced from a very simple mathematical process; by which the whole triangle *a b* W can be shown to have its three angles respectively equal to those of the original triangle A B C, so that the sides are proportional to each other. (Euclid, Book VI., Prop. 8.) Hence W *a* is to W *b* as A C is to B C; and W *a* is to *b a* as A C is to A B. Now W *a* represents the whole pressure of the body W in the downward direction; and W *b* that portion of its pressure which causes it to move down the plane, which must be resisted in order to keep the body at rest, and which must be overcome to cause it to ascend. Hence, under all circumstances, the whole weight of the body is to its *pull* down the incline, as the whole length of the incline A C, is to its height B C. And, again, the whole weight of the body is to its pressure perpendicularly to the surface of the incline, as the whole length of the incline A C is to the length of its base A B.

355. This enables us to estimate very readily the amount of force, which, putting aside friction, &c., would serve to drag a load up an incline. For we have only to ascertain how many

times the height of the incline is contained in its length, and we shall have the proportion of the power, acting in the direction of the incline, to the weight to be dragged up. Thus, supposing that the three inclines represented in the adjoining figure have the proportion of 1½, 2, and 3, to the perpendicular height;

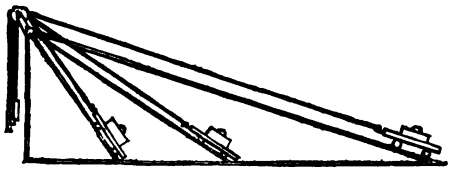


Fig. 107.

then a power of 1 lb. acting over the pulley at the top, would balance 1½ lb. on the steepest, 2 lbs. on the middle one, and 3 lbs. on that least inclined; or a weight of 12 lbs. on the first would require 8 lbs. acting over the pulley to balance it, whilst it would be balanced by 6 lbs. on the second, and by 4 lbs. on the third. In the construction of roads and railways, it is necessary to have constant regard to this principle. The proportion of the height of the inclined plane to its length is there termed its *gradient;* thus, if it be said that the gradient is 1 in 100, it is understood that the road rises 1 foot in height for every 100 feet of its length. On such a road, therefore, the additional load which the engine or horse will have to draw, will be the 100th part of the actual weight that is being moved. Thus, if a railway-train weigh 100 tons, and it can be moved along a dead level with a force that would raise 1 ton through a perpendicular, that force must be *doubled*, to move it up an inclined plane whose gradient is 1 in 100, and *tripled* to cause it to ascend one whose gradient is 1 in 50.

356. In the inclined plane, as in all other simple and complex machines, what is gained in power is lost in velocity. This is easily understood. For, in Fig. 107, the power that draws the weight up the least steep of the inclines, causes it to descend, by its own descent through the whole height of the incline, only one-third of the length of the plane; and must therefore descend through three times the perpendicular height, in order to raise the weight to that height. In the same manner, the second plane being twice as long as the perpendicular, the power which

descends in the direction of the latter must move through twice its length, in order to elevate the weight to the whole height of the plane; whilst the power, which in the first plane only exceeded one-third of the weight, must here be more than one-half. Hence the inclined plane is a means by which we can raise a certain weight to a given height, by a power which may be as small as we please, but which must act for a proportionably longer time in order to produce the required effect.

357. The mode of exhibiting the action of the inclined plane, which is represented in the adjoining figure, is of interest in reference to the operation of the screw; which power will be shown to have much the same relation to the inclined plane, as the wheel-and-axle has to the lever. The weight is here fixed in such a manner, as to be capable of being moved only in a vertical direction; and instead of being drawn up the inclined plane, the latter is drawn under it by a cord pulling in the direction of its base. When the force is thus applied, its proportion to the weight must be that which is borne by the height of the plane, not to its length, but to its base.

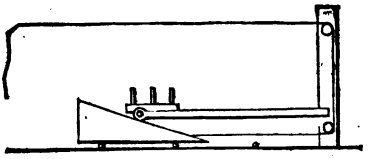


FIG. 108.

358. It necessarily follows, from the principles which have been explained, that any two inclined planes will have to each other a proportional advantage, expressed by the length of each required to raise a weight to the same height. Thus, let A B and B C be two planes, having the same elevation BD; but the length of the plane A B being three times B D, whilst B C is only twice B D. Then, if two weights W and w were connected by a cord that passed freely over a pulley at B, those two weights would be in equilibrium when W should be to w in the proportion of

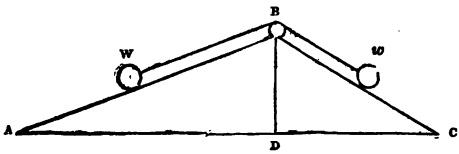


FIG. 109.

3 to 2. For a weight of 3lbs. at W would be balanced by a weight of 1 lb. freely suspended in the perpendicular B D; and this weight of 1 lb. would balance a weight of 2 lbs. on the plane B C. Hence these two weights, W and *w*, being in equilibrium with the same weight, will balance each other.

359. A very simple and elegant mode of expressing these properties of the inclined plane, is to suppose a heavy chain, of equal size throughout, to be supported on an inclined plane, without friction, and to pass over a pulley at its top. Then a portion of chain lying along the whole length of either plane (Fig. 109) will be exactly counterpoised, under all circumstances, by the portion hanging down in the direction of the perpendicular B D. For the weights of two such portions will be to each other as their lengths; and it has been shown that the weights that will balance each other, the one pressing on the plane, the other hanging in the perpendicular, are to each other as the length of the plane to its height. And, again, if similar portions of chain were to lie, without friction, on the double inclined plane A B C, the portions A B and B C would balance each other, being both equivalent to the portion B D hanging in the perpendicular direction.

360. The power of the inclined plane is more affected by friction, than is that of the lever, wheel-and-axle, or pulley; for whilst the friction in those instruments is merely that of the points on which they move, the friction in the inclined plane is that of the weight itself over the whole surface. The amount of this friction will of course depend upon the nature of the surfaces in contact. Thus, if we were to lay a book with a rough-grained cover sideways upon a board covered with cloth or unplaned, we might raise this to a considerable inclination before the book would begin to slide down; thus showing that the resistance occasioned by friction, is to the actual weight of the body, as the height of the plane at which the body begins to slide down is to its length. If the body, however, be smooth, and the surface of the plane also smooth, the angle at which it will slide down is of course greatly diminished; and it comes to be extremely small when the body is made to roll, instead of

sliding down the incline. The difference produced by friction is well seen in the common locking of a carriage-wheel, in order to retard the motion of the vehicle down a steep hill; for though the inclination be such as to cause the carriage to run rapidly down the hill if its wheels are left free, the fixing of one wheel, or the interposition of a "shoe," produces a degree of friction so great, that the carriage would remain at rest, even on a very steep incline, if not drawn down by the power of the horses. But as the descent of a body down an incline is regulated by precisely the same law of gravitation, as that which causes the ordinary descent of falling bodies, a carriage once put in motion downwards by a force sufficient to overcome the friction, may continue to descend, through the increased force which it acquires in consequence of its increasing speed.

361. We have hitherto considered the action of the inclined plane only on the large scale in which it is introduced into the construction of roads, &c., and in which the base of the plane corresponds with the surface of the ground. But the same principles operate, upon a smaller scale, in a great variety of modes. Thus the common chisel is an inclined plane; in which, when it is used for cutting, the pressure of the substance that is being divided, on the two sides of its edge, constitutes the weight or resistance; whilst the power is the force with which the chisel is made to go through it; and this force operates in the direction of the base of the plane, or the length of the chisel. Some other cutting instruments have the same action; but, in general, they may be rather compared to the wedge, the nature of whose power will be presently explained.

362. The inclined plane has been very effectively applied in the construction of some forms of printing-press; to supersede the screw and the system of levers employed in the Stanhope and Columbian presses (§.302 and 305). In one of these, the power is obtained by means of an inclined plane, which is forced by lever power between a pair of rollers; one of which is fixed, and the other connected with the platten, so that the latter is forced down as the inclined plane is driven between them. By making the inclination less in that part which is forced in last,

greater power may be gained at the latter part of the *pull;* whilst, the inclination being greater in the part of the plane first inserted, the quickness with which the rollers are separated and the platten moved down, is greater at *that* part of the pull. This, as already stated, is an important point to be attended to in the construction of the printing-press (§.301). In this press, the inclined plane works in a case filled with oil; and thus the friction is greatly diminished.

The Wedge.

363. The wedge may be regarded as consisting of two inclined planes applied base to base; and the peculiarity of its effect is rather due to the *mode* in which its power is applied, than to the simple mechanical advantage obtained by its construction. The latter is very easily computed on the principles already stated. For, let A B C be a wedge, formed by the union of the two inclined planes A B and B C along the line of their bases, B D; and let this wedge be driven into a piece of timber in order to split it: its resistance will be acting in a direction perpendicular to the sides of the wedge, at the points of contact, that is, in the lines *b e*, *a f:* these forces may be resolved (§.174) into the forces *b d*, *a c*, acting perpendicularly to the back of the wedge, and the forces *c f* and *d e*, acting in contrary directions along the back. These last will counterbalance each other; and the force, therefore, which must be applied to the wedge in the direction of its length, in order to counterbalance a given resistance at *a* or *b*, will be to that resistance as *a c* is to *c f*, or as *b d* is to *b e*,—which, on the principle already mentioned (§.354), will be as the side C D of the triangle B C D, or half the *back* of the

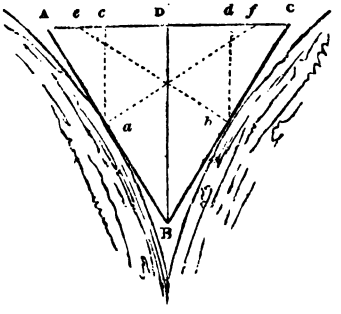


FIG. 110.

wedge, is to the side B C or the *side* of the wedge. But as the resistance is applied at both the points *a* and *b*, the force required to neutralize that resistance will be to the whole resistance itself, as the whole back of the wedge is to one of its sides. Hence the more acute the angle A C B,—or in other words, the sharper the wedge,—the shorter will be its back in proportion to its length, and the greater will be its power.

364. But it is evident that the amount of friction against the sides of a wedge must be enormous; and it is, in fact, such as would resist a very large amount of direct pressure applied upon its back; so that the real action of a wedge is very different from that which might have been inferred from theory. This, indeed, is evident from the fact, that an ordinary wedge, once driven in, will seldom be forced out by any pressure of the surfaces which it is separating, unless both it and the surfaces should be peculiarly smooth. Of course, however, the force which will operate to press it out will depend, not only upon the tendency of the sides to close together, but upon the form of the wedge itself; since it is evident that a thin wedge, which can be made to enter with a force bearing but a small proportion to the resistance to be overcome, will not be pressed out nearly as readily as a thick wedge, which requires a large amount of force to introduce it. Indeed, the pressure upon the sides of a thin wedge will be such as to hold it in with considerable firmness; and it is on this principle that the efficacy of the common *nails*, so extensively used in connecting wood-work, entirely depends. The length of the nail will, of course, have considerable influence on the force with which it will be held by the wood; since the surface over which the pressure is exerted, will be increased as its length is extended. It has been found that a sixpenny nail driven into dry deal, at right angles to the grain of the wood, required a force of 530 lbs. to extract it, when driven to the depth of two inches; but that, when it was driven to the depth of only one inch, a force of 187 lbs. was sufficient to extract it. A similar nail driven to the depth of one inch into dry oak, required for its extraction a force of 557 lbs.; and when driven into dry beech, a force of 667 lbs. It appears that, for

nails of the ordinary tapering form, the force required to extract them, is to that required to force them in, as 5 to 6. The tenacity of nails which have been long imbedded is greatly increased by the roughening of their surfaces through the formation of rust, which almost always takes place to a certain extent, even in very dry situations; and as the nail is itself weakened by the same cause, it very commonly happens that nails, which were quite strong enough to have borne being extracted, soon after they were driven in, break when an attempt is made to remove them at a subsequent period.

365. Another difficulty in estimating the mechanical power of the wedge results from the mode in which force is usually applied to it. This is not by simple *pressure*, but by *impact*,—that is, by a succession of blows upon its back, with a heavy body in motion. This produces results which no steady pressure could accomplish; thus we may with an ordinary hammer drive a wedge into some hard material, which would resist its entrance if it were pressed in with a force of many tons. The reason of this is, that a pressure, however great, necessarily yields at *the moment of impact* to an impinging force, however small (§. 206). The slightest yielding of the sides permits the forward motion of the wedge to that extent, and thus a succession of blows must in time produce an evident result; since the advantage gained by each is followed up by that of the next, provided the wedge have an angle sufficiently small to prevent its being pressed back again.

366. The applications of the wedge, in the various mechanical arts, are extremely numerous. Not only are large strong wedges, driven in by heavy hammers, employed for riving timber or for splitting stone, but all our cutting instruments with two inclined edges, such as knives, axes, sabres, &c., operate on the same principle. Wedges are often used, again, to raise immense weights through a small height; and they have the great advantage over all other mechanical powers, of being applied extremely easily in such cases. Thus, when it is necessary to examine and repair the keel of a ship, she is floated into a dock; and the water being pumped out of this, she is allowed to settle down

upon blocks placed under her keel, being at the same time steadied by shores, or timbers, placed rather aslant against her sides. It is an object to cause her to be entirely suspended upon these shores, in order to remove the blocks from under the keel; and this is accomplished by putting wedges under the lower ends of the former, and causing these to be all driven in at the same time. Though the ship may be thus raised by only an inch or two, she is completely lifted off the blocks, which may be then removed. Another plan is adopted in some dockyards, which is also an application of the principle of the wedge, though in a mode directly contrary to this. The blocks, on which the ship at first rests, are made in three pieces, the middle one of which has a wedge form, with the sides sufficiently inclined towards each other, to prevent its being held between the other two, when a considerable weight is resting upon them. It may be prevented from springing out, however, by a pin which is passed transversely across the three in the position of the dotted line; and the block will then bear the same amount of pressure as if it were solid. Now, after the ship has settled down, and has been propped up on the shores, these blocks are easily removed, one after the other, by simply drawing out the pin from each, and striking the small end of the middle or wedge-piece with a hammer, which will cause it to spring out. This method has the great advantage, of not requiring a large number of men to be brought together, for the purpose of lifting the ship off the blocks; for these being thus removed from under her, she is left supported on the shores, exactly in her previous position.

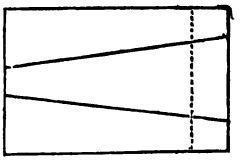


FIG. 111.

367. A successful application of wedges has been made, in restoring to the perpendicular a very tall chimney, which had become considerably inclined in consequence of a defect in the foundation. In the oil-mill, too, the wedge is the principal agent. The seeds from which the oil is to be extracted are placed, after having been reduced to the state of meal by another machine, in horse-hair bags, which are laid between upright planes of

hard wood; between these planes there are large wedges, which are driven down by the successive blows of heavy beams of wood; and the pressure thus produced is so great, as to compress the matter within the bags, into a mass almost as dense as the wood itself. The pressure is removed, by causing one of the beams to fall a few times on the small end of a wedge, placed in the contrary direction. There is a very simple and useful application of the wedge, the operation of which closely resembles the one just described, though its purpose is different. It is often desirable to fix large timbers together with wooden, rather than with iron, fastenings; especially in situations where the latter would be exposed to corrosion through dampness. With this view, a wedge-shaped mortice ($a c, c' a'$, and $b\ c$, $c'\ b'$) in each of the timbers; these mortices are of the same size at their smaller extremities $c\ c'$, which correspond when the timbers are laid together. Two pieces of hard wood are cut out, of such a form as to fit into the sides of this double wedge-shaped hollow, leaving between them an interval, which is somewhat broader at the top than at the bottom; and if a wedge, with sides very slightly inclined, be driven in between them, they will be pressed against the sides of the mortice, with a force so great, that no power can draw the timbers apart. This method is used in bolting together the timbers of the immense wooden bridges which have been erected in America (§.78).

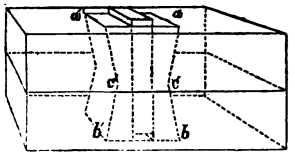


FIG. 112.

The Screw.

368. It is obvious that the action of the inclined plane will be the same, whether it is extended in a straight line or in a curve. Thus a certain inclination of a road will have the same effect upon the draught, whether that road be carried straight up a hill, or winds round and round it. In the same manner, an inclined plane coiled round a central pillar, like a spiral flight of stairs, would have the same mechanical advantage as if it were

extended in a straight line. Such an apparatus is sometimes employed for raising water, or finely-divided solid substances. For this purpose, it is only necessary to make a spiral incline revolve in a hollow cylinder fitted to its outside; and anything which is placed upon it at the bottom will be gradually raised to the top, in exactly the same manner as the weight is raised by a straight inclined plane forced under it (§. 357). This constitutes one form of the Archimedes Screw, sometimes used for raising water;* and it is sometimes employed in flour-mills, as a convenient mode of conveying corn, flour, &c., from one part of the building to another; since the material to be carried may thus be caused to move along a cylinder of any length or any inclination, and is delivered at the top, without any manual assistance, as fast as it is supplied at the bottom.

369. Such is, in fact, the principle of the Screw; which is nothing else than an inclined plane coiled round a cylindrical axis; and which, having thus a circular action instead of a continued movement in a straight line, bears very much the same relation to the inclined plane that the wheel and axle does to the lever. This may be made very apparent by a simple contrivance. Let a triangular piece of paper be cut out, having the

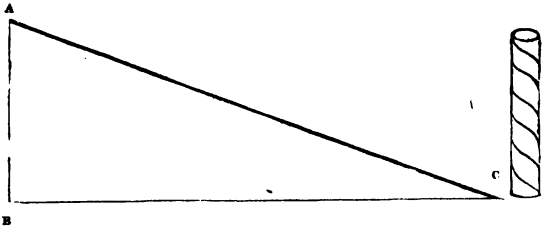


FIG. 113.

form of the inclined plane A B C (Fig. 113), and of any convenient dimensions,—say 18 inches long and 6 inches high; and let a broad black line be drawn along the edge A C. Now, if this piece of paper be coiled round a ruler, beginning at its large end, the black edge A C will be seen to form a spiral path

* See Treatise on HYDRAULICS.

around it, exactly resembling the thread of a screw. This path will commence from the point C, and will occupy on the ruler a length equal to C B. The number of turns which it will make, depends of course upon the size of the ruler; thus, if it be two inches round, the paper will make nine turns upon it,—if three, six turns. The distance between two threads at any point will depend upon the number of turns; for the whole inclination B C being constantly 6 inches, this amount, being divided by the number of turns, will give the space between each thread. If the cylinder were so large, that the whole length A B was required to make one turn round it, then the edge A C would form only one coil of the thread; and, as the point A would then be exactly under C, the distance between the beginning and the end of the coil would be 6 inches. But if the paper take 6 turns round the cylinder, the inclined edge or screw-thread will make 6 coils, and the depth of each will be only 1 inch; or if it take 9 turns, the whole depth of 6 inches will be distributed among 9 coils; and the space between each will only be 6-9ths or 2-3rds of an inch.

370. The power of the screw is usually applied, not by causing the weight or resistance to move along its surface; but by making it revolve within a hollow cylinder, of which the sides are cut with a thread the reverse of its own, so that the two exactly fit together. Now, it is obvious that, if this hollow cylinder be made to turn upon the screw (as in Fig. 115, where it has the form that is termed a *nut*), or if the screw be made to turn in it, the nut will be caused to move over the screw in the direction of its length. Supposing that the power is applied to the circumference of the screw itself, the advantage gained by it (putting friction aside), is estimated in precisely the same manner as in the ordinary inclined plane. For we may consider each turn of the thread as a short inclined plane, of which the length is the circumference of the cylinder, whilst the elevation is the distance between the two consecutive threads, or between the beginning and the end of each thread. Let it be supposed that a weight is attached to the nut, and that it is to be drawn upwards by the revolution of the screw,—

the nut being stationary; then such a force applied to the circumference of the screw, as to give it one turn, will raise the nut by the height of the incline, or the distance between the threads. The force being applied in the direction of the circumference of the screw, which is that of the base of the inclined plane, it will have an advantage equivalent to the excess of the base over the height of the plane (§. 354); that is, to the number of times that the circumference of the screw contains the distance between the threads. Thus, suppose the circumference of a screw to be 6 inches, and the distance between the threads to be $\frac{1}{2}$ an inch; then the former contains the latter 12 times; and a power of 1 lb. applied to the circumference of the screw would balance a weight of 12 lbs. suspended from the nut.

371. But, in practice, the power is never so applied; for a much greater advantage is gained by the application of a lever, either to the screw (as in Fig. 114), or to the nut (as in Fig. 115). The additional power thus gained is calculated on precisely the same principle with that of the wheel and axle; for the centre of the screw being the fulcrum, the distance from that point to the circumference of the screw (that is, its radius) constitutes the short arm of the lever; and the whole length of the bar is the long arm. Thus, if we apply to such a screw as that last mentioned a lever of 3 feet in length, we shall gain a lever power of about 36 to 1; for the circumference of the screw being 6 inches, its diameter will be about 2 inches, and its radius 1 inch; and thus the long and short arms of the lever will be to each other as 36 to 1, so that a force of 1 lb. applied at the end of the lever, is equivalent to 36 lbs. acting at the circumference of the screw. And as 1 lb. acting at the circumference of the screw is equivalent to a

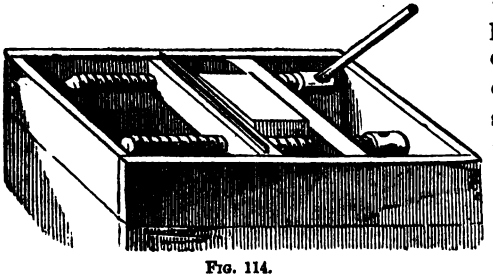


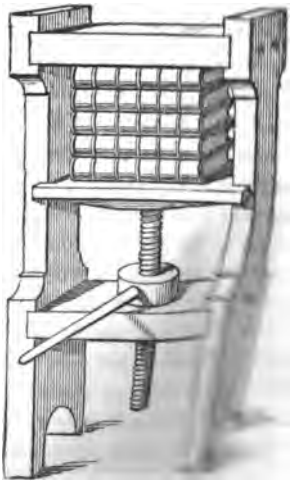
Fig. 114.

weight of 12 lbs. bearing on its thread, so 36 lbs. is equivalent to (36 × 12) 432, the weight supported by 1 lb. applied to the lever.

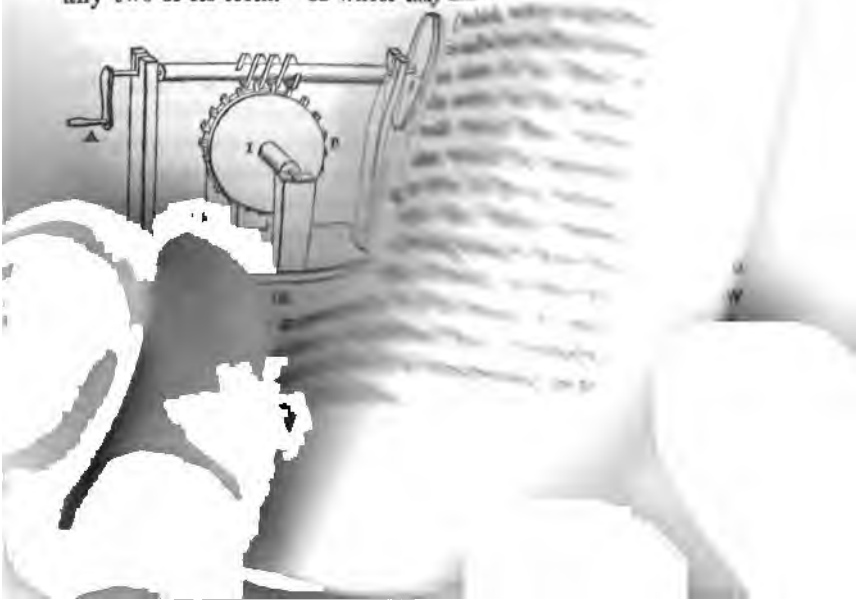
372. The same computation may be made, however, in a shorter way; for we may leave out the middle term, the size of the screw, which is of no consequence when we know the distance between its threads, and the length of the lever which moves it. For if we ascertain the space through which the power moves during one revolution of the screw, and divide this by the distance between the threads, we shall at once obtain the proportion between the resistance and the power. Thus, in the last example, the length of the lever being 3 feet, the circumference of the circle, of which this is the radius, will be something more than 6 times that amount, namely, 18 feet, or 216 inches. This being divided by $\frac{1}{2}$ an inch, the distance between the threads, gives 432, the proportion of the resistance to the power, as before. The simple rule for ascertaining the power of a screw, therefore, is to divide 6 times the length of the lever-arm by the distance between the threads of the screw; but the result obtained by such a calculation is greatly interfered with, by the friction of the surface of the screw-thread against that in which it works. So great is this friction, that a screw has very seldom any tendency to turn backwards; however strong might be its impulse to do so, arising from the weight or resistance acting against it.

373. Still the screw affords a very simple means of gaining a vast mechanical advantage; and it is, therefore, much employed, where a great resistance is to be overcome by a moderate power. This is especially the case in the construction of *presses* of different kinds, in which the screw is applied in a great variety of ways. The accompanying figure represents the common bookbinder's press; in which the screw is fixed [illegible] press-board, whilst the nut, resting on [illegible] round by the lever. In the [illegible] hand, the screw is attached [illegible] to admit of being [illegible] screw [illegible]

such as cotton, into a small compass, for convenience of transportation;—or when it is desired to make an impression by one substance upon another, as in printing or coining. For those purposes in which the most powerful compression is required, however, the screw-press has been very much superseded by the Hydrostatic* or Bramah Press, in which the amount of friction is comparatively small.



374. Another mode of applying the screw, consists in making its threads work against the teeth of a wheel, instead of into a nut; as is seen in Fig. 116. Here the distance between two threads of the screw is equal to the distance between two teeth of the wheel; and thus by one revolution of the screw, the wheel is advanced by an interval equal to the distance between any two of its teeth. A wheel may thus be



show experimentally the proportion between the power and the weight which it will overcome. The endless screw is turned by the winch A; and at the opposite extremity of its spindle is a pulley B, having a radius equal to that of the winch. The screw works into the wheel D, having 48 teeth, and from the circumference of this hangs a weight G. Now it is obvious that 48 turns of the winch A will be required to give one revolution to the wheel D,—that is, to raise the weight G through a space equal to its circumference. Let it be supposed that the space described by the handle A in *one* revolution is equal to the whole of that through which the weight ascends; —then by one turn, it will raise the weight through 1-48th part of the space; and, if the radius of the pulley B be equal to that of the winch A, a weight of 1 lb. hung at C will balance 48 lbs. at G, and a slight addition will cause it to descend so as to raise G. If the radius of the winch A be greater than that of the wheel D, additional power will be gained in the same proportion; and a very great increase of mechanical advantage will be gained, by suspending the weight G from the axle I, instead of from the wheel. For if the wheel have 6 times the diameter of the axle, then a weight of 48 lbs. at G will be equivalent to 288 lbs. suspended from the axle; but there will, of course, be a loss of velocity in the same proportion. It is by an endless screw that the vanes of a smoke-jack are usually made to act upon the wheel which causes the revolution of the spit: the power applied is very small; but, as it acts with considerable velocity, it gives sufficient motion to the wheel.

375. As the power of an ordinary screw depends upon the proportion between the circumference of the circle described by its power and the distance of its threads, it is obvious, that, by increasing the length of the lever, or diminishing the distance of its threads, we might increase our power to any required extent. But here the same practical difficulties occur, as in the case of the wheel and axle; for a great extension of the arm of the lever is attended with obvious inconveniences; and in proportion as the distance between the threads of the screw is diminished, they must themselves become thinner and weaker. The principle of

the Chinese wheel and axle (§.325) has been applied in a very ingenious manner to the screw. It consists of two screws, of which one works within the other; whilst the outside one turns in a fixed nut or cross-bar, as in the common press; and the inside one, being fixed to the press-board, rises into the outer one as the latter is turned round. If the threads of the two screws were at equal distances, it is obvious that the press-board would not be made to descend by the downward motion of the outer screw; since the inner screw would move up within it to just the same amount. But by making a certain difference between the distance of the threads in the two screws, any amount of movement which we desire may be given to the press-board. Thus, supposing that the outer screw have the distance of an inch between each thread, whilst the inner screw has only 3-4ths of an inch; the outer screw descends an inch by one turn through its fixed nut, but the inner screw rises into it 3-4ths of an inch by the same turn; so that the press-board is only made to descend through the difference,—1-4th of an inch; and the same power is thus gained, as if the turns of the screw had only the distance of 1-4th of an inch from each other.

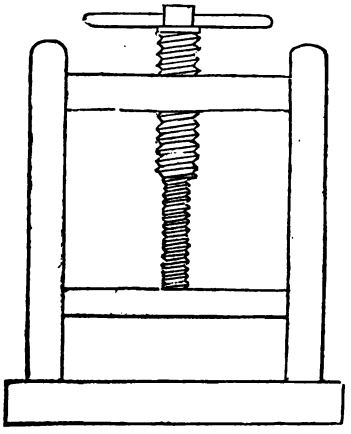


FIG. 117.

376. The more nearly the distance between the threads of the two screws approaches to equality, the greater will be the power gained, since the real motion is only to the amount of the difference between them. Thus, supposing that the outer screw has an inch between the threads, or 24 threads in two feet, whilst the inner screw has 25 threads in the same length, or a distance of 24-25ths of an inch between the threads,—the difference of distance, or the amount of motion produced by one

turn of the double screw, will be no more than 1-25th of an inch; so that, if the lever were 6 feet long, the power gained, according to the principle already stated (§.372) will be no less than ($36 \times 12 \times 25 =$) 10,800. To this kind of increase there are no limits, since we can make the threads as nearly equal as we please, and on a scale proportionate to the amount of strength required. This compound screw is termed Hunter's screw, after its inventor, the celebrated surgeon. As now applied in practice, however, its construction is somewhat different, although the principle is the same. If a single long cylinder have a thread of any size cut upon it for half its length, and the other half have a thread a little different, it is evident that two nuts, or cross-pieces, made to receive the two parts of the screw, will be made to approach each other, or to recede from one another, as the screw is turned, by an amount equal to the difference between the threads. For, supposing the threads equal, both nuts are made to advance along the screw as it is turned, to the same amount; and their distance from each other remains the same. But, if the lower thread be coarser than the upper, its nut will move on faster than that of the other, and will tend to overtake it; whilst, if the screw be turned in the contrary direction, the nuts will be made to separate from each other to the same amount.

377. The screw is not used, however, only as a means of gaining mechanical advantage; for it serves many other important purposes. Wherever a very small amount of motion has to be produced, and its quantity exactly measured, the screw is the instrument employed. Thus, in the instruments termed micrometers, which are applied to microscopes and telescopes, for the purpose of measuring the size of objects seen through them, a screw is used, either to move the object across a fixed thread, or to move the thread across the object; so that, by the amount of motion given, the diameter of the object may be known. This is easily ascertained by employing a very fine-threaded screw; and by marking divisions upon the head by which it is turned. Thus, suppose the screw to have 50 turns to the inch, and the circle round the head to be divided into 200 parts, each of these divisions will be equivalent to 1-10,000th part of an inch; since

one whole turn of the screw will move the object but 1-50th part of an inch; and 1-200th of a turn will move it but the 1-200th of that amount. The same principle is employed in drawing very fine lines upon glass or other hard substances; for, supposing the lines to be drawn by a diamond-point, fixed in a frame which only allows it to be moved backwards and forwards,—if the object be moved by the micrometer screw after each line has been drawn, the next may be drawn at any the most minute distance from it, which may be measured with the greatest nicety. In this manner lines have been drawn on glass at a distance of only 1000th of an inch from each other; and on steel at a distance of only 1-10,000th of an inch. These last were for the purpose of giving to a polished steel surface the power of reflecting light after the manner of mother-of-pearl, according to the principles to be explained in the Treatise on Optics. The principle of Hunter's screw may be very conveniently applied in the construction of a micrometer; since the minutest degree of motion required may be thus obtained, without such a degree of fineness of thread, as would make it difficult to cut the screw accurately.

378. Among the familiar applications of the screw may be noticed the vice, the clamp, and the patent corkscrew. The action of the two former requires no explanation; that of the latter may not be at once understood. In the patent corkscrew, there are two screws with unequal threads, one working within the other, as in Hunter's screw; but the threads are cut in contrary directions, and their action is not connected. The inner screw is attached to the handle above, and carries at its lower end the *helix* or spiral, which is to be made to penetrate the cork. This is forced down by the motion of the handle, until it can turn no farther; and it then lays hold of, and drags with it, the cylinder in which it turns. By means of the coarse screw which is cut on the outside of this, in the contrary direction to that of the interior screw, the continued circular movement of the handle causes it to ascend rapidly, carrying with it the interior screw, and the cork which is held at its lower end.

379. The most numerous of all the instances of the applications of the screw, however, are those in which it is made to

bind together two pieces of timber or other material, and its utility for such purposes entirely depends upon the amount of friction, which is generated by the pressure against its surface and thread. For, supposing the friction to be altogether removed, every screw on which there is any strain would slowly turn out of its socket, and allow the separation of the parts which it is intended to hold together. Screws which are to be inserted into a material that can be compressed, such as wood, are usually made smaller near the point than near the top; and thus they act upon the principle of the wedge also, being driven into a hole that is at first too small for their large extremity, but the resistance being gradually overcome. The increased pressure thus created serves to hold the screw much more firmly in its place, than if it were of a cylindrical form, and were driven into a hole just large enough to receive it. Only this last form, however, can be given to those screws which are to be inserted into metals and other incompressible materials; but here there is much more strength in the interior thread by which the screw is embraced. In instruments which are used to penetrate wood, &c., such as gimlets or augers, the screw is brought to a point at its extremity, so as to enter easily, and then widens rapidly; the combination of the action of the wedge with that of the screw, in such an instrument, is very apparent.

CHAPTER XII.

OF FRICTION.

380. ALLUSIONS have been several times made, in the preceding pages, to the resistance to motion occasioned by *friction*, or the rubbing of the surfaces, of which one is moving over the other. It is intended here to examine the nature of this resistance somewhat more in detail, and to state the laws by which it is governed.

381. It has been shown (§ 32) that great resistance to motion may occur, when the surfaces are extremely smooth, especially if they are of the same kind; in consequence of the homogeneous attraction, which occasions a tendency in the particles of the two surfaces to adhere, when brought into very close contact with each other. This is certainly one cause of friction; and it seems to explain the well-known fact, that there is more friction between two smooth surfaces of the same kind—two of brass, for instance,—or two of steel—than there is between two surfaces of different kinds—steel and brass, for example. It has been found that where steel works against gun-metal, (a kind of very hard brass,) the same weight may be moved with a force of $15\frac{1}{2}$ lbs., which it would require 22 lbs. to move, when steel works against cast-iron. Hence we see the importance of so combining different metals, in the construction of machinery, as to reduce the friction of their moving surfaces to as low an amount as possible. It is for this reason that we almost always see iron spindles working in brass sockets; and even when the frame in which the spindle works may be of iron, the hole is lined or *bushed* (as it is termed) with brass. It

is, in part, for the same reason, that, in clock and watch-work, the wheels are of brass, but the pinions of steel.

382. The resistance produced by friction, however, is generally caused, in part, by the roughness of the rubbing surfaces, which prevents their free motion on one another. It is impossible to make any surface perfectly free from such roughness; and the minute elevations on one, locking (as it were) into the depressions in the other, cause an obstruction to the force that moves them, which varies with the degree of roughness. Hence, even in the simplest machines, there must be a certain amount of loss by friction; and in those of a complex nature, that loss is very greatly increased. Yet this very friction performs purposes in the economy of nature, which far more than compensate us for the loss which it thus occasions. "Were there no friction," it has been well remarked, "all bodies on the surface of the earth would be clashing against one another; rivers would dash with unbounded velocity; and we should see little else besides collision and motion. At present, whenever a body acquires a great velocity, it soon loses it by friction against the surface of the earth; the friction of water against the surfaces it runs over soon reduces the rapid torrent to a gentle stream; the fury of the tempest is lessened by the friction of the air on the face of the earth; and the violence of the ocean is subdued by the attrition of its own waters. Its offices in works of art are equally important. Our garments owe their strength to friction; and the strength of ropes, sails, and various other things, depends on the same cause; for they are made of short fibres pressed together by twisting, and this pressure causes a sufficient degree of friction to prevent the fibres sliding one upon another. Without friction, it would be impossible to make a rope of the fibres of hemp, or a sheet of the fibres of flax; neither could the short fibres of cotton have ever been made into such an infinite variety of forms as they have received from the hands of ingenious workmen. Wool, also, has been converted into a thousand textures for comfort or for luxury; and all these are constituted of fibres united by friction. In fine, if friction retards the motion of

machines, and consumes a large quantity of moving power, we have a full compensation in the numerous and important benefits which it insures to us."

383. As an example of the advantageous application which may be made of the resistance occasioned by friction, we may advert to the various kinds of *drags* used to retard the motion of a coach down a hill, or to bring a rapidly-moving railway-train to rest (§ 152). A very ingenious drag has been contrived for the former purpose, which deserves special notice. It consists of a circle or collar of wood, divided into three pieces, which are jointed together; and this is fixed on the nave of the wheel, in such a manner, that it may either stand off at a short distance from its surface, or may be made to embrace it closely, causing friction sufficient to render the motion of the wheel difficult, or even to resist it altogether. The first position is seen in the left-hand figure; and the second in the right. The wooden collar is made to embrace the nave, by means of a chain attached to the bolt *b c*; and this chain is connected with the breeching of the horse in such a manner, that, when the vehicle is descending a hill, and the horse in resisting its descent bears upon its breeching, the chain is pulled, and the collar tightened, so as to drag the wheel. Both the wheels may be fitted with this apparatus; and the dragging of either or both of them may be prevented, by inserting a small pin into the shaft, which shall prevent the bolt from being drawn forwards when the horse bears on its breeching.

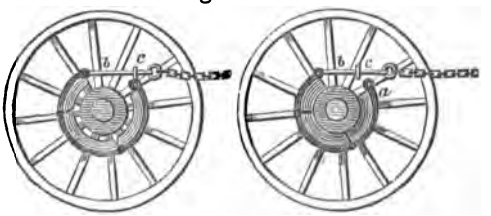


FIG. 118.

384. Numerous experiments on the effects of friction have been made at different times, and by various persons; and, although there is a good deal of discrepancy in their results, the following principles seem to be well established by them.—I. The amount of friction for hard substances is proportional to the pressure employed. Thus, if one piece of cast-iron be laid

upon another, both having flat surfaces, it will be found that the force necessary to move the upper one over the lower will be almost exactly proportional to its weight—being doubled, tripled, or halved, as the weight is doubled, tripled, or halved. This rule holds good for hard metals, with almost perfect exactness, until the pressure amounts to 32 lbs. on the square inch; but there is a considerable deviation from it, in the case of pressures exceeding that limit. There is but little irregularity in the case of hard woods, even up to a greater pressure than this. A piece of Norway oak having a surface of 2 inches square, being pressed upon another with a weight of 1 cwt., exhibited a friction of 14 lbs., 5 oz.; whilst, under a pressure of 4 cwt., its friction became 56 lbs., 7 oz.,—thus differing from four times its previous friction by only 13 oz., although the pressure was then 112 lbs. on the square inch. The case is very different, however, with regard to fibrous substances, such as cloth; for *their* friction *diminishes* as the pressure is increased; probably because their texture is thus rendered closer, so that their character approaches nearer to that of hard substances. The friction of metallic substances for pressures beneath 32 lbs. on the square inch, is, upon the average, about one-sixth of the weight;—that is, a force equal to one-sixth of its pressure is required to move a mass of metal having a smooth surface, over the surface of another. The friction is found to be least, when brass and wrought-iron work together; its proportion to the weight being nearly as 1 to $7\frac{1}{3}$. When steel works upon steel, the proportion of friction to the weight is as 1 to $6\frac{6}{7}$. And when cast-iron works upon cast-iron, it is as 1 to nearly $6\frac{1}{2}$. The friction of the softer metals upon one another is much greater. Thus, when brass works on brass, its friction is to the weight as 1 to $5\frac{3}{4}$; and when tin works on tin, it is as 1 to $3\frac{1}{3}$.

385. II. The second principle is, that the amount of friction remains the same, when the whole pressure is the same, whatever be the extent of the surfaces in contact, provided their character remains the same. This is a very important property, and may be regarded as well established. It is evident that, when a weight rests upon a large surface, the pressure upon any given

portion of that surface is far less, than if the whole weight rests upon a surface of smaller dimensions. Thus, if a weight of 100 lbs. rest upon a surface of 8 square inches, the pressure per square inch is only $12\frac{1}{2}$ lbs.; whereas, if the weight rested upon a surface of only 2 square inches, the pressure would be 50 lbs. per square inch. As the friction is proportional to the pressure, it follows that the friction of any given extent of surface, loaded with a certain weight, will be the same as that of a surface four times as large, but having a pressure only one-fourth as great on each quarter. This is found to be almost precisely the case, in regard to hard surfaces of the same character; but it does not hold good in reference to fibrous substances, such as wood, if the direction of its fibres be changed. Thus the friction of a piece of cast-iron, laid upon another, is very nearly the same, whether it be made to slide upon its flat side or upon its edge. But if a block of oak be made to slide upon an oak table, in such a manner that the fibres of the two are in the same direction, the friction is greater than it is, when the fibres of the block and those of the table run in opposite directions.

386. III. The third principle is also one of great importance. It may be thus expressed. The friction of a body, when in a state of continuous motion, bears the same constant proportion to the pressure, whatever may be the rapidity of the motion. Hence the friction of any machine is the same, whatever may be the velocity of its motion; and its influence may be regarded as a *uniformly retarding* force. This does not hold good, however, in regard to cloth and other fibrous substances, the friction of which is actually diminished by increased velocity. Neither does the principle apply in any case to the force required to change the state of a body from one of rest to one of motion; for the resistance occasioned by friction is usually much greater under such circumstances, than it is when the body is already in motion. The *friction of quiescence* (as it is termed) may, however, be changed into the *friction of motion*, by the slightest *jar* or shock, which shall produce the most imperceptible movement of the surfaces of contact. Of this we frequently see illustrations, in the case of bodies which are held by friction in insecure

positions—as a book lying upon an inclined surface—but which are made to fall by any tremulous motion, such as the shaking of the floor by a person walking across it. The same is noticed in the launching of a ship: the *ways* form an inclined plane, down which the ship slides readily when once set in motion; but, although all the props that previously kept her up are removed, she frequently does not commence her descent, until the people on board have produced a *jar* by jumping on her deck. When both surfaces are of wood, the friction of quiescence is generally about one-half more than the friction of motion; but when both surfaces are of metal, the difference is much less, and indeed can scarcely be said, in some instances, to exist at all.

387. The resistance produced by the friction of metallic surfaces is diminished, as every one knows, by interposing oily or greasy substances between them; and it is a matter of great practical importance to ascertain what substances are best adapted for this purpose. No general principles can be laid down in regard to their use; for the employment of them entirely destroys the constant proportion, which has been stated to exist between the friction and the pressure, in other instances. When oil is interposed between the surfaces, it is found to diminish the friction to a very great degree, so long as the load is not heavy; but when the pressure is much increased, the friction increases in a far higher proportion; so that it is nearly as great, for a very heavy load, as when no oil is employed. Thus, when an axle of yellow brass worked in a collar of cast-iron, without oil, its friction was about 1-4th of the pressure, whatever the amount of the load. When the surfaces were oiled, the proportion of friction was reduced, whilst the pressure was only $\frac{1}{2}$ cwt., to 1-37th of the weight; but when the pressure was increased to 11 cwt., the proportion of friction rose to 1-6th, or little less than when no oil was employed. But when a harder unguent was employed, such as tallow, soft-soap, or anti-attrition composition, the proportion of friction under heavy pressures is much less; so that it seems to be a general principle, that, the heavier the pressure, the harder should be the unguent; whilst, with light pressures, the more fluid unguents may be

employed with advantage. Hence, for large machinery, tallow, lard, &c., may be most advantageously used; whilst for clock and watch-work, in which the least viscidity would check the movement, and the pressure is very light, only the finest oils can be employed. It is remarkable that the interposition of water between surfaces of wood *diminishes* their friction *when in motion*, if the fibres of one piece have a direction perpendicular to those of the other, from about one-third to one-fourth of the pressure; whilst it increases their *friction of quiescence*,—that is, increases the force requisite to produce the first movement,—from rather more than one-half to nearly three-quarters of the pressure.

388. The laws of friction have an important bearing on the construction of carriages; particularly as to the direction in which the power of the horse should be applied. It is found by experiment, that this power will produce a much greater effect, when it is made to pull the carriage somewhat upwards (or from the ground) as well as forwards, than when it pulls simply forwards; and the angle at which the power may be best applied is determined in the following simple manner. Let the carriage be placed upon a road having just such an inclination, that whilst at rest it will remain at rest, and that when put in motion it will continue to descend uniformly; its downward tendency at this inclination, therefore, is exactly equal to the friction, and is counterbalanced by it. Now such an inclination is exactly that at which the power may be most advantageously applied, when the carriage is moving on a horizontal surface; for the power will then have the tendency to lift the carriage from the ground, to just such a degree as may diminish its friction as much as possible, without any of it being uselessly expended. Hence, the rougher the road, and the more friction there may be in the axles of the wheels, the greater should be the inclination at which the draught is applied. In the railroad, in which the friction on the road is trifling in comparison with the pressure, the power is most advantageously applied horizontally, or in the line of the road.

389. Many contrivances may be adopted to reduce friction,

where it opposes an undesirable resistance to motion; but the principle in all of them is the same,—that of substituting a *rolling* for a *rubbing* motion. It is in this that the superiority of the wheeled-cart or carriage over the sledge consists. The friction of the latter upon ice or snow is not too great to be readily overcome; but upon a common road it is enormously increased; whilst the interposition of even very small wheels or rollers at once diminishes it. There are few persons who have not witnessed the assistance given by rollers, laid under a beam of wood or a block of stone, in facilitating its movement along the ground. In America it is not uncommon to move whole houses by a contrivance of this kind. The earth is gradually dug away beneath the foundations, and the walls are supported on horizontal beams as the earth is withdrawn, until the whole house rests upon a timber frame-work. This is then drawn by capstans and pulleys, over *ways* previously prepared; the friction being diminished by placing rollers or balls between the moving surfaces. The largest mass ever thus moved, is probably the block of stone, on which stands the colossal statue of Peter the Great at St. Petersburg. It is estimated to weigh 400,000 lbs.; and its size may be judged of by the figure on its top in the accompanying diagram, representing the mode in which the moving power was applied; this figure is that of a drummer, who was placed there to regulate the movements of the men employed. The mass of stone was supported on each side upon two vast beams, which were grooved to receive the friction balls; on these, one beam was made to move over the other with the least possible friction. By these means, this enormous block was drawn for a distance of several miles, by the combined efforts of a large number of men.

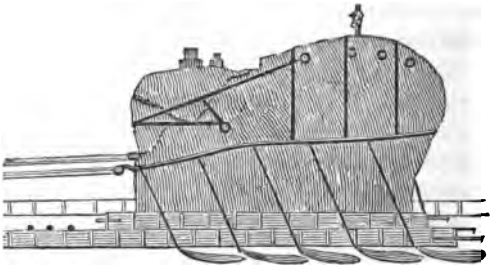


FIG. 119.

390. A similar plan is applied to the axles of wheels, whose friction it is desired to diminish to the smallest possible amount. Instead of making them turn in a socket, or collar, they are made to rest upon two other wheels, placed as in Fig. 120; and when put in motion, the axle *rolls* upon the edges of these, instead of rubbing. This is the contrivance employed to diminish the friction of the pulley in Atwood's machine (§ 247). Or the axle may be entirely surrounded by a set of rollers, which line, as it were, the socket in which it works.

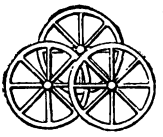


FIG. 120.

In either case the actual amount of friction is the same as that which would have existed, had the axle turned in a socket in the usual manner; since the pressure distributed over the pivots of the wheels or rollers, on which the axle rests, produces altogether exactly the same friction, as it would do when sustained by a single axis, on the principle just now laid down. But the force necessary to overcome that friction is applied much more advantageously; for the friction is now transferred from the axle itself, to the pivots of the friction-wheels. These last are turned round very slowly, by the rolling of the axle upon their edges; but as there is a gain of power proportional to the loss of velocity, a very small force is expended by the axle, in overcoming the friction of the pivots of the friction-wheels. Thus, we will suppose the diameter of the friction-wheel to be 6 inches, and that of its pivot to be $\frac{1}{2}$ an inch; a power of 1 applied to the wheel will overcome a resistance of 12 at the axle; so that the resistance to the motion of an axle resting on two such wheels is only 1-12th of the combined friction of their pivots, or 1-12th of that which the friction of the axle itself would have produced.

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